HAND WRITTEN NOTES:

OF

CIVIL ENGINEERING

SUBJECT:

STRUCTURAL ANALYSIS
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Structure Analysis

T F S  G N A T F

5

8 to 10 marks

Obj  Conv.

20-25 question  50-60 Marks

Nature of Qns.  Easy to score

Type of Subject  Conceptual

History of Teaching  50
Syllabus

- Indeterminancy & Stability → obj
- Influence line Dia & Effect of → obj + conj
- Rolling load
- Arches → obj

- Method of Analysis → Strain Energy
  \[ \{ \text{moment dist. method} \quad \text{obj + conj.} \]
  \[ \{ \text{slope deflection method} \quad \text{Matrix Method} \]

- Trusses → Determine
  \[ \{ \text{obj + conj} \]
  \[ \{ \text{undetermined} \]

Ref. Book

1. Theory of st. → by S. Ramamurtham → Strain Energy
  \[ \{ \text{Moment dist.} \]
  \[ \{ \text{Trusses} \]

2. Str. Analysis → Slope defn
  \[ \{ \text{I.S. Negi & Tandig} \]
  \[ \{ \text{strain energy} \]
  \[ \{ \text{TLD} \]

3. Str. Analysis → Indeterminacy & Stability
  \[ \{ \text{Gupta & Pandey} \]
  \[ \{ \text{matrix method} \]

4. Structure Analysis → R.C. Hibbler → Basic Concept
Indeterminacy & Stability of Structure

Stability

\[ \downarrow \]

External

\[ \downarrow \]

Related to the Support

Internal

\[ \downarrow \]

Related to the Shape

Condition

of Str.

External Stability:

Large displacement of the entire structure at the supports and joint is not permitted. However, small elastic deformation may occur at free ends. In movement rigid body displacement there should be enough reaction at the supports. At these arrangement should be adequate for plane structure (2-D), there should be min. of 3 independent reactions at the support which should be:

1. Non parasite
2. Non-concurrent
3. Non-Trivial

Displacement if reaction is small which may permit large \( \theta \) that it is called trivial.

\( \text{eqn} \) spring support with small stiffness
Note: All the support reaction are (11) than linear unstability will occur Reaction are parallel.

Linear unstability

If all reaction are Concurrent than angular unstability will occur.

Angular / Rotational unstability

For stability of 3D body (Plane is x-y & loading plane) there should be loading plane in

1) x-direction ($Ax = 0$)
2) y-direction ($Ay = 0$)
3) about z-axis ($\theta = 0$)
To prove that above displacement following condition of stable equilibrium should be studied.

1. \( \Sigma F_x = 0 \)
2. \( \Sigma F_y = 0 \)
3. \( \Sigma M_z = 0 \)

For 3D structure (Space structure):

There should be no displacement. There should no rigid displacement in following direction.

\( (\text{ax} = 0, \text{by} = 0, \text{cz} = 0, \text{ax} = 0, \text{ay} = 0, \text{az} = 0) \)

In order to prevent rigid body displacement for static equilibrium of space structure following 6 - condition should be satisfied:

- \( \Sigma F_x = 0 \)
- \( \Sigma F_y = 0 \)
- \( \Sigma F_z = 0 \)
- \( \Sigma M_x = 0 \)
- \( \Sigma M_y = 0 \)
- \( \Sigma M_z = 0 \)

Internal Stability:

No part of the structure should move relative to the other part, show as to pressure. Give to different pressure shape of structure. However small elastic deformation may occur due to presence of local geometric enough number of members and appropriate.
Arrangement is required. There should be no formation of condition of mechanism.

Mechanism means rotation collapse which may be complete or partial.

Mechanism may occur 3 - Colinear in a row hence rotation is permitted.

Diagram showing:
- Mechanism/Collapse
- Partial Collapse

Internal stability also called Geometric Stability.

In Truss Geometric Stability occurs due to deflection, deficiency may off member and inadequate arrangement.

Diagram showing:
- Incomplete (Geometrically)
- Complete (Internal)}
Overall stability:

For overall stability, external stability is complex. In some cases, overall stability may occur when the structure is externally over stiff and internally is unstable. That is, extra external reaction may prevent internal deformation.

No. of External Reactions = 4

Internally, it is deficient to 81st order but overall & externally, it is over stiff. To 1st order, it is stable.

Externally, deficient to 1st order but internally, over stiff to 1st order. But overall, it is unstable.
It is desirable to have internal and external stability both for overall stability.

Ext. → stable
Int. → stable
Overall → stable

Stable & over stiff.

External - Stable
Internal - unstable
(Because left panel is deficient)

Overall - unstable
Unstable because all reaction are concurrent.

In 3D structure, for external stability, there should be a minimum 6-reaction, which should be non-coplanar, non-concurrent, non-trivial.

Indeterminacy:

\[ D_s = D_{se} + D_{si} \]
Static indeterminacy: These structures which can be analyzed by using condition of static equilibrium alone are called statically determinate structure. But if the number of equations are not enough to compute all the reactions, then it is called statically indeterminate / hyperstatic / redundancy structure.

1. External Static indeterminacy

   It is related to the support condition. If all the support reactions can be computed by using condition of static equilibrium alone, then the structure is statically determinate externally.

   Let \( n_e \) be the total number of external supports.

   Reaction of all joint (No. of independent reactions)

\[
\begin{align*}
HA & \rightarrow \text{A} \\
& \uparrow R_A \\
& \downarrow R_B \\
\text{At} & = 3 \\
HA \rightarrow & \text{A} \\
& \downarrow R_A \\
& \uparrow V_B \\
V_B &= R_B \cos \theta \\
W_{V_B} &= R_B \sin \theta \\
\left( \frac{R_B}{V_B} = \tan \theta \right) \Rightarrow \text{depend}.
\end{align*}
\]
\[ D_{se} = 2e - 3 \quad \text{for} \quad 2D \text{ structure} \]
\[ = 2e - 6 \quad \text{for} \quad 3D \text{ structure} \]

<table>
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<th>Type of support</th>
<th>No. of Reaction</th>
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<td>1. Fix support</td>
<td>HA</td>
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<tr>
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<td>RA</td>
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<td>MA</td>
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<tr>
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<td>( Pe = 3 )</td>
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<tr>
<td>2. Hinge support</td>
<td>HA</td>
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<td></td>
<td>RA</td>
</tr>
<tr>
<td></td>
<td>( Pe = 2 )</td>
</tr>
<tr>
<td>3. Roller support</td>
<td>HA</td>
</tr>
<tr>
<td></td>
<td>RA</td>
</tr>
<tr>
<td></td>
<td>( Pe = 1 )</td>
</tr>
<tr>
<td>4. Guided Roller support</td>
<td>HA</td>
</tr>
<tr>
<td></td>
<td>RA</td>
</tr>
<tr>
<td></td>
<td>MA</td>
</tr>
<tr>
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<td>( Pe = 2 )</td>
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Types of support

1. Fix support / Builtin support
2. Hinge support / Ball & socket joint
3. Roller support

No. of reaction:

\[ \begin{align*}
\text{No. of reaction} & = 3 \\
Rx & = 3 \\
My & = 1 \\
Mz & = 1
\end{align*} \]

Note: Only normal to plane
Ex. Find external static indeterminancy for the structure shown in figure.

![Structure Diagram]

Total \( M_e = 3 + 1 + 3 = 7 \)
Total equilibrium condition = 3

\[ D_{se} = M_e - 3 = 7 - 5 \times 3 = 4 \]

When loading is not mentioned then general loading is considered which may have a horizontal and vertical component both therefore at the support horizontal & vertical & moment will be available.

If only vertical load is present then in frame, arches and trusses a horizontal component of zero will be present also moment depending upon support condition.

But in beams due to vertical loading alone horizontal reactions will not be present therefore to determine \( D_{se} \), horizontal reaction may be ingored and equation for horizontal forces may be ingored.
Equations of F.m = 0

\[ \sigma_e = 5 \]

\[ \sigma_e - 2 = 5 - 2 = 3 \]

\[ \delta_e = 3 + 3 = 6 \]

\[ \delta_e = 6 - 8 = 3 \]

\[ \tau_e = 6 + 3 + 6 + 3 = 18 \]

\[ \delta_e = \tau e - 6 \]

\[ \tau e - 6 = 12 \]
In Truss, all joints are hinge, loading is applied only at joints, and self-weight of member is ignored. Hence, all member carries axial force; there fore each member has only one internal reaction, that is, axial force.

If there are \( m \) members than total no. of internal reactions, let there are \( j \) no. of joints. At each joint there are \( 2 \) equations are equilibrium. \( (\Sigma F_x = 0) \)

\( (\Sigma F_y = 0) \)

Total no. of equilibrium equation at all joint \( \rightarrow 2j \)

Out of which \( 2j - 3 \) equations are utilized to determine external support reaction. Hence net available equation to find internal forces \( \rightarrow (2j - 3) \)

\[ D_{si} = \text{Total No. of internal Reaction} - \text{Total available equation for int. reaction} \]

\[ D_{si} = m - (2j - 3) \]
If $D_{si} = 0$ \{internally determinate\}

$D_{si} > 0$ Truss is indeterminate \{Redundant\}
oven stiff

$D_{si} < 0$ internally unstable \{Deficient\}

\section*{Case-2 3-D Truss | Space Truss}

In each member of Truss there is only one internal force (axial force) but at each joint there are 3 equations \{(F_{Ex} = 0, F_{Ey} = 0, and F_{Ez} = 0\}) hence total \# eqn. of all joint $3J$.

Out of which 6 equations are use for external support reaction hence net available equation to two find internal reaction = $(3J - 6)$

$$D_{si} = m - (3J - 6)$$

If $D_{si} = 0$ Truss is internally determinate.
Case 3. Rigid Frame (2-D, 3-D) →

In rigid frames, internal indeterminacy will not exist. If structure has open configuration-like a truss, beams are always internally determinate. To check internal indeterminacy, following thumb rule can be applied:

\[ 2 \]

Rule 1: If structure is internally determinate, then it is not possible to provide a cut anywhere on the structure without splitting the structure into two parts.

If structure is statically determinate, it is impossible to return back at any point without the path it mean in internally determinate structure there will be no cycle loops if internal hinges, links, shear-fissure etc.
2-D rigid frame-

In each member 2-D rigid member there will be 3 internal reaction (Rx, Ry, and Rz) or axial force, shear force and bending moment.

if there is no close loop all the above three internal reaction can be found by providing it cut any section at stabilise free body equilibrium for either left or right position.

\[ Efx = 0, \text{ } Efy = 0, \text{ } \& \text{ } EMz = 0 \]

but if there are close loop then each close loop will have three internal indeterminacy. Hence if No. of close loops are less than total degree of internal indeterminacy

\[ Dsl = 3c \]

\[ Dsl = M - (3J - 6) \]

\[ \left( \frac{\text{No. of close loop}}{\text{No. of cuts required to convert close struture}} \right) \]

\[ \text{Into an open structure} \]
Note: If all joints are not rigid that is sum of the joint are hyloxide due to presence of internal hinges, internal collar, shear collar than some of the internal reaction are released. No. of internal reaction released upon type of joint and no. of member meeting at the joint.

