HAND WRITTEN NOTES:

OF

CIVIL ENGINEERING

SUBJECT:

SURVEYING
Introduction:

Earth: Earth is an oblate spheroid.

Diameter: Poles axes = 12,716.0 km.
Equatorial axes = 12,742.75 km.

Difference = 26.75 km (0.347 less)

Types:

1. Plane Surveying: At earth curvature is not considered.
   (Suitable for small area)

2. Geodetic Survey: At earth curvature is considered.
   (Suitable for large area)

Difference for 13 km length = 1 cm

Difference of total angle

\[ \cos A - \cos (A + B) = 1 \text{ Second } = 0.01^\circ \]
3. **Principle of Surveying:**

   a. Location of a point by measurement from two points of reference.

   ![Triangle diagram](image)

   Chain Survey

   Chain Survey

   Offset Method

   Compass Survey

   ![Offset Method Diagram](image)

   ![Compass Survey Diagram](image)

   Not very common

b. Working from whole to part: first major control points representing whole are fixed and distance are measured with higher accuracy. Then minor details can be taken even with less precision. Error involved in each individual measurement will not be accumulated.

4. **Accuracy and Errors:**

   a. Definitions

   1. **Accuracy:** Degree of perfection obtained in measurement of a quantity is called accuracy. By using precise instrument correct measurement and correct manner of taking measurement.
Precision: Degree of perfection used in the measurement is called precision.

True Error: Difference between true value of a quantity and measurement value of a quantity is called true error.

Discrepancy: Difference between two measured values of same quantity is called discrepancy.

Sources of errors:

1. Instrument: Due to faulty instrument.
2. Personal: Wrong reading/writing of measurement.
3. Nature: Due to change in temperature, humidity, some refraction, local attraction magnetic distortion.

Kind of errors:

1. Mistakes: Human errors due to lack of knowledge, careless.

2. Systematic Errors: (Cumulative error)
   Always have same size and direction under same condition of measurement may be either positive or negative.

3. Accidental Errors: (Compensating errors): There, errors occur's same times in one direction and same times in opposite time and errors compensate each other.

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A Theory of Probability:

- Accidental errors followed a definite rule method law of probability.
  As per this law according to possibility distribution curve of error.
  1. Small errors have higher frequency than large errors.
  2. Positive and negative errors of same magnitude have same frequency.

A True Value: Error value of a quantity.
(AAlmost impossible to measure)
A Most Probable Value: The value of measurement which in chances of being the correct of a quantity than other measurement is called may probable value.
A Principle of least square:
Most probable value of quantity is for which sum of a square of a residual error is min.
Example: when all measurement have equal weight x₁, x₂, x₃, ..., xₙ (equal weight = 1.0)
Residual error:

If most probable value = $x$,

\[ x = x_1 \]
\[ x = x_2 \]

Square \( (x-x_1)^2 \)
\[ (x-x_2)^2 \]

As per principle of least square

\[ y = (x-x_1)^2 + (x-x_2)^2 + \cdots = \min_m \]

\[ \Rightarrow \frac{dy}{dx} = 2(x-x_1) + 2(x-x_2) + \cdots = 0 \]

\[ nx = (x_1 + x_2 + \cdots + x_n) = 0 \]
\[ n \cdot x = \left( \frac{x_1 + x_2 + \cdots + x_n}{n} \right) = 0 \]
\[ x = \left( \frac{x_1 + x_2 + \cdots + x_n}{n} \right) = \text{mean of all} \]

Case II: Unequal weights

<table>
<thead>
<tr>
<th>Squares</th>
<th>measurement</th>
<th>weight</th>
<th>MPV</th>
<th>Residual error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (x-x_1)^2 \times w_1 )</td>
<td>$x_1$</td>
<td>$w_1$</td>
<td>$\uparrow$</td>
<td>$x - x_1$</td>
</tr>
<tr>
<td>( (x-x_2)^2 \times w_2 )</td>
<td>$x_2$</td>
<td>$w_2$</td>
<td>$\downarrow$</td>
<td>$x - x_2$</td>
</tr>
<tr>
<td>( (x-x_n)^2 \times w_n )</td>
<td>$x_n$</td>
<td>$w_n$</td>
<td>$\downarrow$</td>
<td>$\vdots$</td>
</tr>
</tbody>
</table>

Sum of square of residual error:

\[ y = w_1 (x-x_1)^2 + w_2 (x-x_2)^2 + \cdots + \min_m \]

\[ \frac{dy}{dx} = 2w_1(x-x_1) + 2w_2(x-x_2) + \cdots = 0 \]

\[ MPV \Rightarrow x = \frac{\sum w_1 x_1 + w_2 x_2 + \cdots}{\sum w_1} = \frac{\sum w_i x_i}{\sum w_i} \]
The probable error of a single observation:

\[ E_s = \pm 0.6745 \sqrt{\frac{E v^2}{n-1}} \]  \hspace{1cm} (9)

\( E_s \) = difference between any single observation and mean of the series.

The probable error of the mean:

\[ E_m = \pm 0.6745 \sqrt{\frac{E v^2}{n(n-1)}} \]

\[ E_m = \frac{E_s}{\sqrt{n}} \]

\[ \text{Significant figures: } \frac{|\text{max error}|}{\text{most probable error}} \rightarrow 4.6 \]

error 0.05 \( \leq \) max error

0.035 \( \rightarrow \) most probable error

4.6 \( \rightarrow \) most probable error

\[ \text{No of significant figures shows the accuracy of measurement} \]

If there are \( n \) significant figures in a measure, \( (n-1) \) figures are called certain figures

least figure is called uncertain figure.

There are two types of error:

Example: Max error Probable error

4.6 0.05 0.025

5.86 0.005 0.0025
Accumulation of error:

1. For max. error:
   \[ \text{Total error} = \text{Algebraic sum of all errors} = e_1 + e_2 + e_3 \]

2. For probable error:
   \[ \text{Sum} = \text{Root mean square value} \]
   \[ e_T = \sqrt{e_1^2 + e_2^2 + e_3^2} \]

3. Error's in computed result:
   (a) Addition:
      - If \( x \) and \( y \) are two measured values
      \[ S = x + y \]
      \[ ds = dx + dy \]
      - If max. error's are \( \pm \delta x \) and \( \pm \delta y \)
      \[ \text{max. error} \]
      \[ S = (x + \delta x) + (y + \delta y) \]
      \[ \Rightarrow S = (x + y) \pm (\delta x + \delta y) \]
      \[ \delta s = (\delta x + \delta y) \text{ max. error in } S \]

   (b) Probability error:
      - Are \( e_x \) and \( e_y \) sum of probable error's
      \[ e_S = \sqrt{e_x^2 + e_y^2} \]

   Range of \( S \) = \( (S + e_s) \) to \( (S - e_s) \)
(ii) Sub- Function:

\[ S = x - y \]
\[ \frac{ds}{ds} = \frac{dy}{dx} \]

Max. error if \( \pm \Delta x \) and \( \pm \Delta y \) are max. errors.

For \( S \rightarrow S = 8 (-dx) - (-dy) \)
\[ ds = + (\Delta x + \Delta y) \]
\[ = (-\Delta x) - (+\Delta y) \]
\[ = - (\Delta x + \Delta y) \]
\[ \Delta s = \pm (\Delta x + \Delta y) \]

Range of \( S \)
\[ S = (S + \Delta s) \text{ to } (S - \Delta s) \]

Probable error:
\[ \epsilon_s = \pm \sqrt{(\epsilon x)^2 + (\epsilon y)^2} \]

Range:
\[ = (S + \epsilon_s) \text{ to } (S - \epsilon_s) \]

(iii) Multiplication:

\[ S = x \cdot y \]
\[ ds = \frac{dy}{dx} \cdot x \cdot dy + y \cdot dx \]

Max. error
Respective error of \( x \) and \( y \rightarrow \epsilon_x \) and \( \epsilon_y \)

For \( S \rightarrow \)
\[ x \rightarrow y \cdot \Delta y \]
\[ y \rightarrow x \cdot \Delta x \]

\[ S = x \cdot \Delta y + y \cdot \Delta x \] \[ \text{(i)} \]

Range:
\[ (S + \Delta S) \text{ to } (S - \Delta S) \]

Probable error:
Respective error of \( x \) and \( y \rightarrow \epsilon_x \) and \( \epsilon_y \)

For \( S \), error in \( x = \epsilon_x \cdot x \) and \( y = \epsilon_y \).
\begin{align*}
\text{eqn} & = e_s = \sqrt{(y \cdot x)^2 + (x \cdot e_y)^2} \\
\Rightarrow & \quad \frac{e_s}{s} = \frac{1}{xy} \cdot \sqrt{(y \cdot e_x)^2 + (x \cdot e_y)^2} \\
\Rightarrow & \quad \frac{e_s}{s} = \sqrt{\left(\frac{e_x}{x}\right)^2 + \left(\frac{e_y}{y}\right)^2} \\
\Rightarrow & \quad e_s = s \times \sqrt{\left(\frac{e_x}{x}\right)^2 + \left(\frac{e_y}{y}\right)^2} \\
\text{(from eqn) } & \Rightarrow s = \frac{x}{y}. \\
\frac{ds}{d} &= \frac{y}{x^2} \cdot (x \cdot dx - y \cdot dy) \\
\frac{ds}{d} &= \left(\frac{dx}{y}\right)^2 - \left(\frac{x}{y^2}\right) \cdot dy \\
\text{Max error:} & \quad \text{Respective error of } x \text{ or } y \Rightarrow 8x \text{ and } 8y \\
\text{For } s & \Rightarrow \frac{1}{y} \cdot \frac{dx}{dx} \\
\text{In } y & \Rightarrow \frac{x}{y^2} \cdot \frac{dy}{dy} \\
\Rightarrow & \quad \frac{ds}{d} = \left[\frac{dx}{y} + \frac{x}{y^2} \cdot dy\right] \\
\text{Range (Std's) to (S-Std's)} \\
\text{Probable error:} & \quad \text{Respective error of } x \text{ and } y \Rightarrow e_x \text{ and } e_y \\
\text{For } s & \Rightarrow \text{error in } x = \frac{1}{y} \cdot e_x \\
\text{In } y & \Rightarrow \frac{x}{y^2} \cdot e_y \\
\text{eqn} & = e_s = \sqrt{\left(\frac{e_x}{y}\right)^2 + \left(\frac{e_y}{y^3}\right)^2} \\
\Rightarrow & \quad e_s = \frac{y}{x} \cdot \sqrt{\left(\frac{e_x}{y}\right)^2 + \left(\frac{x \cdot e_y}{y^2}\right)^2} \\
\Rightarrow & \quad \frac{e_s}{s} = \frac{y}{x} \cdot \sqrt{\left(\frac{e_x}{y}\right)^2 + \left(\frac{x \cdot e_y}{y^2}\right)^2} \\
\Rightarrow & \quad \left[\frac{e_s}{s}\right] = \sqrt{\left(\frac{e_x}{x}\right)^2 + \left(\frac{e_y}{y}\right)^2} \quad \text{-- } \theta
A quantity $S$ is equal to sum of two measured quantities $x$ and $y$.

\[ S = 5.698 + 3.649 \]

Find out probable error, max error, most probable limits max limits of $S$.

\[ x = 5.698 \quad \text{o.e.e.s} \quad dx \quad 0.0025 \text{lb} \]
\[ y = 3.649 \quad \text{o.e.e.s} \quad dy \quad 0.0025 \text{lb} \]

max error probable error.

**Solution:** → 1) max. error

\[ (\text{for } S) \quad \delta S = \delta x + \delta y \]
\[ \delta S = 0.005 + 0.005 \]
\[ \delta S = 0.0055 \]

Range max. range = $S (S+\delta S)$ to $S - \delta S$

\[ = 9.329 + 0.0055 \quad \text{to} \quad 9.329 - 0.0055 \]
\[ = 9.329 \quad \text{to} \quad 9.323 \]

2) most probable error:

\[ \varepsilon_S = \sqrt{\varepsilon_x^2 + \varepsilon_y^2} \]

\[ \varepsilon_S = \sqrt{(0.0025)^2 + (0.0025)^2} \]
\[ \varepsilon_S = 0.0025 \text{lb} \]

Probable range of $S$

\[ = S + \varepsilon_S \quad \text{to} \quad S - \varepsilon_S \]
\[ = 9.329 \quad \text{to} \quad 9.326 \]
\[ = 9.3305 \quad - 0.0025 \]
\[ = 9.335 \quad \text{lb} \]
Calculated the max. probable error in range of a computed quantity.

\[ S = \frac{9.58}{4.6} \]

**Solution:**

\[ x = 9.58 \quad S_x = 0.005 \]
\[ y = 4.60 \quad S_y = 0.05 \]

\[ S = 2.0826 \]

\[ e_x = y \frac{dx}{dx} - x \frac{dy}{dy} \]
\[ e_y = \frac{dx}{y} - \frac{xe}{y^2} dy \]

**Max. error**

\[ e_s = \frac{S_x}{y} + \frac{S_y}{y^2} x \]
\[ e_s = \frac{0.005}{4.6} + \frac{9.58 \times 0.05}{4.6^2} \]
\[ e_s = 0.0237 \]

Range of S:

\[
\begin{array}{c}
2.0826 \\
+ 0.0237 \\
\frac{2.1063}{2.0589}
\end{array}
\]

**Probable error**

\[ e_s = \sqrt{\left( \frac{S_x}{x} \right)^2 + \left( \frac{S_y}{y} \right)^2} \]
\[ e_s = 0.0836 \sqrt{\left( \frac{0.0025}{9.58} \right)^2 + \left( \frac{0.025}{4.6} \right)^2} \]
\[ e_s = 0.0113 \]

Range of S:

\[
\begin{array}{c}
2.0589 \\
- 0.0113 \\
\frac{2.0476}{2.0713}
\end{array}
\]

Any
**Fundamental Definition:**

(*Linear Measurement*)

**Scale:** Scale is ratio of distance plotted on drawing to distance on the ground.

*Example:* 1 cm = 1 km  
1 cm = 1000 m  
1 cm = 1000x100 cm

\[ R.F = \frac{1}{100000} = \text{Representative Fraction} \]

**Types:**

1. **Plane Scale**
2. **Diagonal Scale**
3. **Vernier Scale**
4. **Chord Scale**

**Plane Scale:** It measures up to two dimensions only.

A scale 1 cm = 4 m to be prepared.

\[ R.F = \frac{1}{400} \]

![Diagram](image)

- In this case, 10 m and make line the two dimensions that can be measured.
Diagonal scale:
it can be measured up to 3 dimensions.
Similar triangle theory is used in diagonal scale.

In three dimensions in above case:
1. 10 m
2. 1 cm (0.1 m)
3. 1 mm (0.001 m)

Vernier Scale:
(a) Direct vernier scale: In this case also up to 3 dimensions can be read.

In case of direct vernier scale:
1. Vernier scale moves in some direction as of main scale.
2. (17-1) parts of main scale is equal to 11 divisions (parts) of vernier scale.
\[ n \cdot U = (n-1)S \]

- \( S = \text{mean scale} \)
- \( U = \text{vernier scale} \)

\[ V = \frac{(n-1)}{n} \cdot S \]

Least count \( \rightarrow \) minimum value that can be used using a scale is called least count.

- \( \text{L.C.} = S - V \)
- \( \text{L.C.} = S - \frac{n-1}{n} s \)
- \( \text{L.C.} = \frac{ns - ns + s}{n} \)
- \( \text{L.C.} = \frac{s}{n} \) - least count of the vernier scale.

(b) Retrograde Scale: \( \rightarrow \) In case of retrograde scale

1. Vernier scale moves in opposite direction as the main scale.

2. \((n+1)\) parts of main scale is equal to dimension (parts) of vernier scale.

\[ n \cdot U = (n+1)3 \]

\[ V = \frac{(n+1)}{n} S \]

Least count \( \rightarrow \) minimum value that can be read on a scale is called least count.
\[ L.C. = V - S \]
\[ L.C. = \frac{(n+1) \times S - S}{n} \]
\[ L.C. = \frac{ns + S - ns}{n} \]
\[ L.C. = \frac{S}{n} \]

A **Shrink Scale**: →
(And Shrinkage factor)

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{scale.jpg}
\caption{Scale 1:500}
\end{figure}

This June 13 represent -
20xS = 100 m
16xS = 80 m

**Original Scale**
1:500
10 cm = 5000 cm

**New Present Scale**
9.5 cm = 5000 cm
1 cm = \( \frac{5000}{9.5} \)
1 cm = 526.316 cm  \[ 1:526.316 \]
Shrinkage Factor: It is the ratio of Shrunken length to original length of the line.

\[ SF = \frac{\text{Shrunken length}}{\text{Original length}} \]  \( \text{(19)} \)

Shrink Scale = Shrinkage Factor × Original Scale.

Example: For given example.

\[ SF = \frac{9.5}{10} \]

\[ SF = 0.95 \]

\[ \text{Shrunken scale} = \text{Shrinkage factor} \times \text{Original Scale} \]

\[ = 0.95 \times \frac{1}{500} \]

\[ = \frac{0.95}{500} \]

\[ = 1.9 \times 10^{-3} \approx 1.9 \times 10^{-4} \]

Problem: Area of a plan in a drawing plotted to a scale 1 cm = 50 m is measured 250.39 cm by planimeter. It was observed that the drawing has shrunk and a line originally 10 cm drawn on the drawing measured only 9.72 cm. Find out the shrunken scale and original area of the plan.