No. of inst reaction replaced at A ⇒

\[ \frac{\delta R_i}{\delta R_i} = 0 \]

A.F. = 0

B.M. = 0

Fig. at B = 1
i.e. A.F. at B = 0

Fig. at C = 1 (see at (-c))
At D: B.m = 0
\[ s_{D} = 1 \]

If more than 2 members meting at internal hinges:

Number of released reaction will be:
\[ 3n - (m' - 1) \]

And:
\[ m' = \text{No. of members meeting at internal hinges} \]

If 3D structure is internal hinges then:

\[ s_{D} = (m' - 1) \]
\[ s_{A.B} = (3 - 1) = 2 \]

Dei = 3C - 9s_{D}

\[ s_{D} = \text{Total No. of Internal Reaction released at Hybrid Joint} \]
3D - Rigid Frames

At each close loops, there are
6 internal if no. of close loops = total no. of
internal indeterminacy

\[ D_i = 6C \]

and if sum of the
joints are hybrid

\[ D_i = 6C - 6r \]

for 2D Internal
Hinge. \( r_{11} = 6 \text{ (m-1)} \)

\[ m = \text{No. of members meeting} \]

at hybrid joints.

Overall static indeterminacy: -

1st method

\[ D_s = D_e + D_i \]

\[ D_s = 0 \]

Structure is determinate. Provided are
Non-parallel, Non-concurrent, Non-self
and there is no condition of mechanism

\[ D_s > 0 \]

stable & indeterminate (overstiff &

Desirable

indeterminate str. are more economical

than determinate because thinner

section is required.

\[ D_s < 0 \]

Unstable structure.

Method 2

\[ (2 \text{-} \text{3 Euler})' = \]

\[ D_s = \text{Total reaction (Internal} \]

\[ \text{reaction} \]

\[ - \text{Total Eqn equatin.} \]

\[ D_s = m + ne - 2j \]
3. 3D Truss:

\[ D_S = m + 9k - 3J \]

2. 2D rigid frame/beams:

\[ D_S = \text{Total reaction} - \text{Total equation at All joints} \]

\[ D_S = (3m + 9e - 3J) \]

When all joints are rigid.

\[ D_S = (3m + 9e - 3J) - 9r_i \]

When one of the joints is hybrid.

4. 3D rigid frames/beams:

\[ D_S = (6m + 3e - 6J) \]

When all joints are rigid.

\[ D_S = (6m + 3e - 6J) - 9r_i \]

When some joints are hybrid.

\[ \text{total No. of released reaction} = 9r_i \]
Q. Determine $P_s$ for plane truss shown in figure.

1st method:

$$D_{ce} = 3e - 3$$

$$\Rightarrow (2 + 2 + 2) - 3$$

$$D_{ce} = 6 - 3 = 3$$

$$D_{si} = m - (2 + 3)$$

$$\Rightarrow 4 - (2 + 4 - 3)$$

$$\Rightarrow 4 - 5 \Rightarrow 1$$

$$P_s = D_{se} + D_{si}$$

$$\Rightarrow 3 - 1 = 2$$

2nd method:

$$D_s = m + 3e - 2f$$

$$\Rightarrow 4 + 6 - 2 \times 4$$

$$\Rightarrow 10 - 8 = 2$$
For 2D frame shown in Fig. Find ES

1st method:

\[ g_1e = 2 + 1 = 3 \]

\[ Dse = g_1e - 3 \]

\[ \Rightarrow 3 - 3 = 0 \]

\[ Dsi = 3(9x4 - 1) \Rightarrow 12 - 1 = 11 \]

\[ Ds = Dse + Dsi \]

\[ \Rightarrow 0 + 11 \Rightarrow 11 \]

2nd method:

\[ Ds = 3m + 3e - 3f - 9x1 \]

\[ \Rightarrow 3x1 + 3 - 3x10 - 1 \]

\[ \Rightarrow 39 - 30 - 1 \]

\[ \Rightarrow 11 \]
for 2d rigid frame shown in fig. Find D3

\[ \text{Hinge } g_{21} = 3(2-1) = 3 \]

\[ \text{Hinge } g_{21} = 3(2-1) = 6 \]

\[ \text{Method } 1 \]

\[ D_{se} = 3e - 6 \]
\[ = 18 - 6 = 12 \]

\[ D_{sl} = 6c - 32a \]
\[ \Rightarrow 6 \times 1 - (3 + 3) \]
\[ \Rightarrow 6 - 6 = 0 \]

\[ D_c = D_{se} + D_{sl} = 12 - 3 = 9 \]

\[ \text{2nd Method} \]

\[ D_c = 6m + 3e - 6J - 32a \]
\[ = 6 \times 9 + 18 - 3 \times 9 - 9 \]
\[ = 9 \]
q. for 2D rigid frame find $D_s$?

Frst method:

$g_{le} = 3 + 2 + 3 + 3 = 12$

$D_{sc} = g_{le} - 3$

$= 12 - 3 = 9$

$D_{si} = 36 - 9l_{g}$

$= 36 - 2 = 2$

$D_s = D_{sc} + D_{si} = 9 + 1 = 10$

2nd method:

$D_s = 3m + g_{le} - 3J - g_{i}$

$= 3 \times 16 + 12 - 3 \times 15 - 5$

$= 48 + 12 - 45 - 5$

$= 10$
Find $D_s$ shown in Figure.

1st method:

$$re = 3 + 1 + 2 = 6$$
$$dse = 9 - re = 9 - 6 = 3$$
$$ds1 = 3 - dse = 3 - 3 = 0$$

$$D_s = dse + ds1 = 3 + 0 = 3$$

2nd Method:

$$D_s = 3m + 9 - re - 3.5$$
$$= 3 	imes 8 + 6 - 3 	imes 8$$
$$= 18 + 6 - 24$$
$$= 6$$

Kinematic indeterminacy: "Kinematic Indeterminacy (DI) is referred to the total No. of available degree of freedom at all joint."

or

Kinematic indeterminacy (DI) = Total No. of unanalyzed displacement component at all joints."
A fully locked structure all joint kinematically determinate because it has no degree of freedom at any joints.

A fixed beam at both end is kinematically determinate (fully locked) but statically indeterminate (3rd order).

A  

B

Notes: The displacement at joints are elastic displacement. If a member is in extreme inextensible, then in any direction, then displacement in that dirn will not be available at the joint.

<table>
<thead>
<tr>
<th>Joint Type</th>
<th>No. of Displacement</th>
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| ① 2D rigid joint | 3:  
\[ \text{Ax} \]  
\[ \text{Ay} \]  
\[ \text{Az} \] at joint B |
| ② 2D True joint/pin joint | 2:  
\[ \text{Ax} \]  
\[ \text{Ay} \] at joint B |
(3) 3D Rigid joint

(4) 3D Truss joint

kinematic degree of determinacy:

(a) For 2D plane truss:

\[ \Delta x = 2J - \delta_e \]

At each joint, there may be two displacement \( \Delta x, \Delta y \) out of which some of the joints are supported. In the direction of support reaction displacement are not permitted. Hence, net available displacement is available degree of freedom.

23 - \( \delta_e \)
For 3D Truss:

\[ D_k = 3J - 9k_e \]

2D Rigid Frame:

\[ D_k = 3k - 3J - 3k_e \]

Note: If some of the joint are hybrid, some of the internal reaction are reduced degree of freedom will increased. Hence \( D_k \) will be

\[ D_k = 3J - k_e + r_f \]

\( r_f \) = Reduces due to hybrid joint

Some of the members of the member are axial rigid than due to axially rigidly linear axial displacement in the direction of member at some of the joint will not be available, hence degree of freedom will get reduced.

Let \( m \) axial displacement are not available at joint due to axial rigid

\[ D_k = 3J - k_e + r_f - m \]
\[ D_k = 3J - xe \]
\[ \Rightarrow 3 \times 2 - 4 = 2 \]
\[ DBx \]

\[ D_k = 3J - xe \]
\[ = 3 \times 2 - 5 \Rightarrow 1 \to DB \]

It AB is Axially Rigid

\[ DBx = 0 \]

\[ \theta_k = 3J - xe - m \]
\[ \Rightarrow 3 \times 2 - 4 - 1 \]
\[ \Rightarrow a \to DB \]

Because in this case, DB is not available. Hence, Dk will remain 1. Hence the member is axially rigid.

3D Rigid Frame

\[ D_k = 6J - xe - \theta_k \]

\[ \text{when all joints are rigid} \]
\[ D_k = 6J - xe + \theta_k \]

\[ \text{when some of the joints are rigid} \]

\[ \text{and } D_k = 6J - xe + \theta_k - m \]

If some of the members are axially rigid

\[ m = \text{No. of Axial displacement prevent due to axial rigidity} \]
Degree of freedom at hybrid joint:

\[ A_1 = \text{Axial displacement} \]
\[ A_2 = \text{Shear displacement} \]
\[ A_3 = \theta B_1 \text{ for BA at B} \]
\[ B_4 = \theta B_2 \text{ for BC at B} \]

\[ A_{1-3} \text{ and } A_2 \text{ are Axial displacement} \]
\[ A_3 = \text{Shear displacement} \]
\[ B_4 = \theta \text{ Angle displacement} \]
Find degree of kinematic indeterminacy & represent all displacement at joints.

\[ D_k = 3 \cdot J - 3 \cdot \text{re} \]
\[ 3 \times 10 - 6 = 24 \]

For the rigid frame shown in Figure find \( D_k \) of indeterminacy assuming beams as axial rigid.

\[ J_e = 8 + 3 + 2 = 8 \]

\[ D_k = 3J - 3J_e + 6 \cdot 3^0 - 2 \]
\[ = 3 \times 9 - 8 - 1 \]
\[ D_k = 27 - 12 = 15 \]
Note: In an elastic frame, if connected by rigid bracing than all the braced joint linear displacement will not be permitted, (Ax=0, Ay=0) member are elastic there are rigid braces.
INFLUENCE LINE Diagram

Note: 1) In a simply supported beam or girder if a concentrated load moves to the right maximum b.m. occurs always below the position of load. And in a fixed beam if a concentrated load moves the maximum b.m. always occurs at the support.

2) The distance between two adjacent point of contraflexure is called focal length.

3) The portion of a beam or girder in which shear force is constant is known as shear span.
Influence line diagrams represent variations of the stress function such as reaction, shearing force, bending moment, slope or deflection at a point when unit concentrated load moves from one end to the other end.

Note: ILD for reaction, SIF and bending moment in determinate beam is linear whereas for indeterminate beam it is non-linear.

2. The dimension of ordinate of ILD for reaction and SIF is dimensionless whereas as bending moment dimension of ILD is meter.

Application of Influence Line:

6. ILD can be used to find position of live load which will produce max. volume value of a stress function. (load should be placed at that point whose ordinate of its is max).

Max. (+ve) SF at C will occur when load is placed just to the left of C.
ILD can be used to study the effect of moving load on the C structure.

ILD can be used to find total value of a stress function for a given load system in.

Ex.

\[ \text{ILD for } M_c \]

Total B.M. at C due to given loading

\[ \Rightarrow (w \times \text{Area of ILD below } J1) + p_1 y_1 + p_2 y_2 \]

\[ M_c \Rightarrow (w \times A + p_1 y_1 + p_2 y_2) \]

Muller Breslau Principle

(Applicability for determinate & indeterminate)

It gives qualitative influence line dia. to a stress function (reactions, of & B.M.)

"The ILD for any stress function in a structure is represented by its deflected shape obtained by removing the constraints offered by that stress function & introducing a directly related generalized displacement in the form that stress function."

Muller Breslau Principle (Applicability for determinate & indeterminate)
"Influence line dia. any stress function can be obtained by given unit displacement in a form of that stress function and removing that stress function the resultant shape of beam I LD"

Eq. I LD for Re in a simply supported beam

![Diagram of a simply supported beam with loads and reactions](image)

MPL for Re

I LD for Re can be obtained by removing Re and giving unit displacement of B in the direction of Re. The resultant deflection shape of beam will be I LD:

![Diagram showing the deflection shape](image)
4. Using 1st principal draw influence line diagram for RP, RB, SC & MC for simply supported beam span L as shown in figure.
To find $R_A$, take $S = N_b = 0$

$$R_A = 1 - 1(l-x) = 0$$

$$R_A = \frac{(l-x)}{1}$$

when 1 kN load is at A ($x = 0$)

$R_A = 1$
When 1 KN load is at RB \((x=a)\)

\[ R_B = 0 \]

Eld for RB:

Take \( EMA = 0 \)

\[ R_B x 1 - 1 \cdot x = 0 \]

\[ R_B = \frac{x}{5} \]

When \( x = 0 \), \( R_B = 0 \)

When \( x = 1 \), \( R_B = 1 \)

* SDL for \( S_B \)

Case - I when unit load is on the left of C

\[ S_C = -R_B = -\frac{x}{a} \quad 0 \leq x \leq a \]

\[ S_C = 0 \] when 1 KN load is at a

\[ S_C = -\frac{a}{2} \] when 1 KN load is just to left of C

Case - II when unit load is on the right of C

\[ S_C = +R_B = \frac{l-x}{k} \quad a < x < l \]

\[ S_C = \frac{l-a}{l} \] at \( x = a \)

\[ S_C = 0 \] at \( x = l \)

at \( x = 0 \) \( S_C = 0 \)
Case I: when unit load is in AC

\[ M_c = Ra(x) \]

\[ M_c = \frac{(1-x)}{2} \quad 0 \leq x \leq a \]

If \( x = 0 \) (unit load is at A)

\[ M_c = 0 \]

If \( x = a \)

\[ M_c = a \left( \frac{1}{2} - a \right) \]

Find for MC:

when unit load is in CB

\[ M_c = Ra \cdot a \]

\[ \Rightarrow \left( \frac{1-x}{2} \right) a \quad a \leq x \leq l \]

If \( x = a \)

\[ M_c = \frac{(1-a)}{2} a \]

If \( x = l \), \( M_c = 0 \)

Note: \( M_c \) for a overhanging beam can be obtained by linear extension of \( M_c \) for simply supported beam.
Draw Influence Dia. for RA, RB and RC and AB and also find max. value of above stress function
when a live load of 10kN/m moves left to right which may have any length so as to produce max. value of stress function.
R_n will be maximum when only span AB is loaded

\[ R_{\text{max}} = \text{loading rate} \times \text{area of } \frac{1}{2} \times 10 \times 1 \]

\[ \Rightarrow 100 \text{ kN} \]

loading in BC produces \( - \)ive. Reaction at A.

If entire span is \( - \)ive, ABC is loaded, then net reaction at A will be:

\[ R_{\text{net}} = 10 \times [\text{Net area of } \frac{1}{2} x 20 x 1 - \frac{1}{2} x 10 x 0.2] \]

\[ \Rightarrow 10 \times [10] = 90 \text{ kN} \]

Maxim Reaction at B occurs when load is present on AB.

\[ R_{\text{max}} \Rightarrow 10 \times \frac{1}{2} \times 30 \times 1.2 \]

\[ \Rightarrow 180 \text{ kN} \]

\[ R_{\text{max}} \text{ will occur when at least span BC is loaded.} \]

\[ R_{\text{max}} \Rightarrow 10 \times 1 \times 6.5 \times 1 \]

\[ \Rightarrow 30 \text{ kN} \]
\[
\text{Max} M \text{ at } B \text{ will be } - \text{ves} \text{ i.e. hogging}
\]

when at least span BC is loaded.

\[
M_{\text{max}} = 10 \times \frac{1}{2} \times 4 \times 10
\]

\[
\Rightarrow -200 \text{ kN-m}
\]

Draw a SLD for simply supported girder of length \( L \) for \( RA = RB = SC = MC \) when a unit concentrated moment moves from \( A \) to \( B \) as shown in fig.
\[ \varepsilon MB = 0 \]
\[ RA - 1 + 1 = 0 \]
\[ RA = \frac{1}{l} \]
\[ \varepsilon f_y = 0 \]
\[ RA + RB = 0 \]
\[ RB = \frac{+1}{l} \]

\text{ILD for Sc:}

\[ Sc = -\frac{1}{l} \]

\text{ILD for Mc:}

\text{Case 1: when 1 KN-m to the left of C}
\[ Mc = Ra \times (l - a) \]
\[ Mc = \frac{1}{l} (l - a) \Rightarrow \left( \frac{l - a}{l} \right) \]

\text{Case 1: when 1 KN-m is to the right of C}
\[ Mc = Ra \cdot a \]
\[ \Rightarrow -\frac{1}{l} a \Rightarrow -\frac{a}{l} \]
A cantilever beam AB is supported over a spring as shown in Fig. Draw influence line since fig. for Re and MA when unit concentrated load moves from A to C.

\[ \text{Let internal reaction at } B \text{ is } R_b \]

Downward deflection at B for \( AB = \text{defl}_B \text{ of spring} \)

\[ \frac{1}{3EI} x^3 + \frac{1}{2EI} (1-x) - \frac{R_b L^3}{3EI} = \frac{R_b}{K} \]

\[ \frac{x^3}{3EI} + \frac{x^2}{2EI} \cdot \frac{L-x}{2EI} - \frac{R_b L^3}{3EI} = \frac{R_b}{K} \]

\[ R_b = \frac{4x^2(3L-x)}{9L^3} \]
when \( x = 0 \) \( \rightarrow \) \( Re = 0 \)

\( x = 1 \) \( \rightarrow \) \( Re = \frac{8}{9} \)

\( 8 \frac{1}{3} \) ELD for \( Re \)

ILD for \( MA \)

\( MA = \frac{\pi}{2} \cdot (1 - x) \)

\( \frac{4x^3}{9L^3} (3L-x) - x \)

\( \frac{4ux^2}{9L^2} (3L-x) - x \)

\( \frac{4x^2}{9} (3L-x) - x \)

\( MA = 0 \quad x = 0 \)

\( MA = -\frac{4L}{9} \quad x = 1 \)

\( MA = -\frac{2L}{9} \quad x = \frac{1}{2} \)
Effect of Rolling load:

Case-1

Find maximum shear force and B.M. at a section c when a UDL moves over the girder having its length less than length of girder.

Max. (negative) shear force at c will occur when
- Head of the load is just to c, load moving left to right.

Max. (positive) S.F. at c will occur when
tail of the load is just to c, load moving right to left.

Max. b.m. at c will occur average leading to the left of c is equal to average load. Ring left of c. It means section c devise the load in this same ratio as it devise to the span.
Let length of load is 1'

\[ \frac{c}{a'} = 1 - (a_1 - a) \]

\[ a' = \frac{a_1 - a}{1 - a} \]

Note 1: The absolute maximum s. f. will occur at support B when load is just to the left of B.

Max. (time) s. f. occur at A when load is just to be right of A.

(2) due to moving UDL absolute maxm B.M. will occur at the centre of span when avg. loading equal on both side of center mean load is symmetrically placed at but the centre.

Case 2: when a series of concrete wheel load crosses a simply supported girders than ascertainable bending moment occur at the center of span maxm bending moment any section C occur when loading is placed such that average leading two the left of section C is equal to average leading to the right of section C.
\[
\begin{align*}
\text{due to series of wheel load absolute max bending moment occurs below any one of the wheel load and never between two wheel load when the position of load such that centre of the span is located midway but C.G. of load system and load in consideration load under consideration is either that load 2 which is nearest to C.G. in that w_1 or that load which 8 next nearest to C.G. (w_2)}
\end{align*}
\]

If \( w_3 > w_2 \) where \( w_3 \) is nearest load to C.G. than it is certain that max bending moment will occur below \( w_2 \) but if \( w_2 > w_3 \) and \( w_2 \) is nearest to load to C.G. than max bending may occur either below \( w_2 \) or below \( w_3 \) which even is greater.

If C.G. of load system (\( c_1 \)) consider with any of the wheel load say \( w_3 \) than absolute max bending moment will occur \( w_3 \) when \( w_3 \) is located just over centre.
A series of 3 wheel load 5 tonne, 9 and 16 tonne spanning 3 m center to center cross over a simply supported girder with a span of 10 m. When load moves left to right with 6 tonne load leading than find position and magnitude of max. B.M. which may occur anywhere on the girder (that is means absolute max. value.)

\[ 5t = W_3 \]
\[ 9t = W_2 \]
\[ 6t = W_1 \]

Distance of C.G. of loading system \( W \),

\[ \bar{x} = \frac{W_1 \bar{x}_1 + W_2 \bar{x}_2 + W_3 \bar{x}_3}{W_1 + W_2 + W_3} \]

\[ \bar{x} = \frac{6 \times 0 + 9 \times 3 + 5 \times 6}{5 + 9 + 6} \]

\[ \bar{x} = 2.85 \text{ m} \]
The road nearest to \( a_1 \) is \( W_a \), which is higher than \( W_1 \), which is next nearest to \( C-a_1 \) there. For max B.M. will occur below \( W_2 \) when Centre of Span \( C \) is located mid-way between \( W_1 \) and \( W_2 \). Consideration (10)

\[
\begin{align*}
\text{RA} + \text{RA} &= 6 + 5 + 9 \\
\text{RA} + \text{RA} &= 20 \quad \text{(1)}
\end{align*}
\]

\[
\Sigma E A = 0 \\
R A \times 10 - 6 \times 7.925 - 9 \times 4.925 = 5 \times 1.925 = 0
\]

\[
\begin{align*}
R_A &= 10.15 \text{ ft} \\
R_A &= 9.85 \text{ ft}
\end{align*}
\]

B.M. below \( W_2 \)

\[
R A X 4.925 - 5 \times 3 = 30
\]

\[
\overline{R} = 9.85 \times 4.925 - 15 = 0
\]

\[
M_{max} = 33.5 \text{ ft-lb} \quad \text{occur} \quad 4.925 \text{ m from } A
\]
Method 2: Use of R.L.B.