Solution: Shrunken length = 9.2 cm

Original length = 10 cm

Shrinkage Factor = \( \frac{\text{Shrunken length}}{\text{Original length}} \)

\[ = \frac{9.2}{10} \]

\[ = 0.92 \]

Original Scale = 1 cm = 50 m

1 cm = 5000 cm
Shrunk scale: \[ 8.5 \times 0.5 = 0.9 \times \frac{1}{5000} \]

Scale: \[ 1 \text{ cm} = 50.35 \text{ in} \]

Original area: \[ 250 \text{ cm}^2 \times (50.35)^2 \]

\[ = 7344.906 \text{ m}^2 \]

\[ \text{Ans:} \]

Error due to wrong length of chain / paper: \[ L = \text{length (designated) length of paper/chain} \]
\[ L' = \text{wrong length of paper/chain [actual length]} \]
\[ L'' = \text{measured [written] length of a line} \]

\[ L = \text{true or line measured} \]

\[ \text{True x True = Wrong x Wrong} \]
\[ L \times L = L' \times L'' \]

\[ \text{True length of line} L = \left( \frac{L}{L'} \right) \times L'' \]

\[ \text{Ex: If a 30 m chain is actually 30.20 m long, what would be the actual length of a line which is measured 3052 m using above rope.} \]

\[ \text{Solution:} \]
\[ L = 30 \text{ m} \]
\[ L' = 30.20 \text{ m} \]
\[ L'' = 3052 \text{ m} \]
\[ L = \frac{30.20 \times 3052}{30} = 307.35 \text{ m} \]
Formula

1. \( L = \left( \frac{l_1}{L} \right) \times L' \) for length

2. For area
   \( A = \left( \frac{l_1}{L} \right)^2 \times A' \)

3. For volume
   \( V = \left( \frac{l_1}{L} \right)^3 \times V' \)

Tape Correction:

- Correction due to standardization: [Due to wrong length of chain or tape]

Total Correction required

\[ C_a = \frac{\text{Total Length of Line (L')}}{L} \times C \]

\[ C_a = \frac{L_1}{L} \times C \]

\( C_a = \text{Total Correction} \)

\( c = \text{Correction required per chain length} \)

\( L = \text{Designated length of tape/chain} \)

<table>
<thead>
<tr>
<th>Measured Length</th>
<th>Written Length</th>
<th>Error</th>
<th>Correction</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>More</td>
<td>Less</td>
<td>(−)ve</td>
<td>(+)ve</td>
<td>Tape 13 - Long</td>
</tr>
<tr>
<td>30.50</td>
<td>30.0</td>
<td>− 0.50</td>
<td>+ 0.50</td>
<td></td>
</tr>
<tr>
<td>Less</td>
<td>More</td>
<td>(+)ve</td>
<td>(−)ve</td>
<td>Tape 13 - Short</td>
</tr>
<tr>
<td>20.00</td>
<td>20.00</td>
<td>+ 0.50</td>
<td>− 0.50</td>
<td></td>
</tr>
</tbody>
</table>
(b) Correction due to slope ($C_s$)

In case of chaining along a sloping area we need to measured the horizontal distance.

Slope correction ($C_s$) = $L - L$

$C_s = L - L \left(1 - \frac{h^2}{L^2} \right)$

$C_s = L - L \left[1 - \frac{h^2}{L^2} \right]^2$

$C_s = L - L + \frac{h^2}{2L} + \ldots$

$C_s = \frac{h^2}{2L}$

$C_s$ is always positive (due to correction)

(c) Correction due to alignment $\Rightarrow$ (Cal.) $\Rightarrow$

If $h$ is error in alignment.

$L = \text{length of line measured}$

Correction due to alignment:

Cal = $\frac{h^2}{2L}$

(For error always $+$ive) [Correction always $-$ive]
(d) Correction due to Temperature

\[ C_T = L \left( T_m - T_o \right) K a^2 \]

Here:
- \( L \) = length of line measured
- \( T_m \) = temperature at the time of measurement
- \( T_o \) = temperature at the time of standardization

Case (1): IF \( T_m > T_o \)

- Error = (-)ve
- Correction = (+)ve

Case (2): IF \( T_m < T_o \)

- Error = (+)ve
- Correction = (-)ve

(e) Full Correction:

\[ C_p = \frac{\left( P_m - P_o \right) \cdot L}{AE} \]

\( \times \) \( A = c/3 \) area of tape/chain
\( \varepsilon = \) Young's modulus of elasticity of tape/chain
Case 1: $P_m > P_o$

Length increases

$$\text{Error} = \theta \cdot \text{ue}$$

$$\text{Correction} = (+) \cdot \text{ue}$$

Case 2: $P_m < P_o$

Length & $\theta \cdot \text{ue}$ less

$$\text{Error} = (+) \cdot \text{ue}$$

$$\text{Correction} = (-) \cdot \text{ue}$$

Sag correction:

$$\text{Sag correction } C_{\text{sag}} = \frac{w^2 L}{24 P_m^2} = \frac{(\text{wt})^2 L}{24 P_m^2}$$

- $w =$ Total weight of chain
- $L =$ Total length of chain/Pipe

Error, always = (+) ue

Correction, always = (-) ue
Normal Tension: 

If (P_m > P_0) Pull correction is (+) ve. 
Sag correction is (-) ve.

Pull correction and Sag correction neutralize each other.

→ the value of pull (P_m) for which Pull correction is equal to Sag correction is called Normal tension.

\[
\frac{(P_m - P_0) L}{AE} = \frac{w L^2}{2u. P_m^2}
\]

This eq. can be solved for final and error for (P_m).

Claiming on a sloping ground: 

Indirect method: 

Horizontal distance A to C

\[ = J_1 \cos \theta_1 + J_2 \cos \theta_2 \]

Measuring difference of level: 

Horizontal distance = \[\sqrt{J_1^2 - h_1^2} + \sqrt{J_2^2 - h_2^2}\]
Hypotenusal Connection

\[ CH = BC \]

\[ \frac{Am}{Ac} = \cos \omega \]

\[ CH = AC - AB \]

\[ CH = AD - DE - EO - AB \]

\[ CH = 1 \text{ chain} - 1 \text{ chain} \]

\[ CH = 1 \text{ chain} (\text{see} \omega - 1) \]

\[ CH = 100 \text{ links} (\text{see} \omega - 1) \]

\[ CH = 100 \times \frac{\omega^2}{2} \]

\[ CH = 50.50^2 \text{ links} \]

This connection is applied after each chain, and next chaining is started after connection.
Chain Surveying:

1. Main Line: Lines joining main survey stations.
2. Base Line: The longest line that divides the whole area into two parts.
3. Tie Line: A line drawn to collect the information or details of objects nearby.
4. Check Line: To check the accuracy of survey work.
5. Check Condition Triangle: A triangle having no angle.
6. Off Sets and Field Book:
   a. Right Angle Offsets (Square Offset) (at 90°)
   b. Oblique Offsets: at an angle with the survey line.
Limiting length of offset:

Case 2: when error is in measurement of direction only

\[ L = \frac{L_1 \sin \theta}{S} \]

- \( P \) = location of point on the ground for which offset has been taken.
- \( \theta \) = error in angular measurement.
- \( L \) = length of offset.
- \( P_1 \) = point marked on the drawing.
- \( P_2 \) = point marked on the drawing.

Total error due to wrong angle is \( P_2 \) or \( P_1 \) = \( L \sin \theta \)

\[ P_2 = \frac{L_1 \sin \theta}{S} \]

If 1 cm = 5 m is a scale of a drawing,
length of error on drawing = \( \frac{L \sin \theta}{S} \) cm

If error on the drawing is not more than 0.25 cm
(error shall not be reflected on the drawing)

\[ \frac{L \sin \theta}{S} \text{ cm} = 0.025 \text{ cm} \]

\[ L = \frac{0.025 \times S}{\sin \theta} \]

\[ L = 0.025 \times \frac{\text{accuracy}}{S} \rightarrow \text{limiting length of offset} \]
What is the limiting length of offset for a angular error in offset measurement of 3° and scale of drawing:

1. 1cm = 50m
2. 1cm = 1000m

Solution:
Length of error on ground = 1.8m

Length of error on drawing = \[ \frac{1.8 \times \sin 3°}{50} \times 0.025 \text{cm} \]

\[ t = 0.025 \times 5 \times \sin 3° \]

\[ t = 0.225 \text{ cm} \]

For Scale 1cm = 50m

\[ t = 0.025 \times 50 \times \cot 3° \]

\[ t = 23.80 \text{ m} \]

For Scale 1cm = 1000m

\[ t = 0.025 \times 1000 \times \cot 3° \]

\[ t = 477.7 \text{ m} \]

Note: For a 23.8m offset on ground

Length of error on ground

\[ 1.8 \times \sin 3° \]

\[ = 23.80 \times 0.05 \]

\[ = 1.245 \text{ m} \]

Length of error on drawing

Scale = 1cm = 50m

\[ \frac{1.245}{50} = 0.0249 \text{ cm} \]

R.F.: 1cm = 5000cm
Case 3

IF the error is in angle as well as measurement of length:

\[ p_2 \] is point on the ground for which offset is being taken.
\[ \Delta \] is length of offset with error.
\[ \Delta p_1 \] is error in length measurement.
\[ \Delta \theta \] is error in angular measurement.

\[ p_2 \] is point marked on the drawing.
\[ \Delta p_2 \] is error due to angular measurement.

Assumption: In both cases

\[ \angle p_1 p_2 = 90^\circ \]

So total error (length on ground)

\[ p_{p2} = \sqrt{p_1^2 + \Delta p_2^2} \]

\[ \Rightarrow p_{p2} = \sqrt{x^2 + (\Delta \sin \theta)^2} \]

If scale is 1 cm = 50 m

Length of error on the drawing

\[ p_{p2} = \frac{\sqrt{x^2 + (\Delta \sin \theta)^2}}{5} \approx 0.025 \text{ cm} \]
\[ \sqrt{x^2 + (\Delta \sin \theta)^2} = 0.0258 \]

\[ x^2 + (\Delta \sin \theta)^2 = \left(\frac{s}{40}\right)^2 \]

Q: Length of an offset is 30 m. Error in measurements of offset length is 0.20 m. Find max permissible error in laying the direction of offset. If scale of the drawing is 1 cm = 40 m.

Solution:

\[ a = 30 \text{ m} \]
\[ x = 0.20 \text{ m} \]

\[ 1 \text{ cm} = 40 \text{ m} \] Scale

\[ s = 40 \text{ m} \]

\[ \theta = 9 \]

\[ x^2 + (\Delta \sin \theta)^2 = \left(\frac{s}{40}\right)^2 \]

\[ (0.20)^2 + (20 \sin \theta)^2 = \left(\frac{40}{40}\right)^2 \]

\[ 0.04 + 400 \sin^2 \theta = 1 \]

\[ \sin^2 \theta = 0.001 \]

\[ \sin \theta = 0.01 \]

\[ \theta = 0.18^\circ \]

Ans: \( 2^\circ 11^\prime 29^\prime\prime \)
Optical Square: An instrument used to set perpendiculars, control on a line
- Angle of reflected ray (90°)
- Piece of angle with mirrors (45°)
A compass survey:

System of angle measurement:

1. Most widely used system
   - Circumference = 360° degree
   - 1 degree = 60 min
   - 1 minute = 60 sec

2. Hour system
   - Circumference = 24 hours
   - 1 hour = 60 min
   - 1 min = 60 sec

Note: Circumference is according to the earth rotation.

3. Centesimal system
   - Circumference = 400 grade
   - 1 grade = 100 centigrade
   - 1 centigrade = 100 centi-centigrade

Important terms:

1. Bearing: The direction of a line w.r.t. to a given meridian.
   - It is the angle the gives meridian and the line.

2. True meridian: Line joining true North and true South pole.
   - The earth is called True meridian.
   - True North or South Point are the point about which earth is rotating.
(a) True bearing: Bearing measured from true meridian.
(b) Magnetic meridian: Line joining magnetic North and South poles.
(c) Magnetic bearing: Bearing measured from magnetic meridian.
(d) Magnetic declination: Horizontal angle between true meridian and magnetic meridian at a particular place is called magnetic declination.

(1) Western Declination
\[ M \cdot B = T \cdot B + D \cdot W \]
\[ T \cdot B = M \cdot B - D \cdot W \]

(2) Eastern Declination
\[ M \cdot B = T \cdot B - D \cdot E \]
\[ T \cdot B = M \cdot B + D \cdot E \]

(3) Variation in Declination:

Types:
(1) Diurnal variation (Daily)
(2) Annual variation (Annually)
(3) Secular variation (Due to the moment of moon)
(4) Irregular variation.
6. Angle of Dip: If a magnet is hung freely from its c.g. it attains an equilibrium in the direction of the magnetic flux in that area. The angle made with the horizontal direction of magnet is called the dip angle.

Dip angle at any point on earth is the angle between horizontal and direction of magnetic flux.

- Dip angle at equator = 0°
- Dip angle at poles = 90°

7. System of Bearing Measurement:

1. WCB Method (Whole Circle Bearing Method):

   All angles are measured from North and always in a clockwise direction.
O.S.C. Method: (Fundamental System of bearing)

- Angles are measured either from North or from South.
- Can be measured in clockwise or anti-clockwise direction.
- Angles with direction are shown.
- This bearing system is also known as reduced bearing.

*Fore bearing and Back bearing:*

For point A:

Angle at point A = OA = Fore bearing.
Angle at 2nd point B = \( \theta \) = Back bearing

\[ \theta = 38^\circ \]

For line BA

- True bearing = DB
- Back bearing = OA

**Local Attraction:** Bearings of different line are measured using a magnetic needle (in case of magnetic bearing).

Due to the presence of some iron objects near instrument, magnetic needle may get deflected, resulting in wrong readings.

**Problem:**

<table>
<thead>
<tr>
<th>Line (AB, BC, CD, DE, EA)</th>
<th>True bearing (°)</th>
<th>Back bearing (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>32</td>
<td>212</td>
</tr>
<tr>
<td>BC</td>
<td>77</td>
<td>262, -175</td>
</tr>
<tr>
<td>CD</td>
<td>112</td>
<td>287, -175</td>
</tr>
<tr>
<td>DE</td>
<td>125</td>
<td>85, -175</td>
</tr>
<tr>
<td>EA</td>
<td>255</td>
<td>85, -175</td>
</tr>
</tbody>
</table>

**Solution:**

<table>
<thead>
<tr>
<th>Line</th>
<th>Bearing (°)</th>
<th>Correction</th>
<th>Corrected bearing (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>32</td>
<td>0</td>
<td>32°</td>
</tr>
<tr>
<td>BA</td>
<td>212</td>
<td>0</td>
<td>212°</td>
</tr>
<tr>
<td>BC</td>
<td>77</td>
<td>0</td>
<td>77°</td>
</tr>
<tr>
<td>CD</td>
<td>262</td>
<td>-5°</td>
<td>262° - 5° = 257°</td>
</tr>
<tr>
<td>DE</td>
<td>112</td>
<td>-5°</td>
<td>112° - 5° = 107°</td>
</tr>
<tr>
<td>SE</td>
<td>257</td>
<td>0</td>
<td>257°</td>
</tr>
<tr>
<td>EA</td>
<td>125</td>
<td>0</td>
<td>125°</td>
</tr>
<tr>
<td>ED</td>
<td>85</td>
<td>0</td>
<td>85°</td>
</tr>
</tbody>
</table>
Here all differences of $FB/BB$ are 200° except for line $BC$ and $CD$ so station 'C' is affected from local attraction.

<table>
<thead>
<tr>
<th>EA</th>
<th>265°</th>
<th>0</th>
<th>265°</th>
</tr>
</thead>
<tbody>
<tr>
<td>AE</td>
<td>85°</td>
<td>0</td>
<td>85°</td>
</tr>
<tr>
<td>Method</td>
<td>7.8 in</td>
<td>9.8 in</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>N20E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>S20W</td>
<td></td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>N65E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>245</td>
<td>S65W</td>
<td></td>
<td></td>
</tr>
<tr>
<td>95</td>
<td>S95E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>275</td>
<td>N85W</td>
<td></td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>S70E</td>
<td></td>
<td></td>
</tr>
<tr>
<td>290</td>
<td>N70W</td>
<td></td>
<td></td>
</tr>
<tr>
<td>255</td>
<td>S78W</td>
<td></td>
<td></td>
</tr>
<tr>
<td>73</td>
<td>N73E</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Problem: Following bearings were taken using a compass. Find out the correct bearing.

Solution: When bearings C and D are free from linear attraction.