The forces for R.L.B below W3 are:

\[ \frac{y_1}{y_2} = \frac{1.925}{5.075} \Rightarrow y_1 = 1.02 \]

Total B.M. below W3 is:

\[ \sum (c_1 y_1 + c_2 y_2 + c_3 y_3) = 6 \times 1.02 + 9 \times 2.499 + 5 \times 0.976 \]

\[ = 33.5 \text{ ft-m} \]

5. Point loads of 10 kN, 20 kN, 12 kN, 16 kN, and 20 kN spaced at 5 m centre-to-centre are to be applied over a simply supported girder of 80 m. The load moves left to right. With 20 kN load leading, then calculate position and magnitude of first max B.M. which may occur anywhere on the girder.
$x = 20 \times 0 + 16 \times 5 + 12 \times 10 + 18 \times 15 + 10 \times 20$

$= 20 + 16 + 12 + 15 + 10$

$x = 8.286 \text{ m}$

$w_3$ is nearest to $c_0$ but $w_2$ is less than $w_3$ but where $w_3$ is next in nearest to $c_0$. Hence, absolute max will occur either below $w_3$ or below $w_0$ which ever gives greater value.

Case 1: Consider max bending moment occurs at $w_3$.

The centre span should lie at c.o. of load system. Local under consideration $w_3$.

Diagram showing load distribution and bending moments at various sections.
\[ RA + RB = 20 + 10 + 12 + 12 + 10 = 70 \]

\[ \Sigma M = 0 \]

\[ RB \times 80 = 20 \times 49.143 - 16 \times 44.143 - 12 \times 39.143 - 12 \times 34.143 \]
\[-10 \times 39.143 = 0 \]

\[ RB = 25.753 \text{ KN} \]
\[ RA = 70 - 25.753 = 34.25 \text{ KN} \]

B.M. below \( W_2 \)

\[ \rightarrow RA \times 39.143 - 10 \times 10 - 12 \times 5 \]
\[ = 34.25 \times 39.143 - 100 - 60 \]

B.M. below \( W_1 \)

\[ = 1180.65 \text{ KN-m} \]

**Method 2:** Consider max. B.M. occur below \( W_2 \)

Hence, centre of span to be located between G. of Local System and \( W_2 \).
\[ y_2 = \frac{a(1-a)}{2} \rightarrow y_2 = 1.643 \times (30-4.1649) \]
\[ y_2 = 1.643 \times 25.837 \]
\[ y_2 = 42.966 \]

\[ y_1 = 33.357 \]
\[ y_2 = 38.357 \Rightarrow y_1 = 17.367 \]

\[ y_3 = 36.643 \Rightarrow y_3 = 17.568 \]
\[ y_2 = 41.649 \]

\[ y_4 = 15.1715 \]
\[ y_5 = 12.774 \]

**Total B.M. below w_2**

\[ w_1 y_1 + w_2 y_2 + w_3 y_3 + w_4 y_4 \]
\[ = 20 \times 17.367 + 16 \times 19.966 + 12 \times 17.568 + 12 \times 15.1715 \]
\[ + 10 \times 12.774 \]
\[ = 1187.33 \text{ kN} \cdot \text{m} \]

The absolute bending moment is greater of the above 3 values i.e. 1187.33 kN.m which occur at a distance 41.643 m from a fixed 16 kN (w_2)
1. Load \( L \times \frac{1}{2} \times 16 \times 8 \)
   \[ \Rightarrow 2 \times 24 \]
   \[ = 48 \text{ m} \]

2. \( y_2 = \frac{18 \times c}{24} = 4.5 \)
   \[ y_1 = \frac{y_1}{y_2} \Rightarrow \frac{y_1}{6} = 3 \]

3. \( M_c = 3x y_1 + 4x y_2 + 5x y_3 + 6x y_4 \)
   \[ M_c = 3x(18) + 4x(4.5) + 5x(37.5) + 6x(25) \]
   \[ \approx 262.25 \]

4. \( y_3 = \frac{16}{10} \)
   \[ \frac{16}{10} \times 4.5 = 3.75 \]

5. \( y_4 = \frac{16}{8} \times 3.75 = 6.75 \)
\[ y_3 = 4.5 \]

\[ y_1 = \frac{1}{2} \times 4.5 \times 6 = 0.75 \]

\[ y_2 = \frac{3}{6} 

y_2 = 2.25 \]

\[ y_4 = 14 \times 4.5 = 3.5 \]

\[ \frac{18}{18} \]

\[ M = 3 \times 0.75 + 2.25 \times 4 + 5 \times 4.5 + 6 \times 3.5 \]

\[ M = 54.75 \]

indeterminate the TD Not (linear) than A

\[ A \]
17

\[ MA = R \cdot l - 1 \cdot x \]

\[ \Rightarrow f(x) = l - x \]

\[ \Rightarrow = \left[ x - \frac{1}{n}(x) \cdot l \right] \]

19

20

G(i)

Only C statement is correct and all others are wrong.
Text on the page:

Than determine structure and cry.

\[ R = \frac{150 \times 5 + 100 \times 5}{150 \times 100} = \frac{500}{250} = 2 \]

\[ R_R = 150 \]

\[ R_A = 100 \]

\[ M_{max} = 1 \times R_A \times 4 \]

\[ = 100 \times 4 = 400 \text{ kN-m} \]

I Ud for Reaction of R
Case II
\[150 \times 1.5 + 150 \times 1\]
\[= 300\ \text{max}\ 0\text{m}\]

Case III
\[150 \times 1.5 - 125\ 0\text{m}\]

Case IV
\[-150 \times 0.3 + 100 \times 0.2\]
\[= -25\ \text{m}\]

Then, 150 kN out of G.M. than,
\[-100 \times 0.2 = -30\]

\[150 \times 5 = 750\]

\[P = 25\]

\[\alpha = 150^\circ\]

Diagram:

1. \[\angle KRA\]
2. \[\angle LRA\]
3. \[\angle LRA\]
4. \[\angle LBA\]
5. \[\angle LBA\]

\[S_b = RA - 1\]

\[BA + x - 1 (L-x) = 0\]

\[RA = \frac{L-x}{L}\]

\[S_b = RA + 1\]

\[= L \frac{L-2}{L} - 1 - x - \frac{x}{2}\]

\[\alpha + S_b = 1\]
Arches

3-Hinged  2-Hinged  Fixed

A & B support Hinges  Indeterminate to  Indeterminate

determinate or
Abutment Hinge
or
Springing

Arches structure able to resist S.F., B.M. & Axial
Thrust all as compare to simply supported beam
of equal span upon under equal load free hinged
arch requires smaller section because in Arch.
Horizontal reactions are developed which reduce
the B.M. because B.M. produce by vertical reaction
is sagging and B.M. produce by horizontal reaction
is hogging.
BM at x-x in Beam = RA \cdot x

BM at x-x in Arch = -RA \cdot x - HA \cdot y

Arch Moment = Beam Moment + H Moment

Generally for long span arch structure are economical because net BM moment developed in arches is less than net BM developed in beam hence thinner section is need however for small span beams are preferred.

In case of multi story structure and from aesthetic consideration arches are less preferred.
Analysis of three hinged arches.

Total reaction for total equilibrium.

Total Reaction = 4

Total Equation = 3 + 1

\[ \Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M_z = 0 \quad M_e = 0 \]

\[ \psi = 0 \quad \text{stable & determinate} \]

If there is no horizontal load, then \( \Sigma F_x = 0 \)

\[ H_A - H_B = 0 \]

\[ H_A = H_B = H \text{ (say)} \]

Following 3 types of forces are developed.
1. B.M. at $x-x$

$$M_x = R_A \cdot x - H_A \cdot y$$

2. $s.f. \ at \ x-x /$ Radial Shear

$$S_x = U_x \cdot \cos \theta - H_x \cdot \sin \theta$$

3. Normal Thrust / Axial thrust ($N_x$) $x-x$

$$N_x = V_x \cdot \sin \theta + H_x \cdot \cos \theta$$

$$\tan \theta = \frac{dy}{dx}$$

For Parabolic Arch

$$y = 4h \left( \frac{l-x}{l} \right)$$

$$\frac{dy}{dz} = \tan \theta = \frac{4h}{l^2} \left( 1 - 2x \right)$$
To compute $V_x$ & $H_x$ at $X-X$ consider free body equilibrium on both left of $X-X$

$\sum F_x = 0$

$H_A - H_x = 0$

$[H_A = H_x]$ (23)

$\sum F_y = 0$

$R_A - V_x = 0$

$[R_A = V_x]$

Case-1. Three Hinge Parabolic Arch subjected to UDL over entire span.

$H_A = H$

$R_A$

$H_B = H$

$R_B$

$\sum F_x = 0$

$H_A - H_B = 0$

$H_A = H_B = H$

$\sum F_y = 0$

$R_A + R_B - wL = 0$ - (ii)
\[ \Sigma MB = 0 \]

\[ R_A x 1 - \omega x 1 \times \frac{L}{2} = 0 \]

\[ R_A = \frac{\omega L}{2} \]

\[ R_B = \frac{\omega L}{2} \]

Taking moment about C:

\[ R_A \cdot \frac{l}{2} - H \cdot \frac{h}{2} - \omega \cdot \frac{L}{2} \cdot \frac{l}{4} = 0 \]

\[ \frac{\omega L^2}{4} - H \cdot \frac{h}{2} - \frac{\omega L^2}{8} = 0 \]

\[ H = \frac{\omega L^2}{8h} \]

\[ y = \frac{y_h}{l^2} x(1-x) \]

\[ H = \frac{\omega L^2}{8h} \]

\[ R_A = \frac{\omega L}{2} \]
\[ M_x = R_d \cdot x - H \cdot y - \frac{w \cdot x \cdot x}{2} \]

Put the value

\[ M_x = \frac{w \cdot l \cdot x - w \cdot l^2}{2} \cdot \frac{x(1-x) - \omega \cdot x^2}{8} \]

\[ M_x = \frac{w \cdot l \cdot x - w \cdot l^2}{2} \cdot \frac{x^2}{2} - \frac{\omega \cdot l^2}{8} \]

\[ M_x = 0 \]

"I'm on a Hinge Parabolic Arch Subjected to u.d.l. over entire span Bending moment at any section is zero." (M_x = 0)

\[ \varepsilon_{F_x} = 0 \]

\[ H - H_x = 0 \]

\[ H = H_x = \frac{w \cdot l^2}{8} \]

\[ \varepsilon_{F_y} = 0 \]

\[ R_q = -\omega \cdot x \cdot V_x = 0 \]

\[ V_x = R - \omega \cdot x = \frac{w \cdot l}{2} - \omega \cdot x \]

Radial Shear SF

\[ S_x = V_x \cdot \cos \theta - H_x \cdot \sin \theta \]

\[ = \left( \frac{\omega l}{2} - \omega \cdot x \right) \cdot \cos \theta - \frac{w \cdot l}{8} \cdot \sin \theta \cdot \sin \theta \]
\[ S_x = \cos \theta \cdot \omega \left[ \frac{1}{2} - x - \frac{l^2}{8h} \tan \omega \right] \]

\[ y = \frac{4h}{l^2} x(x - l) \]

\[ \frac{dy}{dx} = \tan \omega = \frac{4h}{l^2} (l - 2x) \]

\[ S_x = \cos \theta \cdot \omega \left[ \frac{1}{2} - x - \frac{l^2}{8h} \tan \omega \right] \]

\[ S_x = \cos \theta \cdot \omega \left[ \frac{1}{2} - x - \frac{l^2}{8h} \right] \]

\[ S_x = 0 \]

"it means 3-Hinge Parabola arch with all on with entire span is free from S.F and B.M."

\[ S_x \text{ and } M_x = 0 \]

However, Normal Thrust will exist

\[ \text{Case 2: 3-Hinge semi-circle Arch Subject to pull over entire span.} \]

\[ w/l \]

\[ Ha = H \rightarrow R \]

\[ R_a = R \]

\[ H = R \]
\[ EFy \]
\[ R_A + R_B = -w \cdot 2R = 0 \]
\[ R_A + R_B = 2wR \]

\[ \Sigma MA = 0 \]
\[ R_B \times 2R - w \times 2R \cdot R = 0 \]
\[ R_B = wR \]
\[ R_A = wR \]

moment about A \[ M_C = 0 \]
\[ R_A \times R - H \cdot R - wR \cdot R = 0 \]
\[ wR \cdot R - HR - \frac{wR^2}{2} = 0 \]
\[ HR = \frac{wR^2}{2} - \frac{wR^2}{2} = wR^2 \]

\[ H = \frac{wR}{2} \]

Horizontal reaction is \( \frac{1}{2} \) of vertical reaction.

BM of \( x-x \):
\[ M_x = R_A \cdot x - H \cdot \frac{y}{2} - wR \cdot R \cdot \frac{x}{2} \]

\[ \frac{y}{R} = \sin \theta \]
\[ \theta = \theta \sin \theta \]
\[ R \cdot x = \cos \theta \]
\[ R - x = R \cos \theta \]
\[ x = R (1 - \cos \theta) \]

\[ M_x = R \rho A_x \cdot x - H y \cdot \omega \cdot x \cdot \frac{x}{2} \]

\[ = \omega R \cdot R (1 - \cos \theta) - \omega R (R \sin \theta) \cdot \frac{\omega R^2 (1 - \cos \theta)^2}{2} \]

\[ M_x = \omega R^2 \left[ 1 - \cos \theta - \sin \theta \cdot \frac{1}{2} (1 - \cos \theta)^2 \right] \]

\[ = \omega R^2 \left[ 1 - \cos \theta - \sin \theta \cdot \frac{1}{2} (1 - \cos \theta)^2 + \cos \theta \right] \]

\[ M_x = -\omega R^2 \left[ \frac{1}{2} \sin \theta - \frac{\cos^2 \theta}{2} \right] \]

\[ M_x = \frac{\omega R^2}{2} \left[ \sin \theta (1 - \sin \theta - \cos^2 \theta) \right] \]

\[ M_x = \frac{\omega R^2}{2} \left[ \sin^2 \theta - \sin \theta \right] \]

\[ M_x = \frac{\omega R^2}{2} \left[ \sin \theta - \sin^2 \theta \right] \]

for \( M_{max} \) \( \delta y = 0 \)
\( \delta \theta \)
\[ = \left[ \cos \theta - 2 \sin \theta - \cos \theta \right] = 0 \]
\[ \cos \theta (1 - 2 \sin \theta) = 0 \]
\[ \sin \theta = \frac{1}{2} \quad \text{or} \quad \cos \theta = 0 \]
\[ \theta = 30^\circ \quad \text{or} \quad \theta = 90^\circ \]
\[ M_{\text{max}} = -\frac{wR^2}{2} \left[ \frac{1}{2} - \frac{1}{a} \right] \]

\[ M_{\text{max}} = -\frac{wR^2}{8} \]

\[ \frac{wR^2}{8} \]

\[ \frac{wR^2}{18} \]

\[ \frac{1}{2} \]

\[ \frac{1}{2} \]

\[ \frac{wR}{2} \]

\[ \frac{wR}{2} \]

\[ \frac{1}{2} \]

\[ \frac{wR^2}{18} \]

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\[ \frac{wR}{2} \]
For a semi-circular arch, subject to a concentrated load \( W \) at the crown, find horizontal thrust \( R \) and S.F. and B.M.

\[ \Sigma F_y = 0 \quad RA + RB - W = 0 \]

\[ \Sigma M_A = 0 \]

\[ RB \times 2R - wKR = 0 \]

\[ RB = \frac{w}{2} \quad RA = \frac{w}{2} \]

\[ M_c = 0 \quad RA \times R - H \cdot Q = 0 \]

\[ \frac{w}{2} \times R - H \cdot R = 0 \]

\[ H = \frac{w}{R^2} \]

\[ M_x = RA \times x - Hy \]

\[ = \frac{w}{2} (x - R) \]

\[ M_x = \frac{w}{2} \left[ R(1 - \cos \theta) - Rs \sin \theta \right] \]
\[ M_x = \frac{wr}{2} \begin{bmatrix} 1 - \cos \theta & -\sin \theta 
\end{bmatrix} \]

At \( \theta = 0 \), \( M_x = 0 \)
At \( \theta = \pi/2 \), \( M_x = 0 \)

\[ \frac{dM_x}{d\theta} = \frac{wr}{2} \begin{bmatrix} 0 & 1 + \sin \theta \cos \theta \n\end{bmatrix} = 0 \]

\[ \Rightarrow \sin \theta - \cos \theta = 0 \]
\[ \Rightarrow \theta = 45^\circ \]

\[ M_{\text{max}} = \frac{wr}{2} \begin{bmatrix} 1 - \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} 
\end{bmatrix} \]

\[ \Rightarrow -\frac{wr}{2} \left( -\frac{1}{\sqrt{2}} - 1 \right) \]

\[ \frac{wr}{2} (\sqrt{2} - 1) \]

\[ \frac{wr}{2} \]
Temperature effect on 3-hinged Arches:

\[ \Delta H = \left( \frac{1^2 + 4h^2}{4h} \right) \alpha T \quad \text{Free Expn in Rise of Arch} \]

\[ \Delta = L \alpha T \]

\[ \Delta = 0 \quad \sigma = E \alpha T \]
A 3-hinged Arch is a determinate structure and due to rise in temp., vertical rise \((h)\) well. Increase by \(nh\) which is called free expansion.

There will be two cases.

Case-1: If Arch is initially unloaded but when horizontal and vertical reaction are not present, then due to rise in temp., vertical height will increase by \(nh\) but horizontal and vertical reaction at the supports will remain zero.

If before temp. change Arch is loaded due to which both horizontal and vertical reactions are present due to rise in temp., vertical height increase and hence horizontal thrust is restricted reduced \((H < \frac{1}{h})\)

There will be no change in vertical reaction let Arch is loaded with UDL due to load horizontal thrust \((H = \frac{Wl^2}{8})\); if due to temp. raise by \(T^oC\) 3-hinge horizontal thrust is \(dh\)

\[
\frac{dh}{H} = \frac{dh}{h}
\]
It means \( y \) decrease in horizontal thrust is equal to \( y \) increase in vertical rise.

Generally due to temp. effect determinate structure undergoes free expn hence no reaction are produced due to temp. change provided structure is unloaded but in indeterminate structure due to temp. change reaction are produced.

To hie a 2-hinge arch is an indeterminate structure hence due to temp. change horizontal thrust is induced at the supports even when arch is unloaded. And horizontal thrust will be increase if arch was initially loaded.
Analysis of 2-Hinged Arches

2-Hinge arch - indeterminate structure. Hence, additional compatibility eqn is required. Since support a, b, and c are not movable. Hence, u is total strain energy and since effect of bending moment is much greater than normal thrust and shear force, then u is bending strain energy:

\[ U = \int \frac{M_x}{2EI} ds \]

\[ M_x = R_A x - H_y \]
\[ \Rightarrow M = H_y \]

M = B.M. due to vertical force only this is called beam moment

\[ U = \int \frac{(M-H_y)^2}{2EI} ds \]

\[ \frac{du}{dt} = \int \frac{\partial E}{2EI} (M-H_y) (-y) ds = 0 \]
\[ \frac{\delta w}{\delta H} = - J \frac{M \cdot y \, ds}{E_I} + H \int \frac{y^2 \, ds}{E_I} < 0 \]

\[ H = \int \frac{M \cdot y \, ds}{E_I} \]

\[ H = \int \frac{y^2 \, ds}{E_I} \]

Find horizontal thrust 2—hinge semi-circle beam subjected to a vertical load \( W \) at the crown.

\[ \frac{y}{R} = \sin \theta \]

\[ y = R \sin \theta \]

\[ \frac{R - x}{R} = \cos \theta \]

\[ R - x = R \cos \theta \]

\[ x = B \left( 1 - \cos \theta \right) \]
\[ H = \int \frac{M \cdot dy \, dz}{EI} \]

\[ N = \text{Beam Moment at } x-x [\text{B.M. due to vertical force only}] \]

\[ = 1 \cdot R \cdot A \cdot x \]

\[ = \frac{1}{2} \sin \cdot x \]

\[ db = R \cdot d\theta \]

\[ \theta \]

\[ \theta \]

\[ H = \int_0^{\pi/2} \frac{\omega}{2} R (1 - \cos \theta) R \, d\theta \]

\[ = \frac{\pi R^2}{8} \int_0^{\pi/2} (1 - \cos \theta) \, d\theta \]

\[ = \frac{\pi R^2}{8} \left[ \theta - \sin \theta \right]_0^{\pi/2} \]

\[ = \frac{\pi R^2}{8} \left[ \frac{\pi}{2} - 0 \right] \]

\[ = \frac{\pi^2 R^2}{16} \]

\[ \sin^2 \theta = \frac{1 - \cos \theta}{2} \]

\[ H = \frac{\pi R^2}{2} \left[ \theta + \sin \theta \right]_0^{\pi/2} \]

\[ = \frac{\pi R^2}{2} \left[ \frac{\pi}{2} + \sin \frac{\pi}{2} \right] \]

\[ = \frac{\pi^2 R^2}{4} \]

\[ = \frac{\pi R^2}{2} \left[ 1 - \cos 2\theta \right]_0^{\pi/2} \]

\[ = \frac{\pi R^2}{2} \left[ 1 - 0 \right] \]
Special case-1

If concentrated load \( W \) at radius vector \( r \) as shown in Fig., than Horizontal thrust \( H \) is

\[
H = \frac{W}{\sin^2 \alpha}
\]

![Diagram of circular arch with forces](image)

Special case-II

2 Hinge semi-circular arch subjected to UDL over entire span

![Diagram of semi-circular arch with forces](image)

If UDL acts over half span, or UDL acts over entire span than Horizontal thrust

\[
H = \frac{2WB}{3L}
\]
Given: 2-hinged Parabolic Arch subjected to UDL over

Find Horizontal thrust at the support if moment of
inertia varies as \( \frac{1}{2} \cdot \text{sec}^2 \theta \)
when \( \theta \) is angle of tangent with
the Horizontal

\( I_0 = \text{moment of inertia at the crown} \)

\( I = \text{insec}^2 \)

\[ R_A = R_B = \frac{wL}{2} \]

\[ H = \frac{1}{EI} \int \frac{y^2}{dx} ds \]

\[ H = \frac{1}{EI} \int \frac{y^2}{dx} ds = \frac{wL}{2} \cdot \sec^2 \theta \]

\[ H = \frac{1}{2} \left[ \frac{wL}{2} \cdot \sec^2 \theta \right] \]

\[ H = \frac{1}{2} \left[ \frac{wL}{2} \cdot \sec^2 \theta \right] \]
\[ H = \frac{w e^2}{8 h} \int_0^h x^2 (e-x)^2 \, dx \]

\[ H = \frac{w e^2}{8 h} \int_0^h x^2 d \end{array} \]

which is same as in 3-hinge parabolic arch.