<table>
<thead>
<tr>
<th>AB</th>
<th>75° 51'</th>
<th>250° 20'</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC</td>
<td>115° 30'</td>
<td>296° 35'</td>
<td>x</td>
</tr>
<tr>
<td>CD</td>
<td>165° 35'</td>
<td>345° 35'</td>
<td>190°</td>
</tr>
<tr>
<td>DE</td>
<td>220° 50'</td>
<td>40° 51'</td>
<td>x</td>
</tr>
<tr>
<td>EN</td>
<td>300°</td>
<td>125° 5'</td>
<td>x</td>
</tr>
<tr>
<td>Lines</td>
<td>Bearings</td>
<td>Correction</td>
<td>Corrected bearing</td>
</tr>
<tr>
<td>-------</td>
<td>-----------</td>
<td>------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>AB</td>
<td>75° 5'</td>
<td>+0° 30'</td>
<td>75° 35'</td>
</tr>
<tr>
<td>BA</td>
<td>250° 20'</td>
<td>-1° 15'</td>
<td></td>
</tr>
<tr>
<td>BC</td>
<td>115° 20'</td>
<td>+1° 15'</td>
<td>116° 35'</td>
</tr>
<tr>
<td>CB</td>
<td>296° 35'</td>
<td>0</td>
<td>296° 35'</td>
</tr>
<tr>
<td>CD</td>
<td>165° 35'</td>
<td>0</td>
<td>165° 35'</td>
</tr>
<tr>
<td>DC</td>
<td>305° 35'</td>
<td>0</td>
<td>305° 35'</td>
</tr>
<tr>
<td>DE</td>
<td>224° 50'</td>
<td>0</td>
<td>224° 50'</td>
</tr>
<tr>
<td>ED</td>
<td>00° 5'</td>
<td>+0° 45'</td>
<td>04° 50'</td>
</tr>
<tr>
<td>EA</td>
<td>304° 50'</td>
<td>+0° 45'</td>
<td>305° 35'</td>
</tr>
<tr>
<td>AE</td>
<td>125° 5'</td>
<td>+0° 30'</td>
<td>125° 35'</td>
</tr>
</tbody>
</table>

Problem: There are the bearing of a closed traverse.

\[
\begin{array}{c|c|c}
\text{Lines} & \text{FB} & \text{BB} \\
\hline
\text{AB} & 142° 30' & 322° 30' \\
\text{BC} & 223° 15' & 401° 15' \\
\text{CD} & 287° & 107° 45' \\
\text{DE} & 12° 45' & 193° 15' \\
\text{EA} & 60° & 229° \\
\end{array}
\]

Considered the value as correct, find out the correct bearing of all other lines.

Solution:
<table>
<thead>
<tr>
<th>Lines</th>
<th>Bearing</th>
<th>Corrrection</th>
<th>Correct Bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>142°30'</td>
<td>0</td>
<td>142°30'</td>
</tr>
<tr>
<td>BA</td>
<td>322°30'</td>
<td>0</td>
<td>322°30'</td>
</tr>
<tr>
<td>BC</td>
<td>223°15'</td>
<td>0</td>
<td>223°15'</td>
</tr>
<tr>
<td>CD</td>
<td>49°15'</td>
<td>-1°15'</td>
<td>48°15'</td>
</tr>
<tr>
<td>DE</td>
<td>107°45'</td>
<td>-1°45'</td>
<td>106°</td>
</tr>
<tr>
<td>ED</td>
<td>12°45'</td>
<td>-1°45'</td>
<td>11°</td>
</tr>
<tr>
<td>ED</td>
<td>193°15'</td>
<td>-2°15'</td>
<td>191°15'</td>
</tr>
<tr>
<td>EA</td>
<td>60°</td>
<td>-2°15'</td>
<td>58°15'</td>
</tr>
<tr>
<td>AE</td>
<td>289°</td>
<td>not equal</td>
<td>287°15'</td>
</tr>
</tbody>
</table>

**Method of Internal Angle:**

1. Draw the traverse.
3. Draw a clockwise circular direction.

4. Internal angles:
   \[ A = \text{angle} \]
   \[ B = \text{angle} \]
   \[ C = \text{angle} \]
   \[ D = \text{angle} \]
   \[ E = \text{angle} \]

5. If any angle is 360°, deduct 360°.
6. If any angle is negative, add 360°.

Sum of all internal angles = \((2n-4)\times90°\)

\[ = \frac{(2\times5-4)\times90°}{5} = 54°15' \]

Difference in sum of all internal angle = 1°15'

Correction in each angle = \[ \frac{1°15'}{5} = 0°30' \]

Corrected internal angle:
96°15'
99°15'
101°15'
100°45'
133°
\[ A = AE - AB = 96.15' = \angle A \]
\[ B = BA - BC = 99' = \angle B \]
\[ C = CB - CD = 38.45' = 117' \]
\[ D = DC - DE = 94.45' \]
\[ E = ED - EA = 138' = \angle E \]

Corrected bearing of line: =>

\[ AB = 142.30' \]
\[ + \angle A = 96.15' \]
\[ AE = 238.45' \]
\[ -180' \]
\[ EA = 58.45' \]
\[ + \angle E = 138' \]
\[ ED = 191.45' \]
\[ -180' \]
\[ DE = 11.45' \]
\[ + \angle D = 94.45' \]
\[ BC = 106.30' \]
\[ +190' \]
\[ CD = 296.30' \]
\[ + \angle C = 117' \]
\[ CB = 43.30' \]
\[ A = AB - AE = 191'30" - 53' = 138'30" \]
\[ B = BC - BA = 69'30" - 13' = 56'30" \]
\[ C = CD - CB = 32'15" - 346'30" + 360' = 115'05" \]
\[ D = DE - DC = 262'05" - 210'30" = 51'15" \]
\[ E = EA - ED = 230'15" - 86'05" = 144'30" \]
\[ \text{Total error} = 2'30" \]
\[ \text{Error in each angle} = \frac{2'30"}{5} = 0'30" \]

Correction in each angle: 
\[ \leftarrow 0'30" \]

Corrected angle:
138'
56'
115'15"
51'05'
144'

AB has minor error
Corrected bearing \( AB = 191'30" - 0'05" = 190'45" \)
\[ \text{BA} = 13' + 0'05" = \text{the value is range. Min} \]

Corrected bearing \( AB = 191'30" + 0'45" = 192'15" \)
\[ \text{BA} = 13' - 0'05" = 12'15" \]
AB = 192° 15'

\( \angle A = 180° \)

AE = 54° 15'
+190°

EA = 234° 15'

LE = -149°

ED = 85° 15'
+190°

DE = 265° 15'

-\( \angle D = -145° 45' \)

DE = 213° 30'

CD = -33° 30'

LC = -145° 15'
+360° \rightarrow \text{Because the value is negative, add 360°.}

CL = 208° 15'

-180°

BL = 68° 15'

-\( \angle B = -56° \)

BA = 12° 15'

+190°

AB = 193° 15'
Traverse - Latitude and departure of different lines of a traverse.

![Diagram showing a closed traverse with points A to E]

- Latitude is the projection of a line on N-S direction:
  \[ L = D \cos \theta \]

- Departure is the projection of line on E-W direction:
  \[ D = L \sin \theta \]

<table>
<thead>
<tr>
<th>Line</th>
<th>WCA</th>
<th>Latitude</th>
<th>Departure</th>
</tr>
</thead>
<tbody>
<tr>
<td>l_1</td>
<td>0 ≤ θ ≤ 90°</td>
<td>( N \theta_1 E + l_1 \cos \theta_1 )</td>
<td>( l_1 \sin \theta_1 )</td>
</tr>
<tr>
<td>l_2</td>
<td>( S \theta_2 E - l_2 \cos \theta_2 )</td>
<td>( -l_2 \sin \theta_2 )</td>
<td></td>
</tr>
<tr>
<td>l_3</td>
<td>( S \theta_3 W - l_3 \cos \theta_3 )</td>
<td>( -l_3 \sin \theta_3 )</td>
<td></td>
</tr>
<tr>
<td>l_4</td>
<td>( N \theta_4 W + l_4 \cos \theta_4 )</td>
<td>( -l_4 \sin \theta_4 )</td>
<td></td>
</tr>
</tbody>
</table>
Independent co-ordinate:

IF co-ordinate of different points are measured w.r.t. to a fixed origin is called independent co-ordinate.

Properties of a closed triangle:

1. Sum of all latitudes and departure should be zero:
   \[ d_1 \cos \alpha_1 + d_2 \cos \alpha_2 + \cdots = 0 \quad (1) \]

2. \[ d_1 \cos \alpha_1 + d_2 \cos \alpha_2 + \cdots = 0 \quad (2) \]
(2) Sum of all internal angle of a closed traverse
\[ = (2n-4)90' \]

(3) Sum of external angle of a closed traverse
\[ \text{AE + BE} = (180 - \alpha) + (180 - \beta) \]
\[ = n \times 180 - (\alpha + \beta) \]
\[ = 2n \times 90 - (2n-4) \times 90 \]
\[ = 2n \times 90 - 2n \times 40 + 4 \times 90 \]
\[ = 360' \]

To close Error: In case of a closed traverse

If the first point of the closed traverse is not same as the that point, the error is called closing error. \( A \rightarrow A' \) is the error

\[ A' \rightarrow \text{Correction} \]
Adjustment or closing errors:

1. \( E_L = 0 \)
2. \( E_D = 0 \)

- Sum of all internal angles is \( (2n-4) \times 90^\circ \)

Methods for balancing a closed traverse:
1. Bowditch method.
2. Transit method.
4. Paxis method.

**Bowditch method:** This method is suitable when linear and angular measurements have been measured with equal precision. Changes of error are done in linear/ angular measurements.

As per Bowditch:

1. Error in linear measurement: \( \sqrt{1} \)
2. Error in angular measurement: \( \sqrt{n} \)
$$EL = \text{Total Error in Latitude} \quad \text{(Sum of all Latitude with Singh)}$$

$$ED = \text{Total Error in Departure} \quad \text{(Sum of all Departure with Singh)}$$

$$EL = \text{Sum of all Length of different line}$$

Correction in Latitude of a particular line

$$C_L = EL \times \frac{L_l}{EL} \quad \text{(1)}$$

Correction in Departure

$$C_D = ED \times \frac{D_l}{ED} \quad \text{(2)}$$

2. **Transit method**: This method is suitable when angles measurement are more precise than linear measurement.

Error

\[
\begin{align*}
EL &= \text{Sum of all Latitude (with Singh)} \\
ED &= \text{Sum of all Departure (with Singh)} \\
L_l &= \text{Total Latitude without Singh} \\
D_l &= \text{Total Departure without Singh}
\end{align*}
\]

Correction in Latitude of a particular line

$$C_L = EL \times \frac{L_l}{L_T}$$

$$C_D = ED \times \frac{D_l}{D_T}$$
Graphical method:

all this correction is marked and the most point is same direction.
A closed traverse has the following length and bearing:

<table>
<thead>
<tr>
<th>Line</th>
<th>Length</th>
<th>Bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>300 m</td>
<td>Roughly East</td>
</tr>
<tr>
<td>BC</td>
<td>93 m</td>
<td>17° 8'</td>
</tr>
<tr>
<td>CD</td>
<td>-</td>
<td>270°</td>
</tr>
<tr>
<td>DA</td>
<td>84.6 m</td>
<td>1°</td>
</tr>
</tbody>
</table>

Find out missing data

**Solution:**

![Diagram of a square with bearings and distances labeled]

<table>
<thead>
<tr>
<th>Bearing</th>
<th>Latitude</th>
<th>Departure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roughly</td>
<td>300.55°</td>
<td>300.55°</td>
</tr>
<tr>
<td>17° 8'</td>
<td>0</td>
<td>3.2</td>
</tr>
<tr>
<td>270°</td>
<td>0</td>
<td>0.50</td>
</tr>
<tr>
<td>84.6</td>
<td>86.39</td>
<td>1.50</td>
</tr>
</tbody>
</table>

**EL = 0**

**ED = 0**

From **EL = 0**

300 cos30° = 1155

cos30° = 0.866025

⇒ \( \theta = 86° 41' 22" \)
\[ \Sigma D = 0 \]
\[ 700 \text{sm} + 3.43 + 1.50b = x = 0 \]
\[ x = 8 - 700 \sin 86.0/22 + 4.92b \]
\[ x = 204.60 \text{m} \]

**Problem:** A closed traverse has following readings. Find out the missing data.

<table>
<thead>
<tr>
<th>Line</th>
<th>Length</th>
<th>Angled with</th>
<th>Latitude</th>
<th>Departure</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>250 m</td>
<td>85°</td>
<td>31.79</td>
<td>349.05°</td>
</tr>
<tr>
<td>BC</td>
<td>−x</td>
<td>40°</td>
<td>0.766x</td>
<td>0.643x</td>
</tr>
<tr>
<td>CD</td>
<td>150</td>
<td>390°</td>
<td>114.907</td>
<td>−96.918</td>
</tr>
<tr>
<td>DE</td>
<td>330</td>
<td>295°</td>
<td>92.976</td>
<td>−199.39</td>
</tr>
<tr>
<td>EF</td>
<td>140</td>
<td>0°</td>
<td>140.8580</td>
<td>140.8580</td>
</tr>
<tr>
<td>FA</td>
<td>200</td>
<td>160°</td>
<td>−107.94</td>
<td>68.149</td>
</tr>
</tbody>
</table>

**Solution:**

\[ \Sigma L = 0 \]
\[ 700 \cos 86.0/22 + 140 \cos 0 + 41.733 = 0 \] \[ \text{--- (1)} \]

\[ \Sigma D = 0 \]
\[ 0.643x + 140 \sin 0 + 21.648 = 0 \] \[ \text{--- (2)} \]

From eq. (1)
\[ 140 \cos 0 = 0.766x - 41.733 \] \[ \text{--- (3)} \]

From eq. (2)
\[ 140 \sin 0 = 0.643x - 21.648 \] \[ \text{--- (4)} \]
\[ \cos \theta = \frac{0.746 + 411.738}{0.641 + 21.696} \]

\[ 19600 = x^2 + 91.772x + 2210.192 = 0 \]

\[ x^2 + 91.772x - 17389.8 = 0 \]

\[ x = 9.374 \text{ m} \]

From eq. 3

\[ \cos \alpha = \frac{173.54}{140} = 0.81098 \]

From eq 60

\[ \sin \alpha = \frac{81.92}{140} = -0.58575 \]

\[ \tan \alpha = \frac{\sin \alpha}{\cos \alpha} = 0.7315 \]

\[ \alpha = 35.45^{\circ} 52^{\prime} \]

\[ \text{Bearing of line (line 1) m 3rd 0º) } \]

\[ 180 + 0 = 180 + 35.45^{\circ} 41.52^{\prime} \]

\[ = 35.45^{\circ} 41.52^{\prime} \]

\[ \text{Ans} \]
A Leveling :

1. Reduced Level: Reduced level of point on earth surface is the elevation of that point relative to a fixed location (mean sea level) or to the length of certain bench marks.

2. Back Sight (Reading): First reading taken after setting up the instrument (leveling instrument) at a particular location.

3. Foresight: Last reading taken from an instrument reading location after which the instrument is being changed is called foresight reading. Last reading after which survey work is closed is also a foresight.

4. Intermediate Sight: All other reading than Rs and Fs are intermediate sights.

5. Sight of Instrument: The elevation of line of sight at particular instrument location.

H of mth location = RL of Bm + BS of
\[ \text{N. of next station} = \frac{H_{T}}{10} - F_{S} = F_{S} + B_S \]

6. Rise and Fall: \( \Rightarrow \) 1st reading - 2nd reading

(+)ve \( \Rightarrow \) Rise
(-)ve \( \Rightarrow \) Fall

Table from some instrument station:

<table>
<thead>
<tr>
<th>Point</th>
<th>Back side (BS)</th>
<th>Intermediate side (IS)</th>
<th>Rise side (FS)</th>
<th>Height of instrument (HI)</th>
<th>R.L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Bin</td>
<td>1.60</td>
<td></td>
<td></td>
<td>101.60</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.20</td>
<td></td>
<td>2.50</td>
<td>99.40 + 1.30 = 100.40</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.30</td>
<td>2.50</td>
<td>2.20</td>
<td>97.30 + 1.50 = 98.80</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.50</td>
<td>2.50</td>
<td>1.80</td>
<td>96.30 = 98.70 - 2.50</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2.10</td>
<td></td>
<td>1.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3.70</td>
<td></td>
<td>0.60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td>0.70</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Rise and fall method:

<table>
<thead>
<tr>
<th>Level Book</th>
<th>Rise</th>
<th>Fall</th>
<th>RL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.60</td>
<td>99.40</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>99.10</td>
<td></td>
</tr>
</tbody>
</table>

\( \text{Check:} \rightarrow \text{Add all back side reading =} \)

\( \text{Add all front side reading =} \)

\( \text{The calculated difference between two readings =} \)

\( \text{Then add all rise =} \)

\( \text{Add all fall =} \)

\( \text{Difference between rise and fall =} \)

\( \text{Note: } EBS - EFs = \text{Last RL - 1st RL =} \)

\( E \text{Rise} - E \text{Fall} \)
In running fry levels from a bar of RL 130-750 the following readings were obtained:

<table>
<thead>
<tr>
<th>BS</th>
<th>IS</th>
<th>FS</th>
<th>Rate</th>
<th>Fall</th>
<th>RC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>-</td>
<td>0.965</td>
<td>0.235</td>
<td>0.763</td>
<td>120.417</td>
</tr>
<tr>
<td>1.295</td>
<td>20.555</td>
<td>0.135</td>
<td>0.790</td>
<td>119.624</td>
<td></td>
</tr>
<tr>
<td>1.182</td>
<td>1.150</td>
<td>0.378</td>
<td>0.20</td>
<td>119.80</td>
<td></td>
</tr>
<tr>
<td>0.965</td>
<td>1.945</td>
<td>0.417</td>
<td>0.20</td>
<td>119.80</td>
<td></td>
</tr>
<tr>
<td>0.49</td>
<td>1.755</td>
<td>0.317</td>
<td>0.20</td>
<td>113.00</td>
<td></td>
</tr>
<tr>
<td>0.117</td>
<td>0.170</td>
<td>0.514</td>
<td>0.20</td>
<td>119.60</td>
<td></td>
</tr>
<tr>
<td>0.514</td>
<td>0.717</td>
<td>0.20</td>
<td>119.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.717</td>
<td>0.20</td>
<td>119.20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.117</td>
<td>0.20</td>
<td>118.70</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.092</td>
<td>0.20</td>
<td>118.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>check: BS = 0.140, FS = 0.417, Rate = 0.790, Fall = 119.624</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Page from the bar position of the instrument 20 cm at 10 cm interval one to be set out on a uniform descending gradient of 180 m. Work out the staff readings for setting the top of pages fall the level book and carry out arithmetic checks.
Seven pages are to set each at distance = 10 m
at gradient 6\(^{\circ}\) to 1000 pages

\[ \text{Total height} = \frac{10 \times 6}{150} = 0.20 \text{ m} \]

The 1st pages R.L = 52 120.0 m

Check

1. \( IS = E_{FS} - E_{BS} = 6.722 - 4.772 = 1.95 \text{ m} \)

2. \( RL = \text{Last RL} - \text{Last RL} \)
   \[ = 120.750 - 119.20 \]
   \[ = 1.5 \]

3. \( E_{rise} + E_{fall} = \)
   \[ = 0.803 + 2.75 = 1.95 \text{ m} \]

Then OK Ans
Problem: Fill the missing data of a level book

<table>
<thead>
<tr>
<th>Point</th>
<th>BS</th>
<th>SS</th>
<th>FS</th>
<th>Rise</th>
<th>Fall</th>
<th>R-L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.125</td>
<td></td>
<td></td>
<td>×</td>
<td>×</td>
<td>123.66</td>
</tr>
<tr>
<td>2</td>
<td>2.265</td>
<td>2.32</td>
<td>1.80</td>
<td>1.325</td>
<td></td>
<td>125.805</td>
</tr>
<tr>
<td>3</td>
<td>×</td>
<td></td>
<td>2.32</td>
<td></td>
<td>0.555</td>
<td>124.95</td>
</tr>
<tr>
<td>4</td>
<td>1.04</td>
<td>1.925</td>
<td></td>
<td>0.40</td>
<td></td>
<td>125.350</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td>2.655</td>
<td>0.735</td>
<td></td>
<td>124.65</td>
</tr>
<tr>
<td>6</td>
<td>1.625</td>
<td></td>
<td>3.205</td>
<td></td>
<td></td>
<td>123.45</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td>3.625</td>
<td></td>
<td>-2.05</td>
<td>120.595</td>
</tr>
</tbody>
</table>

| J = 125.00 - Rise (1.325) |
| J = 123.66 |
| B = 3.125 - 1.325 = 1.80 |
| A = 2.32 - 0.055 = 2.265 |
| K = 125.005 - 0.055 = 124.95 |
| F = 125.350 - 124.95 = 0.40 |

Rise
| C = 2.32 - 0.40 = 1.92 |
| G = 1.92 - 2.655 = -0.735 (Fall) |
| L = 125.35 - 0.735 = 124.615 |
| D = 3.205 - 2.165 = 1.04 |
\[ M = 120.615 - 2.165 \]
\[ M = 122.45 \]
\[ H = 1.620 - 3.625 = -2.005 \text{ (fall)} \]
\[ N = 122.45 - 2.005 = 120.445 \]
\[ I = 122.59 - 120.445 = 2.145 \]
\[ E = 3.625 - 21.45 = 1.48 \]

Check:
1. \[ E_F - E_B = 2.14 - 0.05 \]
   \[ = 1.09 \]
2. \[ (E_F - E_R = 2.14 - 3.67 \]
   \[ = 1.09 \]
3. \[ 1^{st} PL - Last PL = 123.68 - 122.590 \]
   \[ = 1.09 \]
   
   - than old PL

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Problem: Following consecutive reading were taken from dumpy level.

<table>
<thead>
<tr>
<th>BS</th>
<th>W</th>
<th>FS</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.25</td>
<td>4.92</td>
<td>6.52</td>
</tr>
<tr>
<td>4.62</td>
<td>3.50</td>
<td>3.97</td>
</tr>
<tr>
<td>6.3</td>
<td>3.23</td>
<td>8.57</td>
</tr>
<tr>
<td>1.50</td>
<td>2.00</td>
<td>4.25</td>
</tr>
</tbody>
</table>

The level was shifted after 4th, 6th, 9th, and 13th reading. If R.L of 1st point B.M. = 100.00

Fill the level book and find out R.L of different point.

Solution:

<table>
<thead>
<tr>
<th>Point</th>
<th>BS</th>
<th>W</th>
<th>FS</th>
<th>Rise</th>
<th>Fall</th>
<th>R.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Reciprocal leveling: reciprocal leveling is used.

1. Find out any errors in the leveling instrument. (Line of sight may not be in horizontal direction, or with bubble in the centre.)

2. To eliminate the effect of earth curvature and refraction.

In case of reciprocal leveling, two points A and B at a distance about 200-300m is selected. Staff reading are noted keeping Instrument first first near A and then near B.

When Instrument is at A:
Reading at A = \( h_A \)
Reading at \( B = h_B \)

- When Instrument is at \( B \)
  - \( \text{reading at } A = h_A \)
  - \( \text{reading at } B = h_B \)

- If Instrument is Faulty
  \[ h_B - h_A \neq h_B' - h_A \] \( \text{(6)} \)

- When Instrument is at \( A \)
  \( h_B = \text{Correct reading} \)

- Correct reading at \( B \) should be \( h_B + e_R - x \)

- Correct difference of level b/w \( A \) and \( B \)
  \[ H = (h_B + e_R - x) - h_A \] \( \text{(7)} \)

- When Instrument is at \( B \)
  \( h_B' = \text{Correct reading} \)

- Correct reading at \( A \) should be \( h_A' + e_R' - x \)

- Correct difference of level b/w \( B \) and \( A \)
  \[ H' = h_B' - (h_A' + e_R' - x) \] \( \text{(8)} \)

- \( e_R' = e_R \) or \( H' = H \)

- Add eq. (7) and (8)
  \[ H + H' = h_B + e_R - h_A + h_B' - h_A' - e_R' + x \]

- \( 2H = (h_B - h_A) + (h_B' - h_A') \)
Correct difference of level between A and B

\[ H = \frac{(h_B - h_A) + (h'_B - h'_A)}{2} \]

**Problem:** For a reciprocal leveling following reading was taken:

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Reading</th>
<th>A</th>
<th>B</th>
<th>difference</th>
<th>Correct diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>[1.95 \ h_A]</td>
<td>[2.76 \ h_B]</td>
<td>[0.91]</td>
<td>[0.94]</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>[0.925 \ h_A]</td>
<td>[1.955 \ h_B]</td>
<td>[1.03]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Distance between point 1 and 2 is 250 m. IF R.L of A is 130.50 m. Find out corrected R.L. of B. Find out the error in line of sight of instrument. Neglect errors due to earth curvature and refraction.

**Solution:**

3. Difference of reading when instrument is at A

\[ h_B - h_A = 2.76 - 1.95 = 0.91 \]

4. Difference of reading when instrument is at B

\[ h'_B - h'_A = 1.955 - 0.925 = 1.03 \]

5. Correct difference of level between A and B

\[ H = \frac{(h_B - h_A) + (h'_B - h'_A)}{2} = \frac{0.91 + 1.03}{2} \]

\[ H = 0.94 \text{ m} \]
Correct Reading at A = 1.85

when Instrument is at A

\[ \text{R.L. of A} = 120.50 \text{m} \]
\[ \text{R.L. of B} = 120.50 \text{m} - 0.97 \text{m} \]
\[ = 119.53 \text{m} \]

B is at lower elevation than A.

Error in line of collimation

\[ = 2.82 - 2.76 \]
\[ = 0.06 \text{ m} \]

\[ \tan \theta = \frac{0.06}{250} \]
\[ \theta = \frac{1}{2166.67} \]
8. Correction due to curvature and refraction:

1. Correction due to curvature (Earth curvature):
   \[ \text{Level line} \Rightarrow \text{A line parallel to earth surface (curved line)} \]

2. Horizontal line: Tangent line at earth surface at any point.

---

In triangle ORB:

\[ R^2 + d^2 = (R + c)\,^2 \]
\[ R^2 + d^2 = R^2 + c^2 + 2R\,c \]
\[ d^2 = c\,\left(2R + c\right) \]
\[ \Rightarrow \text{Correction due to curvature} \]

\[ C_c = \frac{d^2}{2R} \]

\[ R = \text{Radius of Earth or} \]
\[ \text{Radius of curvature} \approx 6370 \, \text{km} \]
Correction due to earth curvature

\[ C_e = \frac{d^3}{2 \times 6370} \times 1000 = \text{meter} \]

\[ \Rightarrow C_e = 0.07946 \; d^2 \; \text{m} \]  \( \textcircled{71} \)

Here \( d \) = distance \( m, \text{km} \).

For staff reading

\( \text{error} = (+) \; \text{ve} \)

\( \text{Correction} = (-) \; \text{ve} \)

For reduced levels calculated using center staff reading

\( \text{error} = (-) \; \text{ve} \)

\( \text{Correction} = (+) \; \text{ve} \)

Generally

\[ C_c = (-) \frac{d^2}{2R} = 0.07946 \; d^2 \]

\( \textcircled{80} \) Correction due to refraction \( \Rightarrow \)
This correction is required due to refraction. The value is \((y_7)\) of curvature

\[ c_R = \frac{1}{7} \times \text{correction due to curvature} \]

\[ c_R = \frac{1}{84} \times \frac{d^3}{2R} \]

\[ \Rightarrow c_R = \frac{d^3}{14R} \]

\[ \Rightarrow c_R = \frac{1}{7} \times 0.6479649 d^2 \]

\[ \Rightarrow c_R = 0.01121 d^2 \]

\[ c_R \text{ is always (+) ve} \]

Combined correction: \(\Rightarrow\)

Due to curvature and refraction

\[ = \left( \frac{d^3}{2R} \right) + \frac{1}{7} \left( \frac{d^3}{2R} \right) \]

\[ = -\frac{6}{7} \times \frac{d^3}{2R} \]

\[ = -\frac{6}{7} \times 0.01121 d^2 \]

\[ = 0.06728 d^2 \]

\( \times d = \text{distance in km} \)
Note

\[ c_c = (-) \frac{d^2}{2R} = -0.07849d^2 \]

\[ c_R = (+) \frac{1}{4} x \frac{d^3}{2R} = 0.01121 \text{ d}^2 \]

\[ c = (-) \frac{5 \times d^3}{7R} = (-) 0.06728 \text{ d}^2 \]

\[ R = 6371 \text{ km} \]

4. **Distance of Visible Horizon**

A person at \( h \) height from sea level can see the point at sea surface \( d \) distance away.

Combined formula for correction due to earth curvature and refraction is used.

\[ c = h \]

\[ c = \frac{6}{7} \frac{d^3}{2R} = h \]

\[ d = \sqrt{\frac{105}{6} Rh} \]

\[ c = 0.06728 \text{ d}^2 \]
\[ d = \sqrt{\frac{h}{0.06728}} \]

\[ d = 3.8555 \sqrt{h} \]

If \( h = \) in meter,
\[ d = \text{in ft} \]

\[ h = \text{in meter} \]
\[ d = \text{in ft} \]

Problem 10b

An observer standing on the deck of a ship just see a light house. The top of light house is 49 m above sea level and the height of observer eye is 9 m above sea level.

Find the distance of observer from light house.

Solution:

Distance of observation to light house = \( d_1 + d_2 \)

\[ = 3.855 \sqrt{h_1} + 3.855 \sqrt{h_2} \]

\[ = 3.855 (\sqrt{49} + \sqrt{49}) \]

\[ = 3.855 \times 3 \times 4.7 \]

\[ = 36.55 \text{ ft m} \]
Sensitivity of a bubble tube:

Experiment:

1. Fix the instrument at a location take bubble in the centre at on a staff key.
2. Now turn the telescope such that bubble in division.
   \[ l = \text{length of one division} \]
Total moment of bubble = nL

3) Radius of curvature of bubble tube = R

4) Staff reading after rotation = s₁

5) Staff intercept s = s₁ - s

Total angle

$$\theta = \frac{s}{2R} = \frac{nL}{R} \quad \text{(A)}$$

Sensitivity of a bubble tube is the angle of rotation for one division moment of bubble

$$\Rightarrow \quad \alpha = \frac{\theta \cdot \rho}{\eta} = \frac{s}{nD} = \frac{L}{R} \quad \text{(B)}$$
Problem: If a bubble tube has sensitivity of \(110 \text{ deg/m}\) and the length of one division of bubble tube is 2 mm. Find out the error in staff reading on a staff kept at 300 m distance. Caused due to bubble to division out of the centre. Find out radius of curvature of bubble.

Solution:

\[ \psi = \frac{25 \text{ deg}}{2 \times 180} = \frac{25}{60 \times 60} \times 180 = \frac{25}{306265} \text{ rad.} \]

\( \eta = 2 \)

Length of one division \( l = 2 \text{ mm} \)

Sensitivity

\[ \psi = \frac{\text{arc}}{\eta l} = \frac{1}{R} \]

Error in staff reading

\[ S = \eta l \psi = 2 \times \left( \frac{25}{306265} \right) \times 300 \]

\[ S = 0.0727 \text{ m} \]

\[ S = 7.27 \text{ mm} \] (in m)

\[ \Rightarrow \psi = \frac{1}{R} = \frac{1}{0.0727} = 8.2 \text{ mm} \]

Radius of curvature \( R = 16.5012 \text{ m} \) (mm)
Problem: The reading taken on a staff kept at 200 m from an instrument with bubble at centre was 1.85 m. Bubble is moved by 5 division out of the centre and an staff reading observed was 1.96 m. Find the sensitivity of bubble tube. What is the radius of curvature of bubble tube. Length of 1 division of bubble tube is 3 mm.

Solution:

\[ s_1 = 1.95 \text{ m} \]
\[ s_2 = 1.98 \text{ m} \text{ (after 5 divisions movement)} \]
\[ s = s_2 - s_1 \]
\[ s = 1.98 - 1.95 \]
\[ s = 0.03 \text{ m} \]

\[ n = 5 \]
\[ d = 3 \text{ mm} \]

\[ \text{Sensitivity of bubble tube} \quad \delta = \frac{0}{n} = \frac{s}{nd} = \frac{1}{R} \]
\[ \Rightarrow \quad \delta = \frac{s}{nd} = \frac{0.03}{5 \times 0.3} = \frac{0.03}{1.5} = 0.02 \text{ m} \]
\[ \text{Radius of curvature} \]
\[ R = \frac{1}{\delta} = \frac{s}{\delta d} = \frac{0.03}{0.02 \times 0.3} = 5 \]
\[ \Rightarrow R = 5 \times 100 = 500 \text{ mm} \]
Plane Table Surveying:

There are four methods:

1. **Radiation Method:**
   - For collecting the object's details

2. **Intersection Method:**
   - For locating the position of instrument station

3. **Prism method:**
   - For locating the position of instrument station

4. **Re-section:**
   - For measuring the drawing:

**Diagram:**

- Current the table in correct direction (N-S) using a line drawn on the drawing.
- Position of instrument station P is marked over drawing.
- For locating object's position, distances are measured by using any other instrument.
- Lines are drawn towards different objects A, B, and C and position of objects are marked by using a suitable scale.

**Intersection:**

First, the instrument at A is set up at a station P and after orienting the table,

Lines are drawn towards different objects, i.e., A, B, and C.
Then another station O is marked by measuring the distance and by drawing a line to distance and by drawing a line from P towards O.

The instrument is charged to O and after orienting the table by either compass or back orientation line are again drawn toward at Bande.

Intersection points are the location of objects.

3. Traversing:

Different station of a closed traverse are marked by distance and orientation of table by back orientation.

Other details are collected either by notation or intersection.
Resection is the process of determining the position of instrument station after orienting the table in correct direction.

Methods:
1. Orientation by compass.
2. Orientation by back sighting.
3. Three point problem.
4. Two point problem.

Three point problem: (using the known location of free points on the drawing)
1. By tracing paper method.
2. By Basels graphically method.
3. By Lahman's rule method.
Tracing paper method:

1. Place a tracing paper on the drawing. Select any point on the tracing paper and draw three lines towards A, B and C.

2. Now rotate the tracing paper over drawing dish so that the marked position a, b and c comes directly over the three lines drawn in tracing paper.

3. Transfer the location of intersection point on the tracing drawing. This is the correct position of instrument station. Draw any one line towards the corresponding object. This is the correct correlation.
Ressel's Graphical Method:

Step 1

After setting the table at station P, rotating the table till make line \( \overline{ba} \) from b towards c.

Step 2

Rotating the table such that line \( \overline{ab} \) is now towards c. Draw a line from a towards c.

Step 3

Intersection point of two lines above is \( P' \).

Step 4

Now rotating the table and make line \( \overline{pc} \) towards c. This is the correct rotation of the table; draw line from A, B, and C. That intersect line \( \overline{pc} \) at a point P that is the location of Instrument Station.
(3) Lehman's method:

- If orientation of the table is not correct, the three lines drawn towards three objects does not intersect at a single point, this a triangle is formed called triangle of error.

This triangle of error to eliminated by using Lehman's Rules, the solution is also called trial and error solution.

Rules:

1. If triangle of error is within the major triangle, possible location of point is within the triangle.
The point \( p' \) should be chosen such that its distance from \( a, b, \) and \( c \) are proportional to \( pA, pB, \) and \( pC. \)

The point \( p' \) should be chosen such that it is to the same side of all the three rays \( pA, pB, \) and \( pC. \)

**Two point problem:**

**Procedure:**
1. A and B are two points with which location are marked as \( a \) and \( b. \)
2. fixed the table at a station near \( p. \) Placing the tab.
approximately draw line OA and BB that intersect at Q.

3. From Q, draw line toward P to measure distance PQ and plot the position of P'.

4. Now bring the table at P and orient the table by back orientation toward Q. In this position orientation of table is same as it was at Q.