There bending moment and shear force will be zero at any section.

Two-hinged parabolic arch subjected to concentrated load \( w \) at the corum moment of inertia \( I = \frac{1}{12} b e^2 \)

\[ H = \frac{25 w e^2}{128} \]

Two-hinged parabolic arch subjected to concentrated load \( w \) at the corum moment of inertia \( I = \frac{1}{12} b e^2 \)


* Temperature Effect in 2-Hinged Arch *

Due to rise in temp. the horizontal thrust induced given by

\[ H = \frac{E I dt}{\int y^2 \, ds} \]

**Special Case 1**

For 2-hinged semi-circular arch sub. to rise in temp. by \( T^C \)

\[ H = \frac{Y E I dt}{K R^2} \]

for semi-circular arch

\( EI \) is constant.

**Special Case 2**

For 2-hinged parabolic arch

\[ H = \frac{15 E I s}{8 h^2} \]

In 2 percent of merida at the center.
Reaction locus is the locus of point of the intersection of 2 resultant reactions at the support when a point load moves from one end to the other end. For a 2-hinged semi-circular arch, reaction locus is a straight line, whereas for a 2-hinged parabolic arch, reaction locus is a curve.

Parabolic Arch

\[ y = \frac{1.6h}{1.28} \frac{x^2}{x^2 + 1.6h^2} \]
Imagine a structure A, C, D, E & B subjected to loads W₁, W₂, W₃ at joints and all joints A, C, D, E & B to pin connected like a truss. If loads act on the joints of truss, there will be only axial force. No bending moment and shear force. The shape of the above structure is similar to a Funicular Polygon.

A linear arch/theoretical arch is an imaginary structure which has pin joint members having shape of a Funicular Polygon. The member of linear arch carry only axial compression force & No S.F. & B.M.
since Funicular polygon depends on loading and with the shape in position of load, shape of funicular polygon change hence it is difficult to construct a funicular polygon.

Eddy’s Theorem: If linear arch is superimposed over the given arch, then at any section on the given arch, the intensity of the intercept with given arch & linear arch.

\[ M \times \Delta y \]

Given arch
maxm B.M. will occur below occur load at is
at a distance x.

\[ y = \frac{4h}{x(x-h)} \]

\[ M_x = RA - x - Ha - y \]

for max \( M_x \)
\[ \frac{dM}{dx} = 0 \]
\[ M_{xx} = \text{Max} \quad \begin{bmatrix} x = 0.211 \end{bmatrix} \]

2 tonsm
\[ \tan \theta = \frac{4h}{4} \left( \frac{1}{2} \right) \]

\[ \phi = \tan^{-1} \left( \frac{1}{2} \right) \]

\[ M_c = 0 \]
\[ H_b = R_e \cos \theta \]
\[ v_b = R_e \sin \theta \]
\[ t = \frac{v_b}{H_b} \]

\[ M_c = 0 \]
\[ v_b \times 12.5 = H_b \times 5 = 0 \]
\[ \frac{v_v}{H_b} = \frac{5}{12.5} = 0.4 \]
Methods of Analysis

Exact Methods:
- Force Method
- Method of Consistent Deformation
- 

Approximate Methods:
- Displacement Method/Stiffness Method
  - Equilibrium Method
  - Ex.: Cantilever Method
    - Factor Method
    - Portal Method
  - Ex.: Method of Successive Approximation
    - Slope Deflection Method
    - Kane's Method
    - Method of Iteration
    - Stiffness Matrix Method

Method of Minimizing Potential Energy
- Castigliano's Theorem
Difference between force method & stiffness method.

- HFLC (force flexibility compatibility) vs. DISK (displacement stiffness equilibrium).

- Basic unknowns: forces, which may be reaction and member forces.
- Redundant forces: joint displacement (A & B).
- The no. of redundant forces = no. of degree of static indeterminacy.
- The no. of displacement at joint is equal to degree of kinematic indeterminacy.
- In rigid frame & beam, effect of axial force is neglected.

- To determine redundant to compatibility equiv., write no. of compatibility equiv. = degree of static indeterminacy (DI).
- To find joint displacement, joint eqn. conditions and shear eqn. are written. No. of equations is equal to degree of kinematic indeterminacy.

- Force method is suitable when $[Q_s < Q_k]$.
- Displacement method is suitable when $[D_s < D_k]$.

Note:
1. For indeterminate structure to be analyzed by exact method: rigidity, stiffness of member should be known. If indeterminate structure can be analyzed for which stiffness is not known, than approximate method can be used.
If Dr and Dr both are height such as case of multistory building & multiplane than neither force method nor displacement method is appropriate, under such condition approximate analysis is used.

Methods of Analysis of indeterminate structure may be applied for determinate structure also, but simple problem may given lengthy computation & solution.

**Principal of Superposition:**

**Assumption:**

- Material is isotropic, homogeneous, linear elastic in which Hook's law is followed.
- Temp. is constant.
- Supports are unyielding.

**Principal:** In a beam, truss or frame which may be determinate or indeterminate, the resultant stress function due to multiple loading is equal to sum of effect of individual loading.

**Note:** It is valid for these stress function which have linear relationship with loading.
Castigliano Theorem:

Theorem 1: In any structure, the material of which is linear elastic, temp. is constant, and supports are unyielding, then first partial derivative of total strain energy with respect to any displacement component is equal to force applied in the direction of that displacement.

\[ \frac{\partial U}{\partial u} = p \]

\[ \frac{\partial U}{\partial \phi} = M \]

Theorem 2: In a beam, truss, or frame (any structure), the material of which is elastic, temp. is constant, supports are unyielding, the first partial derivative of total strain energy with respect to any force is equal to displacement in the direction of the force.

\[ \frac{\partial U}{\partial p} = \Delta \]

\[ \frac{\partial U}{\partial m} = \varphi \]
\[
\frac{\partial U}{\partial p} = 0
\]

**Principle of minimum potential energy:**

\[
U_{\text{min}} = mgh_{\text{min}}
\]

Statement: "Of all the geometrically compatible states of structure which satisfy the deflection boundary conditions, force equilibrium requirement will have a final stable condition when total potential energy in the system is minimized."

Otherwise:

If a structure is loaded and there are redundant reactions for which infinite solution may be possible, then true solution will be that for which total minimum static potential energy exists.

Let in above frame redundant is \( \Re - \mathcal{R} \). True solution of \( \Re - \mathcal{R} \) is such that

\[
U \text{ is min} \quad \Rightarrow \quad \frac{\partial U}{\partial R} = 0
\]

same as Castiglione theorem.
Betti's Law:

Betti's Law / Raleigh Theorem:

In any structure, the material of which is elastic and follows Hooke's law, in which supports are unyielding, temp. is constant, external virtual work done by P system of forces during the displacement caused by a system of forces is equal to external virtual work done by a system of forces when deformation is caused by P system of forces.

\[ \text{We} = \frac{1}{2} \cdot P \cdot A \]

Where

- \( P \) is the applied force.
- \( A \) is the displacement given in its direction.

Then, work done by P is real work done.
If there is no loss of work in heat of or transmission it is equal to total internal strain energy stored, \( U_i \)

\[
\text{We} = \frac{1}{2} P \cdot \Delta = U_i = \frac{P^2 L}{2AE}
\]

Virtual work

\[
\text{We}^* = P \cdot \Delta^*
\]

\( \Delta^* \) is due to \( P \)

Let \( \Delta_i^* \) is displacement given in the direction of force \( P \) by any other force \( \phi \) then \( \phi \) external work done by \( P \) is \( \text{We}_1^* \)

\[
\text{We}_1^* = P \cdot \Delta_i^*
\]

\( \Delta_2^* \) is displacement given by \( P \) in the direction force \( \phi \) then external virtual work done by \( \phi \)

\[
\text{We}_2^* = \phi \Delta_2^*
\]
According to Raleigh theorem, let's consider law

\[ W_1^* = W_2^* \]

**Special Cases:**

\[ \theta^* \]

Let \( \theta_1^* \) is rotational/ angular displacement in direction of \( M_1 \) (moment) produced by moment \( M_2 \) acting at some other point. And let \( \theta_2^* \) is rotational in direction of \( M_2 \) caused by moment \( M_1 \); then external virtual work done by \( M_1 \) is equal to external virtual work done by \( M_2 \).

Ex. Virtual work done by \( M_1 \) = Ex. Virtual work done by \( M_2 \)

\[ M_1 \theta_1^* = M_2 \theta_2^* \]

If displacement is in direction of force it work done will be tive. If displacement is in opposite direction of force then work done will be (-ive)

\[ W_1^* = M_1 \theta_1^* \]
\[ W_2^* = M_2 \theta_2^* \]
special case II. If $\theta^*$ is angular displacement in the direction of moment in due to a linear force $P$ acting at any point and $\Delta \alpha^*$ is linear displacement in the direction of force $P$ due to moment in acting at some other point, then virtual work done by $m$ is equal to virtual work done by $P$.

$$M\theta^* = P\Delta \alpha^*$$

---

Example: for the semi-circular two-hinge Arch as shown in fig. a moment of 50 kNm at $B$ which produces a displacement of 0.5 cm at $A$. If a concentrated load of 10 kN is applied at $A$, then rotation at $B$ in the arch will be.

A) 0.01  B) 0.1  C) 0.05  D) 0.5
Maxwell Reciprocal Theorem:-

It is a special case of Bell's Law in which \( P = 0 \).
Let \( S_{12} \) is displacement at point 1 in the direction of \( ab \) due to point \( P \) placed at point \( Q \) in the direction of \( cd \).

Let \( S_{21} \) is displacement at point \( Q \) in the direction of \( cd \) due to point \( P \) and in the direction of \( ab \) at point 1. Then,

\[ S_{21} = S_{21} \]
Principal of virtual work:

Case-1 Elastic bodies: The total virtual work done
(External + Internal) for deformable bodies is equal
to zero

\[ W_e^* + W_i^* = 0 \]

Internal virtual work done by internal forces is zero (generally false) because internal deformation is in opposite direction to the internal forces.

\[ |W_e^*| = |W_i^*| = U_i^* \]

Case-2 Rigid bodies: Rigid bodies do not have internal deformation, therefore internal virtual work done is zero; hence total virtual work done is zero.

\[ W_e^* = 0 \]

For the truss shown in fig. Compute
Horizontal Component of displacement at joint E when following movement of support occurs at support A moves horizontally by 0.0025 ft and 0.0075 ft vertically down.

2) Support C moves vertically down by 0.0025 m.

Note that there is no change in the length of any member, because all members are considered axially rigid.
Joint displacement is due to movement of support only.

\[ S_E \rightarrow + \]

\[ S_N \rightarrow (-) \]

To find horizontal displacement at \( E \), sum \& Equate forces due to support settlement. Consider pseudo force \( p \) in horizontal direction at \( E \)

\[ \varepsilon_{Fx} = 0 \]
\[ H_A + P = 0 \]
\[ H_A = P \]

\[ \varepsilon_{Fy} = 0 \]
\[ R_A + R_B = 0 \]

\[ \varepsilon_{Ma} = 0 \]
\[ R_C \times 8 - P \times 3 = 0 \]
\[ R_C = \frac{3}{8} P \]

If joint displacement are given in the direction of reaction forces than external virtual work will be done. For rigid body total external virtual work done is zero.
\[ \delta h_a = -0.0050 \]
\[ \delta h_a = -0.0075 \]
\[ \delta v_c = -0.0025 \]
\[ W_e = 0 \]
\[ R_a \cdot \delta v_a + H_a \cdot \delta h_a + R_c \cdot \delta v_c + p \cdot \delta \varepsilon = 0 \]
\[ -\frac{3}{8} p \cdot (-0.0075) + (-p) (-0.0050) + \left(\frac{3}{8} p\right) (-0.0025) + p \delta \varepsilon = 0 \]
\[ \Rightarrow p \delta \varepsilon = -0.006785p \]
\[ \Rightarrow \delta \varepsilon = -0.006785 \text{ m.} \]
STRAIN - ENERGY METHOD

Redundant is R

\[ \frac{dU}{dR} = 0 \]

Redundant is H

\[ \frac{dU}{dH} = 0 \]

Redundants are H & R

Note: If the structure has redundant reaction then for stable equilibrium its total strain energy w.r.t. redundant forces should be minimum.

\[ \frac{dU}{dH} = 0 \quad \frac{dU}{dR} = 0 \]
The strain energy considered is due to bending moment only and the effect of axial force and shear force is neglected.

\[ U = \frac{\int M_x^2 \, ds}{2EI} \]

if \( H \) is redundant then

\[ \frac{\partial U}{\partial H} = 0 \]

\[ \Rightarrow \int \sum M_x \frac{\partial M_x}{\partial H} \, ds \]

\[ \frac{1}{2EI} \]

\[ \Rightarrow \int \sum M_x \frac{\partial M_x}{\partial H} \, ds \]

\[ \frac{1}{EI} \]

if redundant is \( R \)

\[ \int \sum M_x \frac{\partial M_x}{\partial R} \, ds = 0 \]

\[ \frac{1}{EI} \]

if redundant is \( MA \)

\[ \int \sum M_x \frac{\partial M_x}{\partial MA} \, ds = 0 \]

\[ \frac{1}{EI} \]
\[ U = \sum \int \frac{M_x^2}{2EI} dx \]

Ex. Analysis the portal frame shown in Fig. using strain energy method and draw the B.M.'s.

\[ E \text{ constant} \quad H_A = H \]

\[ \varepsilon_{fy} = 0 \]

\[ RA + Rd - P = 0 \quad RA + Rd = P \]
\[ \Sigma M_O = 0 \]
\[ R_A \cdot L - F \times \frac{L}{2} = 0 \]
\[ R_A = \frac{P}{2} \]
\[ R_D = \frac{P}{2} \]
\[ \Sigma F_x = 0 \]
\[ H_A - H_D = 0 \]
\[ H_A = H_D = H \text{ (say)} \]
\[ \frac{d\nu}{dH} = 0 \]
\[ \sum \int M_x \cdot \frac{dM_x}{dH} \, dx = 0 \]

<table>
<thead>
<tr>
<th>Member</th>
<th>( M_x )</th>
<th>( \frac{dM_x}{dH} )</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>(-H_y)</td>
<td>(-y)</td>
<td>0 to ( L )</td>
</tr>
<tr>
<td>BC</td>
<td>( \frac{P}{2} \times H_L )</td>
<td>(-L)</td>
<td>0 to ( L/2 )</td>
</tr>
</tbody>
</table>

\[ M_x = -H_y \]
\[ M_A = \sum \int M_x \cdot \frac{dx}{EI} \cdot ds + \theta \]

\[ \Rightarrow 2 \int_0^L \frac{(-HY)(-y) \cdot dy}{EI} + 2 \int_0^{\frac{L}{2}} \frac{(L-x-HL)(-L) \cdot dx}{EI} \]

\[ \Rightarrow \frac{3H \cdot y^3}{6EI} \bigg|_0^L + \frac{1}{2EI} \int_0^{\frac{L}{2}} \left[ \frac{pL^3}{2} + \frac{HL^2}{2} - x \right] \frac{dx}{L} \]

\[ = \frac{3H \cdot L^3}{6EI} + \frac{1}{2EI} \left[ \frac{pL^3}{4} + \frac{HL^3}{4} \right] \]

\[ H \Rightarrow \frac{3p}{40} \]

B.M. in AB  \[ M_x = -HY \quad \Rightarrow -3 \cdot \frac{p}{40} \cdot \frac{L}{2} \]

\[ M_A = 0 \]

\[ M_B = -3 \cdot \frac{p \cdot L}{40} \cdot \frac{L}{40} \]
\[ M_x = \frac{P \cdot x}{2} - H L \]

\[ M_B (x = 0) = 0 - H L = -\frac{3}{4} \cdot P L \]

\[ M_F (x = L) = \frac{P \cdot L}{2} - \frac{3}{4} \cdot P L \]

\[ = \frac{1}{4} + \frac{7}{4} \cdot P L \]

Convex

Convex

Concave

Concave

Convex
Q. 3. Identically rods OA, OB, and OC are in a same vertical plane and support loads as shown in fig, focus in all members and calculate, vertical moment of A. Area of each member is a \& all joints is pin connected.

AF = constant

2D Truss

\[ P_3 = m + 5w - 2 \times 1\]
\[ = 3 + 6 - 2 \times 4 \]
\[ = 1 \]

Net forces in member are \( P_1, P_2, P_3 \) in OA, OB, OC respectively.
let \( F_1 \) be redundant force:

\[
\frac{\partial U}{\partial F_1} = 0 \quad \text{(1)}
\]

\[
U = U_{BA} + U_{OB} + U_{OC}
\]

\[
U = \frac{p_1^2 L}{2AE} + \frac{p_2^2 L}{2AE} + \frac{p_3^2 L}{2AE}
\]

\[
\sum F_x = 0
\]

\[
p_3 \cos 30^\circ - p_2 \cos 30^\circ = 0
\]

\[
p_3 = p_2
\]

\[
\sum F_y = 0
\]

\[
p_1 - W - p_2 \sin 30^\circ - p_3 \sin 30^\circ = 0
\]

\[
p_1 - W = \frac{p_2 - p_3}{2} = 0
\]

\[
[p_1 - W = p_2 = p_3]
\]

\[
U = \frac{p_1^2 L}{2AE} + \frac{2(p_1 - W)^2 L}{2AE}
\]

\[
\frac{\partial U}{\partial p_1} = 0 \Rightarrow \frac{\partial U}{\partial p_1} = \frac{2p_1 L}{2AE} + \frac{4(p_1 - W)L}{2AE}
\]

\[
\Rightarrow \frac{2p_1 L}{2AE} = \frac{2W L}{2AE}
\]

\[
\Rightarrow \frac{3p_1 L}{AE} = \frac{2WL}{AE}
\]

\[
\Rightarrow \frac{3p_1 L}{AE} = \frac{2WL}{AE}
\]
1. \[ P_1 = \frac{2W}{3} \] (Tensile) + Tension.

2. \[ P_2 = P_3 = P_1 - W \]
   \[ = \frac{2W}{3} - W \]
   \[ = -\frac{W}{3} \] (Compressive - Tensile)

**Method 1**

Vertical deflection of OA
\[ \Rightarrow \text{Elongation of OA} \]
\[ \Rightarrow \frac{P_1L}{AE} \]
\[ \Rightarrow \frac{\frac{2}{3}P_1L}{AE} = \frac{3P_1L}{3AE} \]

**2nd Method**

\[ U \] is Total Strain Energy Stored.
\[ \frac{dU}{dW} = A_0 \]

**Analysis: a Portal Frame.** Shown in Fig. Use the strain energy method. Draw Bending Moment Diagram.

Bird

Cost = constant

\[ P_1 = 120 \text{ kN} \]

\[ P_2 = -120 \text{ kN} \]

\[ P_3 = 120 \text{ kN} \]

\[ P = +80 \text{ kN} \]

\[ P = -80 \text{ kN} \]
(9e) Reaction = 3
Dse = 9e - 3 = 4 - 3 = 1
Dsi = 0

\[ \begin{align*}
\text{H}_4 + \text{HD} &= 120 \\
\text{H} + \text{HD} &= 120 \\
\text{HD} &= 120 - \text{H}
\end{align*} \]

\[ \text{Epy} = 0 \]

\[ \text{HA} + \text{HD} = 120 \]

\[ (\text{HD}) \text{HA} = 120 \text{NH} \]

\[ \text{Epy} = 0 \]

\[ \text{RA} + \text{RD} = 0 \]  
\[ \text{vii} \]

\[ \sum \text{MA} = \]

\[ \text{RD} \times 6 - 120 \times 4 = 0 \]

\[ \text{RD} = \frac{120 \times 4}{6} \]

\[ \text{RD} = 80 \text{ KN} \]

\[ \text{RA} = -80 \text{ KN} \]

Say H is Redundant Reaction

for stable condition of structure

The True Value of Redundant H

is when total pot. energy of sym is

\[ \min m = 0 \]
\[
\frac{\partial u}{\partial H} = 0
\]

\[
-u = \sum \int \frac{M_x}{2EI} ds
\]

\[
\frac{\partial u}{\partial H} = \sum \int \frac{M_x}{2EI} \frac{\partial M_x}{\partial H} ds
\]

<table>
<thead>
<tr>
<th>Member</th>
<th>(M_x)</th>
<th>(\frac{\partial M_x}{\partial H})</th>
<th>Limit of integration</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>-485 + 0.8Hs</td>
<td>-0.8s</td>
<td>0 to 5</td>
</tr>
<tr>
<td>BC</td>
<td>80(3\theta + 4) + 4H</td>
<td>4</td>
<td>0 to 3</td>
</tr>
<tr>
<td>DC</td>
<td>-(120 - H)y</td>
<td>y</td>
<td>0 to 4</td>
</tr>
</tbody>
</table>

(1) For member AB

B.M. at \(x\):

\[
M_x = R_A \cdot x + H \cdot y
\]

\[
= (-80) \cdot 3 \cos \theta + H \cdot 5 \cdot \sin \theta
\]

\[
= -80 \cdot 3 \cdot \frac{3}{5} + H \cdot 5 \cdot \frac{4}{5}
\]

\[
\text{B.M. max} = -48.5 + 8 \cdot 0.8H \cdot 5
\]
At member BC

Free body diagram:

\[ BA = H \]
\[ RA = -80 \]

\[ M_x = RA (3+x) + H \times y \]
\[ M_x = -80 (3+x) + 4H \]

At the member CD

\[ RD = 8 \]

\[ (120 - H) = H_D \]

\[ M_x = -(120 - H) \times y = 0 \]

Equation:

\[ \sum \int H_x \frac{dM_x}{dH} \, dH = 0 \]
\[ \begin{align*}
H &= 92.33 \text{ kN} \\
M_x &= -485 + 0.8H \\
M_A &= 0 \\
M_B &= -485 + 0.8x \times 88 \cdot 3.3 \times 5 \\
&= +113 \cdot 33
\end{align*} \]

In BM in BC
\[ \begin{align*}
M_D &= 0 \\
M_C &= -126.68
\end{align*} \]
Elastic curve.