5. Now draw a line from P toward B that cuts line QB at B'. QAB is the error in orientation.

6. To correct the error place a flag at R near P in the direction of QAB.

7. Now rotate the table such that line AB comes in the direction of R. This is the correct orientation.

8. Now draw line OA and BB that intersect at P that is the direction of instrument station.
Area

Volume

1. Area:
   Different methods are:
   a) Dividing into a number of triangles.

   
   \[ A = \sqrt{s(s-a)(s-b)(s-c)} \]
   where \( s = \frac{a+b+c}{2} \)

   \[ A = \frac{1}{2}ab \sin C \]
   \[ = \frac{1}{2} bc \sin A \]
   \[ = \frac{1}{2} ca \sin B \]

2. Area computed by offsets measured at equal intervals.
1. Mid-ordinate Method:
   
   \[ \text{Area} = d \left( O_1 + O_2 + O_3 + \ldots + O_n \right) \]

2. Average ordinate rules:
   
   \[ A = \left( \frac{h_1 + h_2 + h_3 + \ldots + h_n}{n} \right) \times (n-1) \times d \]

3. Trapezoidal Rule:

\[ \text{Area of first block} = \frac{h_1 + h_2}{2} \times d \]

\[ \text{Second block} = \frac{h_2 + h_3}{2} \times d \]

\[ \text{Last block} = \frac{h_{n-1} + h_n}{2} \times d \]

\[ \text{Second block} = \frac{h_2 + h_3}{2} \times d \]
Total Area
\[ A = \frac{d}{2} \left[ (h_1 + h_n) + 2(h_2 + h_3) + \cdots + h_{n-1} \right] \]

Repetendial formula
\[ A = d^2 \left[ \frac{h_1 + h_n}{2} + \left( h_2 + h_3 + h_4 + \cdots + h_{n-1} \right) \right] \]

Simpson's Rule:

Area of one pair \((A_1 + A_2)\) = \(\frac{d}{3}(h_1 + h_2 + h_3)\)

Second pair \((A_2 + A_4)\) = \(\frac{d}{3}(h_2 + h_3 + h_4)\)

\(\cdots\)

\(A_{(n-2)} + A_{(n-1)}\) pair = \(\frac{d}{3}(h_{n-2} + h_{n-1} + h_n)\)

Total area:
\[ A = \frac{d}{3} \left[ (h_1 + h_n) + a_1(h_2 + h_4 + h_6 + \cdots) + 3(h_3 + h_5 + \cdots) \right] \]
Note: The above formula can be used for odd number of offsets only if there are even no. of offsets either first block/last block area is calculated using trapezoidal rule and simply added.

Volume: If there are different parallel areas available at equal intervals:

\[ V = d \left[ \frac{A_1 + A_n}{2} + (A_2 + A_3 + \ldots + A_{n-1}) \right] \]

\[ V = \frac{d}{3} \left[ (A_1 + A_n) + 4(A_2 + A_4 + \ldots) + 2(A_3 + A_5 + \ldots) \right] \]

Note: The sine joint in curved line then Simpson's rule is best and the time is joint in a straight line then trapezoidal formula is best.
Problem: An excavation has shape as shown in Fig. Calculate volume of the excavation earth by
(a) Prismatical Rule  (b) Simpson's Rule.

Solution: Prismatical Rule

\[ V = \frac{A_1 + A_2}{2} \times d \]

\[ A_1 = 37 \times 24 = 999 \text{ m}^2 \]
\[ A_2 = 25 \times 15 = 375 \text{ m}^2 \]
\[ d = 6 \text{ m} \]

\[ V = \frac{999 + 375}{2} \times 6 = 4122 \text{ m}^3 \]
8) Simpson's Rule:

Consider another area at centre.

\[ A_1 = 37 \times 27 = 999 \text{ m}^2 \]
\[ A_2 = 31 \times 21 = 651 \text{ m}^2 \]
\[ A_3 = 25 \times 15 = 375 \text{ m}^2 \]
\[ d = 3 \text{ m} \]

Volume

\[ V = \frac{d}{3} \left( A_1 + 4A_2 + A_3 \right) \]
\[ V = \frac{3}{3} \left( 999 + 375 + (4 \times 651) \right) \]
\[ V = 397.8 \text{ m}^3 \]

Problem:

Area of different contours of a reservoir.

<table>
<thead>
<tr>
<th>Contours</th>
<th>Area (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>104</td>
<td>4000 (A₀)</td>
</tr>
<tr>
<td>105</td>
<td>15500 (A₁)</td>
</tr>
<tr>
<td>106</td>
<td>99000 (A₂)</td>
</tr>
<tr>
<td>107-109</td>
<td>160000 (A₃)</td>
</tr>
<tr>
<td>108</td>
<td>850,000 (A₄)</td>
</tr>
<tr>
<td>109</td>
<td>13,500,000 (A₅)</td>
</tr>
<tr>
<td>110</td>
<td>1,040,000 (A₆)</td>
</tr>
<tr>
<td>111</td>
<td>20,200,000 (A₇)</td>
</tr>
</tbody>
</table>

Calculate volume of reservoir by Simpson's formula. Volume below lown may be neglected.
Solution: Volume blue containe 104 and 105 using Trapezoidal formula.

\[ V_1 = \frac{4000 + 15500}{2} \times x_1 \]

\[ V_1 = 950 \text{ m}^3 \]

Volume blue counters 105 to 106

\[ V_2 = \frac{1}{3} \left[ (A_1 + A_7) + 4(A_2 + A_4 + A_6) + \frac{2}{3}(A_3 + A_5) \right] \]

\[ V_2 = \frac{1}{3} \left[ (15500 + 200000) + 4(98000 + 95000 + 1840000) + 2(160000 + 1850000) \right] \]

\[ V_2 = 54059033.3 \text{ m}^3 \]

Total Volume = \( V_1 + V_2 \)

\[ V = 950 + 54059033.3 \text{ m}^3 \]

\[ V = 54155983.33 \text{ m}^3 \]

Ans
Problem: In a proposed reservoir, the area containing within the contours are:

<table>
<thead>
<tr>
<th>Contours</th>
<th>Area (ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>32</td>
</tr>
<tr>
<td>95</td>
<td>96</td>
</tr>
<tr>
<td>90</td>
<td>34</td>
</tr>
<tr>
<td>85</td>
<td>18</td>
</tr>
<tr>
<td>80</td>
<td>15</td>
</tr>
<tr>
<td>75</td>
<td>13</td>
</tr>
<tr>
<td>70</td>
<td>7</td>
</tr>
<tr>
<td>65</td>
<td>2</td>
</tr>
</tbody>
</table>

Solution:

<table>
<thead>
<tr>
<th>Contours</th>
<th>Area (ha)</th>
<th>Avg. Area</th>
<th>Volume (ha-m)</th>
<th>Cumulative Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>32</td>
<td>29</td>
<td>145</td>
<td>650</td>
</tr>
<tr>
<td>95</td>
<td>96</td>
<td>25</td>
<td>125</td>
<td>455</td>
</tr>
<tr>
<td>90</td>
<td>34</td>
<td>31</td>
<td>105</td>
<td>330</td>
</tr>
<tr>
<td>85</td>
<td>18</td>
<td>16.5</td>
<td>97.5</td>
<td>225</td>
</tr>
<tr>
<td>80</td>
<td>15</td>
<td>14.0</td>
<td>70.0</td>
<td>143.5</td>
</tr>
<tr>
<td>75</td>
<td>13</td>
<td>10.0</td>
<td>50.0</td>
<td>73.5</td>
</tr>
<tr>
<td>70</td>
<td>7</td>
<td>45</td>
<td>22.5</td>
<td>29.5</td>
</tr>
<tr>
<td>65</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
1. Capacity of reservoir when it is full at normal level

\[ V = d \left[ \left( \frac{A_1 + A_2}{2} \right) + A_3 + A_4 + \cdots \right] \]

\[ V = 5 \left[ \frac{32 + 2}{2} + \left( 7 + 13 + 15 + 19 + 13 + 26 \right) \right] \]

\[ V = 600 \text{ ha-m} \]

2. Volume when reservoir is 18% full

\[ V = \frac{60}{100} \times 600 = 360 \text{ ha-m} \]

Water level should be \( \approx \) 90 to 95 m

90 m \( \Rightarrow \) 330 ha-m

95 m \( \Rightarrow \) 455 ha-m

By interpolation:

Water level for 360 ha-m volume

\[ = 90 + \frac{95 - 90}{455 - 330} \times (360 - 330) \]

\[ = 91.20 \text{ m} \]

Ans

Remarks: The end area method is a trapezoidal formula for calculation of volume between different areas at equal intervals and parallel to each other.
Area enclosed within a closed Traverse:

Meridian Distance Method:

- Meridian distance method is the distance of mid point of a line from a given meridian.

Meridian distance of line $AB$:

$$m_1 = + \frac{D_1}{3}$$

Meridian distance of line $BC$:

$$m_2 = + D_1 + \frac{D_3}{3}$$

$$= \frac{D_1}{2} + \frac{D_1}{2} + \frac{D_3}{3}$$

$$= m_1 + \left( \frac{D_1}{2} + \frac{D_3}{2} \right)$$

Meridian distance of line $CD$:

$$= D_1 + D_2 - \frac{D_3}{2}$$

$$= D_1 + D_2 + \left( - \frac{D_3}{2} \right)$$
\[ \begin{align*}
    &= \left( \frac{D_1 - D}{2} + \frac{D_1 + D_2}{2} + \frac{D_2 - D}{2} \right) + \frac{D_3}{2} + \left( \frac{-D_3}{2} \right) \\
    &= m_2 + \left( \frac{D_3}{2} + \frac{-D_3}{2} \right) - \frac{D_1}{2} \\
    \text{M.D. of a line} &= (\text{M.D. of point} + \left( \frac{1}{3} \times \text{Depressed} \right) + \left( \frac{1}{2} \times \text{this line} \right) \\
    \text{M.D. of line AD} &= m_3 + \left[ \left( \frac{-D_3}{2} \right) + \left( \frac{-D_4}{2} \right) \right] \\
\end{align*} \]

Area within closed frames:

\[ A = (\pm L_1) \times m_1 + (\pm L_2) \times m_2 + \cdots \]

\[ A = \sum (L \times m) \]

A DMD Method →

(Double Meridian Distance method)
Double meridian distance is the sum of the meridian distance of two extreme points of a line.

\[ \text{DMD of line} \]

\[ AB = M_1 = 0 + \Pi_1 \]

\[ M_1 = \Pi_1 \]

\[ \text{for BC: } M_2 = \Pi_1 + (\Pi_1 + \Pi_2) \]

\[ M_2 = M_1 + (\Pi_1 + \Pi_2) \]

\[ \text{DMD of a line} = \left( \text{DMD of previous line} \right) + \left( \text{Departure of point of line} \right) \]

\[ \text{for CD: } M_2 + \left[ (\Pi_2) + (-\Pi_3) \right] \]

\[ DA = M_3 + \left[ (-\Pi_3) + (-\Pi_4) \right] \]

Area within the closed traverse

\[ A = \frac{1}{2} \sum (L_i \times M_i + L_2 \times M_2 + \ldots) \]

\[ A = \frac{1}{2} \sum (L \times M) \]
\[ \frac{1}{F} = \frac{1}{u} + \frac{1}{v} \quad (1) \]

\[ \frac{1}{u} = \frac{1}{F} - \frac{1}{u'v'} \]

\[ \frac{S}{u} = \frac{u'}{v'} \quad (2) \]

\[ v = \frac{u'd}{S} \]

\[ \frac{1}{u} = \frac{1}{F} - \frac{S}{w} \]

\[ \frac{1}{u} \left(1 + \frac{S}{x}\right) = \frac{1}{F} \]

\[ F \left(1 + \frac{S}{x}\right) = u \]

\[ u = F + \frac{F}{x} \cdot \frac{S}{x} \quad (3) \]

The distance of object from axis of telescope:

\[ D = u + d \]

\[ D = F + \frac{S}{x} \cdot d + d \]

\[ D = \left(\frac{F}{x}\right) S + (F + d) \]

\[ D = kS + c \quad (4) \]

\[ k: \text{ multiplying constant} \]

\[ k = \frac{F}{x} \approx 100 \text{ (generally)} \]
C = Product of constant = E + d ≥ 0 (generally)

→ A telescope which have k=100, and c=0 is called paraxial telescope.

S = Staff intercept

Determination of k and c for a telescope:

Instrument in fixed at a station and staff readings are taken at two location, at distance D1 and D2, where Staff intercept are S1 and S2.

\[ D_1 = kS_1 + c \] \[ D_2 = kS_2 + c \] \( \text{value this first eq} \)

\[ (D_1 - D_2) = k(S_1 - S_2) \]

\[ k = \frac{D_1 - D_2}{S_1 - S_2} \] \( \text{(1)} \)

\[ c = D_1 - kS \]
\[ C = \frac{X_1 - \frac{(X_1 - X_2)}{(S_1 - S_2)}} \]

\[ \Rightarrow C = \frac{D_1 S_1 - D_1 S_2 - D_2 S_1 + D_2 S_2}{(S_1 - S_2)} \]

\[ C = \frac{D_2 S_1 - D_1 S_2}{S_1 - S_2} \rightarrow (B) \]

\[ \Rightarrow \text{for practical Telescope.} \]

\[ \tan \theta_2 = \frac{B / 2}{\cos \omega} \]

\[ B = 0.301^\circ 22.63'' \]

By Distance and Elevation Formula: \[ \rightarrow \]

**Case (i)** when staff is held vertical: \[ \rightarrow \]
(For angle of elevation)
when staff act vertical:

Staff intercept for distance = $s \cos \theta$

Inclined distance

\[ L = ks \cos \theta + c \]

Horizontal distance

\[ AC = d = L \cos \theta \]

\[ d = (ks \cos \theta + c) \cos \theta \]

Distance formula

\[ D = ks \cos^2 \theta + c \cos \theta \]

Vertical distance

\[ V = L \sin \theta \]

\[ V = (ks \cos \theta + c) \sin \theta \]

\[ V = ks \sin \theta \cdot \cos \theta + c \sin \theta \]

\[ V = \frac{ks \sin 2\theta}{2} + c \sin \theta \]

Elevation formula

R.L. of point B

\[ RL \text{ of } BM + s_t + V - h_1 \]
Case (3) Staff held vertical (Angle of depression)

Staff intercept for distance = $S \cos \theta$

Inclined distance

\[ l = k \cos \theta + c \]

Horizontal distance formula

\[ D = l \cos \phi \]

\[ D = k \cos^2 \theta + c \cos \theta \]

elevation formula

\[ V = l \sin \theta \]

\[ V = k \cdot s \cdot \frac{\sin \theta}{2} + c \sin \theta \]

\[ R.L \text{ of point} \]

\[ -R.L \text{ of } BM + s_1 + V - h_1 \]

Combine formula

\[ R.L \text{ of } B = R.L \text{ of } BM - s_1 + V - h_1 \]

We use for angle of elevation, We use for angle of departure.
Case 3: Staff normal (Staff kept perpendicular to line of sight) 

(when angle of elevation)

Staff intercept for distance \( L = S \)

Inclined length

\[
L = kS + c
\]

Horizontal distance

\[
D = L \cos \alpha + h_1 \sin \alpha
\]

Equation formula

\[
V = L \sin \alpha
\]

\[
V = (kS + c) \sin \alpha
\]

R.C of point B

\[
= R.C.E.F. Bm + s + V - h_1 \cos \alpha
\]
Case (c) The angle of depression:

\[
\alpha = \theta
\]

Staff Normal
(Line of sight inclined downward)

- Staff intercept: \( s \)
- Inclined length: \( L = ks + c \)

**Horizontal Distance Formula**

\[
D = L \cos \alpha - h_2 \sin \alpha
\]

\[
D = (ks + c) \cos \alpha - h_2 \sin \alpha
\]

**Elevation Formula**

\[
v = L \sin \alpha = (ks + c) \sin \alpha
\]

**R.C of B**

\[
\text{R.C of BM} + s - v - h_1 \cos \alpha
\]
Problem: To determine the value of multiplying constant and additive constant. Following observations were taken:

<table>
<thead>
<tr>
<th>In. Station</th>
<th>Staff kept at</th>
<th>Distance from Rod</th>
<th>Reading with inclination 0°</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0</td>
<td>200 m</td>
<td>0.95 1.93 2.91</td>
</tr>
<tr>
<td>R</td>
<td>120 m</td>
<td></td>
<td>1.23 1.81 2.39</td>
</tr>
</tbody>
</table>

Solution:

\[
\theta_1 = 2.39 - 1.33 \quad \text{and} \quad \theta_2 = 2.91 - 0.95
\]

\[
\Delta_1 = 1.16 \quad \text{and} \quad \Delta_2 = 1.96
\]

\[
D_1 = k\Delta_1 + C
\]

\[
\Rightarrow 1.33 = k \times 1.16 + C \quad \text{--- (1)}
\]

\[
D_2 = k\Delta_2 + C
\]

\[
\Rightarrow 2.91 = k \times 1.96 + C \quad \text{--- (2)}
\]

\[
\text{From (1) and (2),}
\]

\[
k = \frac{2.91 - 1.33}{1.96 - 1.16} = \frac{1.58}{0.8} = 1.975
\]

\[
k \approx 2
\]

\[
C = 1.33 - 2 \times 1.16 = 1.33 - 2.32 = -0.99
\]

\[
k \times 1.96 + C = 2 \times 1.96 + (-0.99) = 3.92 - 0.99 = 2.93
\]

\[
D_2 = 200 m
\]

Diagram: The diagram shows the setup with points labeled for calculations.
\[ 900 = 0.80 \times k \]
\[ k = \frac{900}{0.80} \]
\[ k = 1125 \text{ pm} \]

\[ C = 120 - 100 \times 1.16 \]
\[ C = 120 - 116 \]
\[ C = 4 \text{ pm} \]

Problem: From a theodolite set-up at station A, readings were taken as in Staff kept at Station B and benchmark M.

<table>
<thead>
<tr>
<th>Instrument Station</th>
<th>Staff Station</th>
<th>Angle</th>
<th>Staff Position</th>
<th>Readings</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>+15°30'</td>
<td>kept vertical</td>
<td>2.35, 2.60, 3.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(upward)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>B, M, RL 200.52 M</td>
<td>-15°</td>
<td>kept normal</td>
<td>1.32, 1.55, 1.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(downward)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If \( k = 100, C = 0 \)

height of instrument at \( A = 1.52 \text{ m} \), find R.C. of A at B. Also find true distance of B and M from A.

Solution:
\[ A \cdot 10 B \]
Staff is vertical
Staff intercept = Sec\( 50^\circ \)
\[ = 0.50 \cdot \cos 50^\circ \]

\[ S = 2.35 - 2.35 = 0.50 \]
Distance formula:

\[ D_1 = k_5 \cos^2 \theta + k_5 \sin^2 \theta \]

\[ D_1 = 100 \times 0.50 \times \cos^2 15.30' \]

\[ D_1 = 41.613 \text{ m} \]

Elevation formula:

\[ V_1 = k_1 \frac{\sin^2 \theta}{2} + k_0 \sin \theta \]

\[ V_1 = 100 \times 0.50 \times \sin^2 31 \]

\[ V_1 = 12.876 \text{ m} \]

From A towards:

\[ S = 1.780 - 1.32 \]

\[ S = 0.46 \]
Distance formula:

\[ D_2 = \frac{k_1 l_2 \cos \theta}{(k_2 + c) \cos \theta - h_2 \sin \alpha} \]

\[ D_2 = 100 \times 0.16 \times \cos 15' - 1.55 \times \sin 15' \]

\[ D_2 = 44.03 \text{ m} \]

Elevation formula:

\[ V_2 = l_2 \sin \alpha \]

\[ V_2 = (k_3 + c) \sin \alpha \]

\[ V_2 = 0.46 \times 100 \times \sin 15' \]

\[ V_2 = 11.91 \text{ m} \]

Rl. of A

\[ = RL \cdot OFB + h_2 \cos \alpha - V_2 - 1.52 \]

\[ = 200.52 + 1.55 \times \cos 15' + 11.91 - 1.52 \]

\[ = 219.41 \text{ m} \]

Rl. of B

\[ = RL \cdot OFA + 1.52 + V_1 - h_1 \]

\[ = 219.41 + 1.52 + 12.876 - 2.60 \]

\[ = 234.80 \text{ m} \]
Problem: To determine the gradient between points A and B, a theodolite was set up at another station C, and the following observations were made:

<table>
<thead>
<tr>
<th>Staff</th>
<th>Vertical angle</th>
<th>Staff reading kept vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>+0°10'00&quot;</td>
<td>1.30, 1.61, 1.92</td>
</tr>
<tr>
<td>B</td>
<td>+0°10'00&quot;</td>
<td>1.10, 1.44, 1.72</td>
</tr>
</tbody>
</table>

If horizontal angle ABC = 85°20', determine the average gradient between A and B.

Solution:

\[ k = 100 \text{ m}, \quad C = 0 \]

\[ S = 1.92 - 1.30 = 0.62 \]

\[ D_1 = kS \cos 20° + 1 \cos 30° \]

\[ D_1 = 100 \times 0.62 \times \cos 20° \]

\[ D_1 = 61.64 \text{ m} \]
\[ V_1 = ks \sin 2\theta + b \cdot e \cdot \sin \theta \]
\[ \Rightarrow V_1 = 100 \times 0.62 \times \sin 2 \times 0.80 \times \frac{1}{2} \]
\[ \Rightarrow V_1 = 4.67 \text{ m} \]

R.L. OF A
\[ = x + y + V_1 = h_i \]
\[ = x + y + 4.67 - 1.61 \]
\[ = (x + y + 3.06) \text{ m} \]

\[ \Rightarrow \text{C towards B} \]

\[ s = 1.72 - 1.10 \]
\[ s = 0.62 \]

Distance \( D_2 = ks \cos^2 \theta + \frac{b}{2} \cos \theta \)
\[ D_2 = 100 \times 0.62 \times \cos^2 0.10' + 4.0'' \]
\[ \Rightarrow D_2 = 62 \text{ m} \]

\[ V_2 = ks \sin 2\theta \]
\[ V_2 = 100 \times 0.62 \times \sin 2 \times 0.10' \]
\[ \Rightarrow V_2 = 0.192 \text{ m} \]
\[ R \cdot L \cdot B = R \cdot L \cdot E + y + y_2 = h_2 \]
\[ = (c+y) + 0.192 - 1.41 \]
\[ = (c+y) - 1.219 \]

\textbf{Difference of R.L. Line A and B:}

E is higher

\[ = R \cdot L \cdot A - R \cdot L \cdot B \]
\[ = \frac{c+y}{2} + 3.06 - \frac{c+y}{2} + 1.219 \]
\[ = 3.279 \text{ m} \]

\textbf{Distance from A to B:}

\[ \cos B = \frac{3.279}{c+y} \]

Apply Sine-Formula:

\[ \frac{D_1}{\sin 35.20'} = \frac{D_2}{\sin A} = \frac{D}{\sin C} \]

\[ \Rightarrow \frac{61.666}{\sin 35.20'} = \frac{62}{\sin A} \]

\[ \Rightarrow \sin A = 35.30' 0.9'' \]

Angle \( c = 180 - (35.30' 0.9'' + 35.30' 0.9'') \)
\[ c = 109.5' 59.10'' \]

\[ \Theta = \frac{8 \cdot D_1}{\sin 35.20'} \times \sin 109.5' 59.10'' \]
\[ \Theta = 100.72\text{ m} \]
Problem 69

Gravitation = \frac{1}{0.240}

\text{any}
**Important Terms:**

1. **Back tangent** $\rightarrow AT_1$
2. **Forward tangent** $\rightarrow BT_2$
3. **Point of curve** $\rightarrow$ Point $T_1$
4. **Point of sagacity** $\rightarrow$ Point $T_2$
5. **Deflection and angle (or intersection angle)** $\Delta$
6. **Radius** $= R$
7. **Centre is at** $O$
8. **Pargant distances** $= VT_1 = VT_2 = R \cdot \tan \frac{\Delta}{2}$
9. **External distances** $= (VC)$ $\Rightarrow$ (Apex distance) $\Rightarrow$
   \[
   VC = R \sec \frac{\Delta}{2} - R
   \]
   \[
   VC = R (\sec \frac{\Delta}{2} - 1)
   \]
(10) Total length of curve (L) =

\[ L = T_1 + T_2 = \frac{2\pi R}{360} \times D \]

(11) Long chord (T_1, T_2) = 2R \sin \frac{A}{2}

(12) Middlinate : \( C_D \)

\[ C_D = R - R\cos \frac{A}{2} \]

\[ C_D = R(1 - \cos \frac{A}{2}) \]

(13) Degree of curve (°) = 

Angle formed at centre by 1 chain length is called degree of curve.

(1) Chain = 30 m

\[ D = \frac{1720}{R} \]

\[ R = \frac{1720}{D} \]

(2) Chain = 30 m

\[ D' = \frac{1146}{R'} \]

\[ R' = \frac{1146}{D'} \]

(7) Setting out a Simple curve on ground:

(1) Offset method:

(1) Perpendicular offsets:

\[ OX = AB = T_1 = 6T_1 - \infty \]

\[ OX = R - \sqrt{R^2 - x^2} \]

(19) Exact formula
Expanding

$$Ox = \frac{x^2}{2R}$$

approximate value

Radial off Set:

$$OX = AB = ON - OB$$

$$OX = \sqrt{R^2 + x^2 - R}$$  \(\text{exact formula}\)
(a) Offset from chord produced:

\[ S_1 = \frac{c_1}{2R} \Rightarrow 2S_1 = \frac{c_1}{R} \]

\[ S_2 = \frac{c_2}{2R} \Rightarrow 2S_2 = \frac{c_1}{R} \]

\[ S_n = \frac{c_n}{2R} \Rightarrow 2S_n = \frac{c_n}{R} \]

Offset:

\[ OX_1 = c_1 \cdot S_1 = c_1 \cdot \frac{c_1}{2R} = \frac{c_1^2}{2R} \]

\[ OX_2 = DF = DE + EF \]

\[ OX_2 = c_2 \cdot S_1 + c_2 \cdot S_2 \]

\[ OX_2 = c_2 \cdot \frac{c_1}{2R} + c_2 \cdot \frac{c_2}{2R} \]
\[ OX_2 = \frac{c_2(c_1+c_2)}{2R} \]
\[ OX_3 = \frac{c_3(c_2+c_3)}{2R} \]
\[ OX_n = \frac{c_n(c_{n-1}+c_n)}{2R} \]

Generally,
\[ c_1 \text{ and } c_n \text{ are of different length, and all other chords } \]
\[ c_2 = c_3 = c_4 = \ldots \text{ and } c_{n-1} = 1 \text{ chain length} \]

\[ OX_1 = \frac{c_1^2}{2R} \]
\[ OX_2 = \frac{c(c+c)}{2R} \]
\[ OX_3 = \frac{c(c+c)}{2R} = \frac{c^2}{R} \]
\[ OX_3 = OX_4 = \ldots = OX_{n-1} \]
\[ OX_n = \frac{c_n(c+c_n)}{2R} \]
Rankine method:

1. One triangle method:

\[
\theta_1 = \frac{360 \cdot c_1}{2\pi R} \text{ degree}
\]

\[
\theta_1 = \frac{360 \cdot c_1 \times 60}{2\pi R} \text{ minute}
\]

\[
\theta_1 = 1718.9 \, \frac{c_1}{R} \text{ minute}
\]

\[
\theta_2 = 1718.9 \, \frac{c_2}{R} \text{ minute}
\]

\[
\theta_3 = 1718.9 \, \frac{c_3}{R} \text{ minute}
\]

\[
\theta_n = 1718.9 \, \frac{c_n}{R} \text{ minute}
\]
Total deflection angle: \\
for point B \\
\[ \Delta_1 = \delta_1 \]

From point C \\
\[ \Delta_2 = \delta_1 + \delta_2 \]

From point D \\
\[ \Delta_3 = \delta_1 + \delta_2 + \delta_3 \]

\[ \Delta_3 = \Delta_2 + \delta_3 \]

\[ \Delta_n = \Delta_{n-1} + \delta_n \]

(b) Two-hinge method: \\
\[ \]
By Bisection of chord =>

First mud coordinate

\[ C_0D = R(1 - \cos \frac{\Delta}{2}) \]

\[ C_1D_1 = R(1 - \cos \frac{\Delta}{4}) \]

\[ C_2D_2 = R(1 - \cos \frac{\Delta}{9}) \]
Problem: For a circular curve of radius 300m, calculate data required to set out a simple curve, if total deflection angle is 75°.

Use:
1. Perpendicular offset method.
2. Radial Offset
3. Offset from Long Chord.

Solution:

\[ VT_1 = VT_2 = R \tan \frac{D}{2} \]

\[ = 300 \times \tan \frac{75°}{2} \]

\[ = 230.10 \text{ m} \]

1. Perpendicular Offset

\[ O_x = R - \sqrt{R^2 - x^2} \]

\[ O_x = 300 - \sqrt{300^2 - 22^2} \]

\[ x = 0 \quad 20 \quad 40 \quad 60 \quad 80 \text{ m} \]

\[ O_x = 0 \quad 0.67 \quad 2.68 \quad 6.06 \]
2. Radial offset

\[ O_x = \sqrt{R^2 - x^2} - R \]

\[ O_x = \sqrt{300^2 - x^2} - 300 \]

<table>
<thead>
<tr>
<th>x (m)</th>
<th>0</th>
<th>30</th>
<th>40</th>
<th>60</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>O_x (m)</td>
<td>0</td>
<td>0.67</td>
<td>2.65</td>
<td>5.94</td>
<td>10.48</td>
</tr>
</tbody>
</table>

Using approximate formula:

<table>
<thead>
<tr>
<th>x (m)</th>
<th>0</th>
<th>30</th>
<th>40</th>
<th>60</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>O_x (m)</td>
<td>0.67</td>
<td>2.67</td>
<td>6.60</td>
<td>10.67</td>
<td>125</td>
</tr>
</tbody>
</table>

3. Offset from long chord:

Distance from mid ordinate = \( x \)

\[ O_x = \frac{CD - (R - \sqrt{R^2 - x^2})}{2} \]

\[ O_x = R(1 - \cos \theta) - (R - \sqrt{R^2 - x^2}) \]

\[ O_x = 61.99 - (300 - \sqrt{300^2 - x^2}) \]

\[ x \quad 0 \quad 20 \quad 40 \quad 60 \quad 80 \quad \text{(upto 100m)} \]

\[ O_x \quad 61.99 \quad 61.32 \quad 59.01 \]

\[ O_x = \sqrt{300^2 - x^2} - 238.01 \]
Problem: ES-1998 (S6)

Two tangent intercept at chainage
50+60 (50 chain and 60 links)
deflection angle being 60°, calculate the necessary
data to set out a circular curve of 30 chain
radius to connect the two tangents point by point with
offset from chords. Take peg interval equal to
100 links with length of chain being 20 m (100 links)

Solution:

C = All chord length except
1st and last = 100 links = 20 m (60)

C1 = 9
Cn = 2

Change of point of intersection
V = 50+60
V = 50 chains + 60 chains
V = 50 x 20 + 60 x 20
V = 1012 m
R = 20 chains
R = 20 x 20 = 400 m

Tangent length
VT1 = VT2 = R \tan \frac{\theta}{2}
\[ VT_1 = \frac{400 \tan \frac{60}{2}}{2} \]

\[ VT_1 = 235.62 \text{ m} \]

Length of curve

\[ \theta = \frac{2 \pi R}{360} \times \Delta \]

\[ \Delta = \frac{2 \pi x 400}{360} \times 61 \]

\[ \Delta = 425.86 \text{ m} \]

change of \( P_1 \) = chainage of \( V \) - \( VT_1 \)

\[ P_1 = 1042 - 235.62 \]

\[ P_1 = 776.38 \text{ m} \]

chainage of \( P_2 \):

\[ P_2 = \text{chainage of } P_1 + \Delta \]

\[ P_2 = 776.38 + 425.86 \]

\[ P_2 = 1202.24 \text{ m} \]

Exact multiple of 20 m

After \( P_1 = 1900 \text{ m} \)

So first chord \( c_1 = 780 - 776.38 \)

\[ c_1 = 3.62 \text{ m} \]

Exact multiple of 20 m

Before \( P_2 = 1200 \text{ m} \)

So last chord \( C_n = 1202.24 - 1200 = 2.24 \text{ m} \)
All other chord length

\[ c = 20 \text{ m} \]

Offsets from chord:

\[ OX_1 = \frac{c_1^2}{2R} = \frac{3.62^2}{2 \times 400} \]

\[ OX_1 = 0.016 \text{ m} \]

\[ OX_1 = 0.016 \text{ m} \]

\[ OX_2 = \frac{c (C_1 + C)}{2R} = \frac{20(362 + 20)}{2 \times 400} \]

\[ OX_2 = 0.59 \text{ m} \]

\[ OX_3 = OX_4 = \ldots = OX_{n-1} \]

\[ = \frac{c^2}{R} = \frac{20^2}{400} = 1 \text{ m} \]

\[ OX_n = \frac{c_n (C_n + C_{n-1})}{2R} \]

\[ OX_n = \frac{2.24 (20 + 2.24)}{2 \times 400} \]

\[ OX_n = 0.062 \text{ m} \]
Problem: 1990 6(b)

Two tangents intersect at Chiangai. If the deflection angle = 40°, compute the data for setting out a 900m radius curve by deflection angle and offset (chord). Take 30m chord length in general case.

Solution:

![Diagram with calculations]

All other chord length
\[ c = 30 \text{ m} \]
\[ c_1 = 8, c_0 = 2 \]

1. Radius \( R = 900 \text{ m} \)
   \[ \Delta = 40° \]

Tangent length
\[ VT_1 = VT_2 = R \tan \frac{\Delta}{2} \]
\[ = 900 \times \tan 20° \approx 185.59 \text{ m} \]

Length of curve
\[ L = \frac{2\pi R}{360} \times \Delta = \frac{2 \times 3.14 \times 900 \times 40}{360} \]
\[ L = 249.25 \text{ m} \]
chainage of tangent point $P_1$

$P_1 = \text{chainage} \; v - v_{P_1}$

$P_1 = 1200 - 145.59$

$\boxed{P_1 = 1054.41 \; \text{m}}$

chainage of tangent point $P_2$

$P_2 = \text{Chainage } P_1 + \ell$

$P_2 = 1054.41 + 249.25$

$\boxed{P_2 = 1333.66 \; \text{m}}$

First chord length

$S_1 = 30 \times 36 = 1080.00 \; \text{m}$

$C_1 = 1080 - 1054.41$

$C_1 = 25.59 \; \text{m}$

$C_n = 1333.66 - 1330 \rightarrow \frac{1333.66}{3030}$

$C_n = 13.66 \; \text{m}$

All other chord length $c = 30 \; \text{m}$

$S_1 = 1718.9 \times \frac{c}{2} = 1718.9 \times \frac{25.59}{400} = 119'58''$

$S_2 = 30'30'' - S_{n-1} = 119'9'9' \times C/12 = 119.9 \times \frac{30}{400} = 20'3'5.5''$

$S_n = 1718.9 \times \frac{C_n}{12} = 1718.9 \times \frac{13.66}{1200} = 0'58'42''$

points

$S \rightarrow \Delta$

$1.11'9'58'' \rightarrow 119'58''$

$2.3'59'53''$

$3.6'7'48''$

$4.5'4'55''$
Problem: Solve last moment
Solution: Loss of Prestress = 20.1,

\[ K = 1 - \frac{20}{100} \]
\[ K = 0.80 \]

Assume depth of slab (considered 1 m width)
\[ D = 500 \text{ mm} \]

Self weight = \( 0.50 \times 1 \times 1 \times 25 \)
\[ = 12.5 \text{ kN/m} \]

\[ M_d = \frac{wD^2}{8} = \frac{12.5 \times 12^2}{8} \]
\[ M_d = 225 \text{ kN-m} \]

Live load \( w_l = 20 \text{ kN/m} \)
\[ M_L = \frac{20 \times 12^2}{90} \]
\[ M_L = 360 \text{ kN-m} \]

1. Section modulus eq.
\[ Z = \frac{(1-K) M_d + M_L}{F_c} \]
\[ Z = \frac{(1 - 0.80) \times 225 \times 10^6 + 360 \times 10^6}{19} \]
\[ Z = 2.992985714 \text{ mm}^3 \]

Depth required
\[ D = \sqrt{\frac{6 \times 2.992985714}{13}} \]
\[ = 416.62 \text{ mm} \]
Self wt. = 0.43 x 1 x 1 x 25
= 10.5 kN/m

\[ \text{Md} = \frac{10.5 \times 12^2}{8} \]
\[ \text{Md} = 18.9 \text{ kN}\cdot\text{m} \]

2. Prestressing force
\[ P = \frac{\lambda \cdot F_c}{2k} = \frac{1000 \times 420 \times 14}{2 \times 0.80} \]
\[ P = 3675 \text{ kN} \]

No. of cable \[ \frac{3675}{225} = 16.33 \approx 17 \text{ nos} \]

3. Eccentricity
\[ e = \frac{(1+k) \text{Md} + ml}{2PK} \]
\[ e = \frac{(1+0.80) \times 18.9 \times 10^6 + 360 \times 10^6}{2 \times 0.80 \times 36.75 \times 10^3} \]
\[ e = 119 \text{ mm} \]
Problem: A post-tensioned prestressed concrete beam of rectangular section 240 mm wide, is to be designed for 25 kN/m live load U.D.L. over an effective span of 12 m. The stress in concrete $f_c$ 17 MPa in compression, 1.4 MPa in tension. Considered loss of prestress = 1.5.  

(1) Calculated minimum possible depth of beam.
(2) Calculated $f$ and $e$.

Solution: Assume depth of beam.

\[ D = 900 \text{ mm} \]

Self weight:

\[ w_0d = 0.8 \times 0.2 \times 1 \times 25 \]
\[ w_0d = 4.96 \text{ kN/m} \]

\[ m_0d = \frac{w_0d \cdot D^2}{2} \]
\[ m_0d = \frac{4.96 \times 900^2}{2} = 86.4 \text{ kN.m} \]

\[ M_1 = \frac{m_0d \cdot D^2}{2} = 25 \times 900^2 \]
\[ M_1 = 450 \text{ kN.m} \]

Section modulus required

\[ z = \frac{(1-K) M_0d + M_1}{(K_c - K_t)} \]
\[ z = \frac{(1-0.95) 86.4 + 450 \times 10^6}{K_c - 0.95 \times 17 - 1.4} \]
\[ z = 2980 \text{ kN.m}^2 \]
Depth required

\[ D = \sqrt{\frac{62.2}{98}} \]
\[ D = \sqrt{62.2 \times 0.37081232} \approx 8 \]
\[ D = 859.50 \text{ mm} \]

Considered \( D = 900 \text{ mm} \)

\( \omega_d = 0.24 \times 0.90 \times 25 \)
\[ \omega_d = 5.4 \]
\[ M_d = \frac{5.4 \times 12^2}{98} \]
\[ M_d = 97.2 \]
\[ 2 = 0.15 \times 97.2 \times 10^6 + 450 \times 10^6 \]
\[ (0.85 \times 17 + 1.4) \]
\[ 2 = 29311041 \text{ mm}^3 \]

Depth required

\[ D_2 = \sqrt{\frac{62.2}{98}} \]
\[ D_2 = 856.00 \text{ mm} \]

Take minimum possible depth = 860 mm

\( \omega_d = 0.24 \times 0.96125 \)
\[ \omega_d = 5.16 \]
\[ M_d = \frac{5.16 \times 12^2}{98} \]
\[ M_d = 92.80 \text{ kNm} \]
3. Proving Force \( P/e \)

\[
F_{\text{sup}} = \left( F_t - \frac{\text{md}}{2} \right)
\]

\[
F_{\text{sup}} = \left( -1.4 - \frac{93.88 \times 10^6 \times 6}{240 \times 960^2} \right)
\]

\[
F_{\text{sup}} = -4.56 \text{ N/mm}^2
\]

\[
F_{\text{emp}} = \frac{F_t + \text{md} + Ml}{K + Kz}
\]

\[
F_{\text{emp}} = \frac{-1.4 + \frac{(93.88 + 4.50) \times 10^6 \times 6}{0.85 \times 240 \times 960^2}}{0.85}
\]

\[
F_{\text{emp}} = 19.94 \text{ N/mm}^2
\]

4. Stressing Force

\[
p = \frac{A \left( F_{\text{emp}} + F_{\text{sup}} \right)}{2}
\]

\[
p = \frac{240 \times 960 \left( 4.56 + 19.94 \right)}{2}
\]

\[
p = 1597 \text{ kN}
\]

5. Asymmetry:

\[
e = \frac{2 \left( F_{\text{emp}} - F_{\text{sup}} \right)}{A \left( F_{\text{sup}} + F_{\text{emp}} \right)}
\]

\[
e = \frac{240 \times 960 \left( 19.94 - 4.56 \right)}{6 \times 240 \times 960 \left( 19.94 + 4.56 \right)}
\]

\[
e = 22.99 \text{ mm}
\]
Work Book
Chapter - 7

Q. 1 (a) (i) 
Q. 2 (a) 
Q. 3 (a) 
Q. 4 (c) 
Q. 5 (c) 
Q. 6 (b) 
Q. 7 (c) 
Q. 8 (b) 
Q. 9 (d) 
Q. 10 (b) 
Q. 11 (c) 
Q. 12 (d) 
Q. 13 (b) 
Q. 14 (b) 
Q. 15 (a) 
Q. 16 (b) 
Q. 17 (b) 
Q. 18 (a) 
Q. 19 (b) 
Q. 20 (c)
Equation 4:

Dead load:

\[ \frac{P}{M} + \frac{P_e}{2} = \frac{M}{l} = 0 \]

\[ P\left( \frac{1}{n} + \frac{e}{2} \right) = \frac{M}{n} = \frac{P\left( \frac{z+e}{2} \right)}{l^2} = \frac{M}{l} \]

\[ P = \frac{Pagn}{\lambda + \epsilon} = \frac{150 \times 300 \times 4.9 \times 10^5}{150 \times 300^2 \times \epsilon - 150 \times 300 \times 575} \]

\[ P = 392 \text{ kN} \]

Equation 5:

\[ \frac{200 \times 300 \times 2}{2} + \frac{200 \times 600 + (5+7)}{2} \]

\[ P = 370 \text{ kN} \]

Equation 6:

40mm

5 mm max. max. 8/12e of aggregate

Equation 7:

\[ V_D = 0.67 \times \beta_D \times \sqrt{P^2 + 0.06 \cdot F_{cp} \cdot f_t} + V_p \]

\[ f_t = 0.24 \sqrt{f_{ck}} = 0.24 \sqrt{205} = 1.61 \]

\[ F_{cp} = \frac{P}{\beta_D} = \frac{200 \times 10^3}{150 \times 300} = 0.09 \]

\[ V_p = 200 \times 8 \times d_{max} = 200 \times 8 \times \frac{78 \times 7}{6000} \]

\[ 200 \times 40 \times \frac{7}{2} = 75 \text{ kN} \]

\[ V_D = 0.67 \times 150 \times 300 \sqrt{(1.61)^2 + 0.06 \times 4.44 \times 1.61} + 7.5 \]

\[ V_D = 244.40 \text{ kN} \]
Case 1. Final $P = 500 \text{kN}$
\begin{align*}
\text{Losses} &= 15\% \\
\text{Initial} \ P_0 &= \frac{500}{0.85} = 588.24 \text{kN} \\
\epsilon &= 75 \\
\Rightarrow \frac{P}{A} + \frac{P_0}{2} &= \frac{588.24 \times 10^3}{230 \times 300} + \frac{588.24 \times 10^3 \times 75}{230 \times 300^2} \\
&= 7.84 + 11.75 \\
\text{Top} &= -3.92 \\
\text{Bottom} &= 19.60 \text{N/m}^2 \\
\end{align*}

**Case 2.** Load balancing concept is applied only for determining structure.

For an indeterminate structure like a continuous beam, concordant profile is used.

![Concordant Profile](image)

For an indeterminate structure, the continuous beam will possess a few Bernoulli's constant. This will reduce the calculation of load at different stages.

**Case 3.**

\[ P = 5000 \text{kN} \]
Problem: Two straight AB and BC meet in an inaccessible point B, and to be connected by a simple curve of R = 600 m. PQ points P and Q were selected on AB and BC such that ∠APQ = 130°, ∠CPQ = 160°, then PQ = 150m. Make necessary calculations to set-out curve by deflection angle method. Chainage of P = 160m. Take chord length = 30 m.

Solution:

\[ \Delta = 90° - (30° + 20°) = 50° \]

In Tangent: BPQ

Apply Sin formula

\[ \frac{150}{\sin 130°} = \frac{BP}{\sin 20°} \]

\[ BP = \frac{\sin 20°}{\sin 130°} \times 150 \]

\[ BP = 66.97 \text{ m} \]

Chainage of B = 160 + 66.97 m

= 1666.97 m
\[ R = 600 \text{ m} \]

\[ \Delta = 50^\circ \]

\[ VT_t = R \times \tan \Delta/2 = 600 \times \tan 25^\circ \]

\[ VT_t = 279.78 \text{ m} \]

Length of curve

\[ L = \frac{2 \pi R \times \Delta}{360} = \frac{2 \times \pi \times 600 \times 50}{360} \]

\[ L = 523.60 \text{ m} \]

Chainage of \( T_1 = 1666.97 - 279.78 \]
\[ T_1 = 1387.19 \text{ m} \]

Chainage of \( T_2 = 1387.19 + 523.60 \]
\[ T_2 = 1910.79 \text{ m} \]

First chord length

\[ C_t = 1410 - 1387.19 \text{ m} \]
\[ \boxed{C_t = 22.81 \text{ m}} \]

\[ C_n = 1910.79 - 1890 \]
\[ C_n = 20.79 \text{ m} \]
Points

<p>| | | | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1</td>
<td>29.81</td>
<td>1° 5' 21&quot;</td>
<td>1° 5' 21&quot;</td>
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<tr>
<td>2</td>
<td>30</td>
<td>1° 25' 57&quot;</td>
<td>2° 31' 18&quot;</td>
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<td>3</td>
<td>30</td>
<td>1° 25' 257&quot;</td>
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<tr>
<td>10</td>
<td>20.49</td>
<td>0° 59' 34&quot;</td>
<td>25°</td>
</tr>
</tbody>
</table>
Problem: Two tangents of a circular curve of 300 m have a deflection angle 90°. To change the position of forward tangent let through 30°, such that deflection is 110° (forward tangent it is rotated of tangency). Calculate the radius of (1) point of curve is not changed.
(2) point of tangency is not changed.
(3) Change of original point calculated change of important point.

Two cases.

Solution:

\[ R = 300 \text{ m} \]
\[ \Delta = 90° \]

Tangent length \( VT_1 = VT_2 = 30 \tan \frac{90°}{2} \]

Case 1: If point of curve is not changed.
The new point of intersection is in triangle \( VV, T_2 \)
Apply Sin

$$\frac{V_{V_1}}{V_{T_2}} = \tan \theta$$

$$\Rightarrow V_{V_1} = V_{T_2} \tan \theta$$

$$\Rightarrow V_{V_1} = 300 \tan \theta$$

$$\Rightarrow V_{V_1} = 109.19 \text{ m}$$

$$\frac{V_{\theta T_2}}{V_{1 T_2}} = \cos \theta$$

$$\Rightarrow V_{1 T_2} = \frac{V_{T_2}}{\cos \theta}$$

$$\Rightarrow V_{1 T_2} = \frac{300}{\cos \theta}$$

$$\Rightarrow V_{1 T_2} = 319.25 \text{ m}$$

New Tangent length

$$V_{1 T_1} = V_{T_1} + V_{V_1} = 300 + 109.19$$

$$V_{1 T_1} = 409.19 \text{ m}$$

$$V_{1 T_1} = V_{1 T_2}$$

$$R_1 \tan \frac{\theta}{2} = 409.19$$

$$\Rightarrow R_1 = \frac{409.19}{\tan \frac{\theta}{2}}$$

$$\Rightarrow R_1 = 226.51 \text{ m}$$
Chainage of $T_1 = 3340.8 - 300$

$= 2940.8 \text{ m}$

Chainage of $V_1 = 3240.8 + 189.19$

$= 3420.99 \text{ m}$

Length of curve

$$\frac{\pi R_1 x D_1}{360}$$

$$= \frac{2 \times \pi \times 286.51 \times 110}{360}$$

$= 550.06 \text{ m}$

Chainage of $T_2 = 2940.9 + 550.06 \text{ m}$

$= 3490.96 \text{ m}$

Case 2: If point of $T_1$ tangency $(T_2)$ is not changed.

$V_1 =$ New point of intersection

So new Tangent length $= V_1 T_2$

$V_1 T_2 = 319.25$

$V_1 T_2 = V_1 T_1$
\[ R_2 \tan \frac{\Delta_1}{2} = 319.25 \]
\[ R_2 = \frac{319.25}{\tan \frac{110}{2}} \]
\[ R_2 = 223.5 \text{ m} \]

change of \( T_1' = \text{change of} \ V_{1,2} = \ V_{1,T_1} \]
\[ = 3349.99 - 319.25 \]
\[ = 3030.74 \text{ m} \]

length of curve \( l = \frac{2\pi R_2 \times \Delta_1}{360} \)
\[ = \frac{2 \times \pi \times 223.54 \times 110}{360} \]
\[ = 429.16 \text{ m} \]

change of \( T_2 = 3030.74 + 429.16 \]
\[ = 3459.90 \text{ m} \]
\[ \text{Compound Curve} \]

Components:
1. \( R_8 \)
2. \( A_L \)
3. \( t_s \)
4. \( t_t \)
5. \( T_1 \)
6. \( T_2 \)
7. \( A_1 \)
8. \( A_2 \)
9. \( L = (A_1 + A_2) \)

Given values are:
1. \( A_1 \)
2. \( A_2 \)
3. \( R_3 \)
4. \( R_L \)

Find out:
1. \( t_s \)
2. \( t_t \)
3. \( t_s \)
4. \( T_1 \)

Solution:
\[ t_s = R_8 \tan \frac{A_1}{2} \]
\[ t_t = R_L \tan \frac{A_2}{2} \]

Value of \( R_8 + R_L \tan \frac{A_2}{2} \)

Value of \( R_8 + R_L \tan \frac{A_2}{2} \)

\[ \ldots \]
In triangle \( \triangle VAC \), apply Sine formula

\[
\frac{VA}{\sin \alpha_2} = \frac{VC}{\sin \alpha_1} = \frac{-AC}{\sin(180-\alpha)} = \frac{(-t_s + t_L)}{\sin \alpha}
\]

\[
\Rightarrow VA = \frac{\sin \alpha_2}{\sin \alpha} (-t_s + t_L) \quad \text{(147)}
\]

\[
\Rightarrow VC = \frac{\sin \alpha_1}{\sin \alpha} (-t_s + t_L)
\]

Total tangent length

\[
R_s = t_s + (t_s + t_L) \frac{\sin \alpha_2}{\sin \alpha}
\]

\[
T_L = t_L + (t_s + t_L) \frac{\sin \alpha_1}{\sin \alpha}
\]

Problem: The following data refer to a compound curve

Total deflection angle = 93°

Degree of first curve = 4

Degree of second curve = 5

Point of intersection = (15.61)

(20 m chain used)

Determine the running distance to tangent point and point of compound curve, given that the latter point is \( (8.36) \) from point of intersection at back angle of 90°3.6 from first tangent.

Solution:

\[
R_1 = \frac{1146}{40} = 28.65 \text{ m} = R_L
\]

\[
R_2 = \frac{1146}{50} = 229.2 \text{ m} = R_S
\]
chainage of P.L. = 45\times 20 + 61 \times 0.20 \\
= 912.20 m

\[
Y_B = (6+24) \\
= 6 \times 20 + 24 \times 0.20 \\
= 124.80 m
\]

\[
\angle A VB = 360 - 290.36' \\
\angle A VB = 69.24'
\]

\[
AB = 290.50 \text{ tan } \frac{\Delta 1}{2} \\
\text{Apply Sin Formula} \\
\frac{AB}{\sin 69.24'} = \frac{124.80}{\sin \angle 1}
\]
\[ \frac{286.50 \sin \frac{\Delta_1}{2}}{\sin 69.2^\circ \cos \frac{\Delta_1}{2}} = \frac{124.80}{\sin \Delta_1} \]

\[ \Rightarrow \frac{286.50 \sin \frac{\Delta_1}{2}}{\sin 69.2^\circ \cos \frac{\Delta_1}{2}} = \frac{124.80}{2 \sin \frac{\Delta_1}{2} \cos \frac{\Delta_1}{2}} \]

\[ \Rightarrow \sin^2 \frac{\Delta_1}{2} = \frac{124.80}{2} \times \frac{\sin 69.2^\circ}{286.50} \]

\[ \Delta_1 = 53.46159.25'' \]

\[ \Delta_2 = 93^\circ - \Delta_1 \]

\[ \Delta_2 = 93^\circ - 53.41' \]

\[ \Delta_2 = 39.19' \]

\[ T_S = t_S + (t_S + t_L) \]

\[ t_L = R \cdot \tan \frac{\Delta_1}{2} \]

\[ \Rightarrow t_L = 286.50 \times \tan \frac{53.41'}{2} \]

\[ \Rightarrow t_L = 144.98 m \]

\[ t_S = 239.20 \times \tan 39.19' \]

\[ t_S = 91.88 m \]

\[ \Rightarrow t_L + t_S = 236.85 m \]

\[ T_L = t_L + (t_S + t_L) \cdot \frac{\sin \Delta_2}{\sin \Delta} \]

\[ T_L = 144.98 + 236.86 \times \frac{\sin 39.19'}{\sin 93'} \]
\[ T_L = 288.92 \text{ m} \]

\[ T_S = 91.08 + 236.98 \times \frac{8111.53}{8100} \]

\[ T_S = 264.92 \text{ m} \]

**Length of curve**

\[ \mu = \frac{2\pi R L}{360} \times \Delta \phi = \frac{2\pi \times 396.50}{360} \times 58.411 \]

\[ \mu = 268.44 \text{ m} \]

\[ \Rightarrow I_2 = \frac{3\pi R S}{360} \times \Delta \phi = \frac{271 \times 229.20 \times 39.191}{360} \]

\[ \Rightarrow I_2 = 157.38 \text{ m} \]

**Chainage**

\[ V = 912.20 \]

\[ \Rightarrow -(T_L) = (\pm) 388.92 \]

**Chainage**

\[ T_1 = 628.20 \]

\[ + \mu = + 268.44 \]

**Chainage**

\[ 891.72 \]

\[ + I_2 = 157.38 \]

\[ \text{Total Chainage} = 1049 \text{ m} \]
First radius

Second radius

Point of reverse curve.

Point of intersection.

Line O₁H is parallel to T₁T₂

O₁F ⊥ T₁T₂

O₂H ⊥ T₁T₂

∠ T₁O₁F = \( \delta_1 \)

∠ T₂O₂G₁ = \( \delta_2 \)

∠ EOF = \( \Delta_1 - \delta_1 \)

∠ EOF = \( \Delta_2 - \delta_2 \)
Relation: →

1. \( S_1 = \Delta + S_2 \)
   \[ S_0 = \Delta = d_1 - d_2 \]

2. \( S_0 = \Delta = \Delta_1 - \Delta_2 \)

3. \( \Delta = S_1 - S_2 = \Delta_1 - \Delta_2 \)

4. \( \Delta_2 - S_2 = \Delta_1 - S_1 \)

\[ T_1 F = R_1 \sin \theta_1 \]
\[ T_2 G_1 = R_2 \sin \theta_2 \]
\[ F G_1 = O_1 H = O_1 O_2 \sin (\Delta_2 - \Delta_2) \]
\[ F G_1 = (R_1 + R_2) \sin (\Delta_2 - \Delta_2) \]

Total Length

\[ \Rightarrow T_1 + T_2 = l = T_1 F + F G_1 + T_2 G_1 \]

\[ \Rightarrow T_1 + T_2 = R_1 \sin \theta_1 + R_2 \sin \theta_2 + (R_1 + R_2) \sin \theta_2 \]

\[ \Rightarrow l = R_1 \sin \theta_1 + R_2 \sin \theta_2 + (R_1 + R_2) \sin (\Delta_2 - \Delta_2) \]

Tangent Length: →

Apply sine formula

\[ \frac{\sqrt{T_1}}{\sin \delta_2} = \frac{\sqrt{T_2}}{\sin (180 - \delta_1)} = \frac{T_1 T_2 \cdot \sin \Delta}{\sin \Delta} \]
\[ V T_1 = \frac{\sin \delta_2}{\sin \delta} \cdot \lambda \]

\[ V T_2 = \frac{\sin \delta_1}{\sin \delta} \cdot \lambda \]

\[ \Rightarrow O_1 F = R_1 \cos \delta_1 \]

\[ \Rightarrow O_2 G_1 = R_2 \cos \delta_2 \]

\[ \Rightarrow O_2 H = (R_1 + R_2) \cos (\delta_2 - \delta_1) \]

\[ \Rightarrow O_2 H = O_1 F + O_2 G_1 \]

\[ \Rightarrow O_2 H = R_1 \cos \delta_1 + R_2 \cos \delta_2 \]

\[ \Rightarrow \cos (\delta_2 - \delta_1) = \frac{R_1 \cos \delta_1 + R_2 \cos \delta_2}{(R_1 + R_2)} \]

\[ \Rightarrow \cos (\delta_2 - \delta_1) = \cos (\delta_1 - \delta) \]
Astronomy:

1. The celestial sphere: The imaginary sphere on which all heavenly bodies like stars, sun, moon, planets, etc., appear to us is called the celestial sphere.

2. Zenith: The point on the celestial sphere exactly above the observer (opposite to the center of the earth) is called the zenith point.

3. Nadir point: The point on the celestial sphere exactly below the observer's position is called the nadir point.

4. Zenith, nadir line: The line joining the zenith and nadir point.

5. Pole: On the celestial sphere, if NP/SP of Earth is extended, we get two points called poles.
   - NCP = North Celestial pole.
   - SCP = South Celestial pole.
6. Celestial horizon: The great circle (that is passing through the centre as centre of earth) perpendicular to zenith nadir line is called celestial horizon.

7. Sensible horizon: It is a circle having centre observer's position. Their centre is an exact perpendicular to zenith nadir line. This circle is parallel to celestial horizon at radius (R = radius of earth).

8. Terrestrial Equator: Equator on earth's surface.

9. Celestial Equator: Equator extended on celestial sphere (great circle) is called celestial equator. It is perpendicular to North and South poles than it is not change.

10. Vertical circle: All those great circles passing through zenith and nadir point are called vertical circle.

11. Observer's meridian: The great (vertical) circle that is passing through North/South pole is called observer's meridian.

12. Prime vertical: The great circle (vertical) passing through zenith/nadir points and perpendicular to observer's meridians is called prime vertical.

13. North and South points: Junction of observer's meridian with celestial horizon are North and South point.
14. East/West point: The junction of prime vertical with celestial horizon are earth/West points.

15. Ecliptic: The great circle of celestial sphere on which the Sun appears to lie in its movement in one year is called ecliptic w.r.t. to earth as centre.

16. First point of Aries / First point of Libra:

\begin{align*}
\text{(Vernal) (equinox)} & \quad \text{(Astral) (equinox)}
\end{align*}
3. Independent Equatorial System: =>
   (Declination and Right Ascension System): =>

Two point of reference

1. Pole
2. Equator

Angle one measure with respect to

1. Equator:
2. First point of aries

- Declination: => Angle above or below equator, along declination circle is called declination.
- Right Ascension: => Angle measured along equator, from the point of aries towards east.
3. Co-deculation angle from pole to star = 90 - δ

4. Dependent equatorial system (Declination and hour angle)
No point of reference:

1. Pole
2. South point

Two planes:

1. Equator
2. Observer's meridian point (for South)

Angle measured:

1. Declination - same as above.
2. Hour angle: measured along equator from south to forward west.
3. Co-declination: (90 - δ) same as above.
Celestial latitude and longitude system:
Terrestrial latitude and longitude

Two point of reference

1. NP of ecliptic
2. First point of Aries
Two planes:
1. Ecliptic
2. Plane passing through first point of Aries

Two angles:
1. Celestial latitude: Angle above or below Ecliptic
2. Celestial longitude: Angle measured along ecliptic starting from first point of Aries.

As per this system, latitude of Earth is always zero.

* Spherical Triangle: *

Angles:
1. Angle between two planes
2. Angle between any two edges formed at center of sphere.

Properties:
1. Any angle is less than 2 x 90 (π)
2. 180 ≤ A + B + C ≤ 540
3. **Sum of any two side > third side**
   \[ a + d > c \]
   \[ a + c > b \]
   \[ a - c > b \]

4. **If sum of any two side = 180°**
   - If \( a + b = 180° \)
   - **Sum of opposite two angle = 180°**
   - \( A + B = 180° \)

5. **The smaller angle is opposite the smaller side**

   **Formula** →

   1. \( \frac{\sin A}{\sin A} = \frac{\sin B}{\sin B} = \frac{\sin C}{\sin C} \)
   2. \( \cos A = \frac{\cos A - \cos B \cdot \cos C}{\sin B \cdot \sin C} \)
   \( \cos A = \cos B \cdot \cos C + \sin B \cdot \sin C \cdot \cos A \)
   3. \( \cos A = \frac{\cos A + \cos B \cdot \cos C}{\sin B \cdot \sin C} \)
   \( \cos A = -\cos B \cdot \cos C + \sin B \cdot \sin C \cdot \cos A \)

   **Napier's Rule** → This is for a right-angled spherical triangle.

   **If any one angle = 90°** (A, B, or C),
   (not side)
\( \alpha, C, B, A, C \)  
\[ \begin{aligned} \alpha & \downarrow (90 - b) \\ \downarrow & \downarrow \downarrow c \\ (90 - c) & (90 - A) \end{aligned} \]

(i) \( \sin c = \tan(90 - A) \tan a = \cot A \tan a \)

(ii) \( \sin c = \cos(90 - B) \cos(90 - C) = \sec B \sec C \)

\[ \begin{aligned} \sin (\text{middle}) & = \tan (\text{adjacent}_1) \times \tan (\text{adjacent}_2) \\ \sin (\text{middle}) & = \cos (\text{opposite}_1) \times \cos (\text{opposite}_2) \end{aligned} \]

(iii) \( \sin(90 - b) = \tan(90 - C) \tan(90 - A) \)
\[ \begin{aligned} \cos b & = \cot C \cot A \end{aligned} \]

(iv) \( \sin(90 - b) = \cos a \cos c \)
\[ \begin{aligned} \cos b & = \cos a \cos c \end{aligned} \]
Total 10 such equations can be formed.

Spherical Excesses formed

By

\[ E = (A + B + C) - 180 \]

Area of Spherical Triangle

\[ A = \frac{\pi R^2 \times E}{180} \]
Different position of Star \( \omega \) to observer's world

1. Star at Elongation:

\[ \text{Angle} = (90 - \alpha), \theta, (90 - \alpha)^\prime, H, (90 - \omega) \]
\[ \sin (\text{middle}) = \tan (\psi_1) \times \tan (\alpha_{52}) \]

\[ \sin (\text{middle}) = \cos (\psi_1) \times \cos (\psi_2) \]

2. **Star of culmination:**

\[ m_1 \quad \text{and} \quad m_2 \]

\[ \downarrow \quad \text{upper} \quad \text{culmination} \]

\[ \downarrow \quad \text{lower} \quad \text{culmination} \]

3. **Star at prime vertical:**

\[ (90-x) \quad (90-\psi) \quad (90-M) \quad (90-O) \]

Apply Napier's rules.
\[ \sin (\text{middle}) = \frac{\tan (\alpha_1)}{\tan (\alpha_2)} \cdot \frac{\cos (\theta_1)}{\cos (\theta_2)} \]

4) Star at Horizon: 

5) Circumpolar Star: Book
Problem: ES-1994

Find the shortest distance between two points A and B on an earth surface, given that latitude of A and B are 15°N and 12°N, and their longitudes are 50°E and 54°E. R = 6370 km.

Solution:

\[ a = 90' - 12.6' = 77.54' \]

\[ b = 90' - 15' = 75' \]

\[ p = 54' - 50.12' \]

\[ p = \frac{24.92}{3.92} = 6.36' \]

\[ p = q \]

\[ \cos p = \frac{\cos a - \cos b \cdot \cos \gamma}{\sin a \sin b} \]

\[ \cos p = \cos a \cos b + \sin a \sin b \cdot \cos \gamma \]

\[ \cos p = \cos 77.54' \cos 75' + \sin 77.54' \sin 75' \cos 6.36' \]

\[ \Rightarrow \cos p = 4.41' 11.6'' \]
Distance \( AB = \frac{2\pi R}{360} \times p \)

\[ = \frac{2\pi \times 6.370 \times 4.\overline{4}11.46'}{360} \]

\[ = 522.1 \text{ km} \]

\text{Ans}

**Problem:** A star having a declination of 50° N how it upper transit in the zenith of the place. Find the altitude of star at its lower transit.

**Solution:**

![Diagram of celestial objects and calculations]
Declination at upper transit

\[ N_1 = 90 - 50 = 40 \]

So \( 2p = 90 - 50 = 40 \)°

at lower transit \( \text{PM}_2 = 40 \)°

Altitude of star at lower transit = \( H_2 M_2 = 2H_2 - 2p - PM_2 \)

\[ H_2 M_2 = 90 - 40 - 40 \]

\[ \boxed{H_2 M_2 = 10 \text{°}} \]

Ans

Problem: Determine the altitude and azimuth of a star if:

1. Declination of the star = 25°30′N
2. Hour angle of star = 15°15′
3. Latitude of observer = 52°N

Solution:
In triangle p2m,

\[ Z = 90 - d = 90 - 35.30' \]

\[ Z = 64.30'. \]

\[ p = 45.15' \]

\[ m = 90 - d = 90 - 35.30' \]

\[ m = 54.30'. \]

\[
\cos P = \frac{\cos p - \cos m \cos Z}{\sin m \sin Z}
\]

\[
\cos p = \cos m \cos Z + \sin m \sin Z \cos P
\]

\[
\cos P = \cos 35.30' \cos 64.30' + \sin 35.30' \sin 64.30' \times 10^{-15}
\]

\[ P = 43.48' = 90 - \phi \]

Altitude \( \phi \):

\[ 490 - 43.48' \]

\[ \phi = 46.55' 30" \]
\[
\cos Z = \frac{\cos \beta - \cos \phi \cos p}{\sin \phi \sin p}
\]

\[
\cos Z = \frac{\cos 64'30'' - \cos 43'48'' \cos 93'0'30''}{\sin 43'48'' \sin 43'0'30''}
\]

\[A = Z = 110' 11' 14''\]

\[\text{Problem: Calculate the Sun's azimuth and hour angle at sunset, at a place of latitude 43'30' when the declination is 25'30'}^tN.\]

\[\text{Solution:}\]

\[\text{At sunset, the angle of Sun is 90'.}\]

\[2\pi = 90 - \delta = 90 - 25'30' = 64'30' = \delta\]
\[ \theta = \text{Latitude} = 42^\circ 30' \]
\[ \phi = 90 - \theta = 90 - 42^\circ 30' = 47^\circ 30' = M \]
\[ \cos \theta = \cos \phi = \frac{\cos \phi - \cos \theta \cdot \cos \phi}{\sin \theta \cdot \sin \phi} \]
\[ A = \frac{\cos 64^\circ 30' - \cos 72^\circ 30' \cdot \cos 90}{\sin 47^\circ 30' \cdot \sin 90} \]
\[ A = 54^\circ 16' 30'' \]
\[ \cos \phi = \frac{\cos \phi - \cos \theta \cdot \cos \phi}{\sin \theta \cdot \sin \phi} \]
\[ \cos \phi = \frac{\cos 90 - \cos 72^\circ 30' \cdot \cos 64^\circ 30'}{\sin 47^\circ 30' \cdot \sin 64^\circ 30'} \]
\[ p = 115^\circ 55' 1'' \]
\[ H \cdot A = 360 - 115^\circ 55' 1'' \]
\[ H \cdot A = 244^\circ 44' 59'' \]
Time:

1. Express following angle into time

\[ 0^\circ 5^\prime 30^\prime\prime \times 3 \] 174

\[ 0^\circ 5^\prime = \frac{0.5}{15} = 5 \text{ hr or } 10 \text{ min } 0 \text{ sec} \]

\[ 30^\prime = \frac{30}{15} = 2 \text{ min } 0 \text{ sec} \]

\[ 41^\prime\prime = \frac{41}{15} = 0 \text{ hr } 0 \text{ min } 3 \text{ sec} \]

\[ 5 \text{ hr or } 42 \text{ min } 3 \text{ sec} \]

2. 19 hr, 26 min, 46.8 sec. Convert into angle

\[ 19 \text{ hr } = 19 \times 15 = 270^\circ 0^\prime 0^\prime\prime \]

\[ 26 \text{ min } = \frac{26 \times 15}{60} = 6^\circ 30^\prime 0^\prime\prime \]

\[ 46.8 \text{ sec } = \frac{46.8 \times 15}{60} = 0^\circ 11^\prime 30^\prime\prime \]

\[ 276^\circ 41^\prime 30^\prime\prime \]
Day = Night = 21 September
Day = Night = 21 March
North east direction = Summer
South east direction = Rainy, Winter

Winter solstice (21 Dec. Night is longest)

First point of Libra
First point of Aries
The end