Analogous to a portal frame shown in fig. Using strain energy method. \( k = \text{constant} \)

\[
\begin{align*}
6\text{kN} & \quad \overline{BC} \\
4\text{m} & \\
4\text{m} & \\
H & = HA \\
M & = Ma \\
T_R & = T_R &
\end{align*}
\]

\( f_I = \text{constant} \)

No. of reactions = 3 + 2 = 5

\( D_S = 5 - 3 = 2 \)

\( E F_x = 0 \quad HA - H_c = 0 \)

\( HA = H_c = H \) \( -1 \)
Consider Redundants are \( H \& MA \)

\[
Ra + Rc = 6 \times 4 = 0
\]

\[
Ra + Rc = 24 \text{ KN} \quad \text{(10)}
\]

\[
MA + Ra \times 4 - H \times 4 - 6 \times 4 \times x = 0
\]

\[
MA + Ra \times y - H = 48 = 0
\]

\[
Ra = 48 - MA + 4H.
\]

\[ Ra = 12 - 0.25MA + 4H. \]

From eqn(9)

\[
Rc = 24 - Ra
\]

\[
Rc = 24 - 12 + 0.25MA + 4H.
\]

\[
Rc = 12 + 0.25 - H
\]

<table>
<thead>
<tr>
<th>Member</th>
<th>( Mx )</th>
<th>( \frac{\partial Mx}{\partial H} )</th>
<th>( \frac{\partial Mx}{\partial MA} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>MA - H \cdot y</td>
<td>- y</td>
<td>1</td>
</tr>
<tr>
<td>BC</td>
<td>( (12 + 0.25MA - H)x )</td>
<td>- x</td>
<td>-0.25x</td>
</tr>
</tbody>
</table>

For member AB:

\[
Mx = MA - H \cdot y
\]
in form \( \tau \) CB

\[
\begin{align*}
6 \text{KN}m & \quad \text{G} \\
\end{align*}
\]

\[
M_x = \tau x - w x x \frac{3}{2}
\]

\[
M_x = (12 + 0.25 MA - H) - \frac{6x^2}{2} = 3x^2
\]

Equation:

\[
\frac{d\theta}{dH} = 0
\]

\[
\sum \left[ M_x \frac{dM_x}{dx} \right] dS = 0 \quad \text{(1)}
\]

\[
\frac{du}{dMA} = 0
\]

\[
\sum \left[ M_x \frac{dM_x}{dS} \right] dS = 0 \quad \text{(2)}
\]

\[
\int (MA - H_y) (-y) dy + \int [ (12 + 0.25 MA - H)x - 3x^2 ] (-x) dx = 0 \quad \text{(1)}
\]

\[
\int (MA - H_y) dx = 0
\]

\[
\int [ (12 + 0.25 MA - H)x - 3x^2 ] [0.25x] dx = 0 \quad \text{(2)}
\]
Solving 18(3)

\[ MA = +3.43 \text{ kN-m} \]
\[ H = 2.57 \text{ kN} \]

\[ M_x = MA - H \bar{y} \]
\[ = 3.43 - 2.57 \bar{y} \]
\[ M_A = +3.43 \]
\[ M_B = +3.43 - 2.57 \times 4 \]
\[ = -6.86 \]

BM in wcb

\[ M_x = (12 + 0.25MA - H) \cdot 3 \cdot \bar{x} \cdot \bar{x} \]

\[ M_c = 0 \]
\[ M_B = -6.86 \]
Q. Determine the B.M. Reaction of the cantilever beam shown in fig.

\[ \Delta = 0 \]

\[ \frac{d\Delta}{dR} = 0 \]

\[ U_{AB} = \int \frac{(Mx)^2}{2EI} \, dx + \int \frac{(Rx)^2 - Lx}{2EI} \, dx \]

\[ U_{BC} = \int \frac{[RAx - \omega x(L - L/2)]}{2EI(2I)} \, dx \]

\[ \omega = \frac{5}{2} \]
A ring of radius R having uniform cross-section (EI constant) is subjected to 2 equal and opposite load at A and B shown in fig. Determine max. Susying & hogging B.M. at A and also find decrease in length of vertical dia. and position of contra flexure with horizontal diameter.

For the analysis let us cut a ring in two equal part. At the level of AB.

\[ H_A = H_B = 0 \]

For True value of Redundant \( M_0 = \) 

\[ \frac{\partial V}{\partial M_0} < 0 \]
$$U = 2 \int_0^{\pi/2} \frac{Mx^2 \, ds}{2EI}$$

$$U = 2 \int_0^{\pi/2} \left[ \frac{\omega}{2} R (1 - \cos \theta) + M_0 \right] R \, d\theta$$

$$\frac{dU}{dM} = \frac{1}{EI} \int_0^{\pi/2} a \left[ \frac{\omega}{2} R (1 - \cos \theta) + M_0 \right] R \, d\theta$$

$$\frac{dU}{dM} = \frac{\omega}{2} \int_0^{\pi/2} \frac{R - \omega \cos \theta + M_0}{2} R \, d\theta$$

$$\frac{dU}{dM} = \frac{\omega}{2} \int_0^{\pi/2} \frac{R - \omega \cos \theta + M_0}{2} R \, d\theta$$

$$\Rightarrow \left[ \frac{\omega R^2 \pi}{2} \right] - \left[ - \frac{\omega R^2 \pi}{2} \right] + \left[ \frac{M_0 R \pi}{2} \right]$$

$$\Rightarrow \frac{\omega R^2 \pi}{2} + \frac{\omega R^2 \pi}{2} + \frac{M_0 R \pi}{2}$$

$$\Rightarrow \frac{\omega R^2 \pi}{2} + \frac{\omega R^2 \pi}{2} + \frac{M_0 R \pi}{2}$$

$$\Rightarrow \frac{-\omega R^2 (\pi - 2)}{2}$$

$$\Rightarrow \frac{M_0}{2} = \frac{-\omega R^2 (\pi - 2)}{2}$$
\[ M_x = \frac{wR}{2} \left[ 1 - \cos \theta + \frac{M_o}{R} \right] - 1 \]

For \( M_x \) to be \( \max \) or \( \min \).

\[ \frac{dM_x}{d\theta} = 0 \]

\[ \frac{wR}{2} (\theta + \sin \theta) + 0 = 0 \]

\[ \sin \theta = 0 \]

\[ \theta = 0 \]

Hence, \( \max \) or \( \min \) bearing \( B \) is at \( A \).

\[ M_o = -\frac{wR}{2\pi} (\pi - 2) \]

\[ \cos \theta = 0 \]

Base (major) \( \max \) sagging:

\[ \frac{wR}{2} \left[ 1 - 0 \right] + \frac{wR}{2\pi} (\pi - 2) \]

\[ = \frac{wR}{2} - \frac{wR}{2} + \frac{wR}{\pi} \]

\[ = \frac{wR}{\pi} \]

For point of contra flexure:

\[ M_x = 0 \]

\[ \frac{wR}{2} (1 - \cos \theta) + M_o = 0 \]

\[ \Rightarrow \frac{wR}{2} \left( 1 - \cos \theta \right) - \frac{wR}{2\pi} (\pi - 2) = 0 \]

\[ \theta = \cos^{-1} \left( \frac{8}{\pi} \right) \]
Vertical deflection of CD

\[ \Delta_{CD} = \frac{dU}{\partial w} \]

\[ u = 4 \int \frac{\frac{N}{2EI}}{\frac{M}{2EI}} \cdot \frac{\pi^2 \cdot ds}{2EI} \]

\[ u = 4 \int \left( \frac{\pi^2}{2} \left( \frac{1}{2} R - \cos \theta + M_0 \right) \right)^2 R \cdot d\theta \]

\[ \Delta_{CD} = \frac{UR^8}{4 \pi EI} \left( \pi^2 - 8 \right) \]

Note: In a closed str., resultant moment member is internal and in open str., redundant member is external.
MOMENT DISTRIBUTION METHOD

is also called:

- Hard - cross method
- Mth. of successive approximation
- Base of stiffness concept

Stiffness:

\[ \theta_B = \frac{ML}{EI} \] when far end is fixed

\[ M = \frac{4EI \cdot \theta_B}{L} \] if \( \theta_B = 1 \), then \( M = k \) (Stiffness

Stiffness of \( BA = \frac{4EI}{L} \), when far end A is fixed

\[ \theta_B = \frac{ \theta_B \cdot m}{3EI} \]

\[ \theta_A = \frac{ML}{6EI} \]

\[ M = \frac{3EI \cdot \theta_B}{L} \]

If \( \theta_B = 1 \), then \( M = k \)

Stiffness of \( BA = \frac{3EI}{L} \) -- when far end A is Hinged.
If one end is free, then stiffness of the member will be zero.

\[ k_1 \text{ Stiffness of } OA = \frac{4EI_1}{L_1} \]  
for end(a) is fixed

\[ k_2 \text{ Stiffness of } OB = \frac{3EI_2}{L_2} \]  
for ceg is fixed

\[ k_3 \text{ Stiffness of } CE = 0 \]  
for c is free

Total stiffness of all members = \( \sum k \)

\[ \sum k = k_1 + k_2 + k_3 \]

\[ \sum k = \frac{4EI_1}{L_1} + \frac{3EI_2}{L_2} \]

Relative Stiffness:

\[ R.S. = \frac{\text{Stiffness}}{4E} \]
R.S. for off \( = \frac{I_1}{L_1} \)  \( \begin{pmatrix} I \end{pmatrix} \) when far end is fixed

R.S. for ob 3 \( I_g \) or 3 \( I \) \( = \frac{1}{L_2} \) \( L \)

R.S. for \( c = 0 \)

\[ M = \frac{M}{2} \]

\[ MA = \frac{1}{M} \]

\( MA = \) Carry over factor

\( \rightarrow \) it is always in the direction of applied moment

At a moment \( M \) is applied at \( B \) (near end) then carry over moment at far end which is fixed will be in the direction of applied moment. The ratio of carry over moment to the applied moment is called Carry over factor

If far end \( A \) is hinged or free, then carry over moment is zero. Hence carry of factor is zero.
1. \[ MA = \frac{1}{2} \text{ carry over factor} \]

2. \[ M_A = 0 \quad c.o.f = 0 \]

3. \[ M_A = \frac{(L/2)}{L} \quad (L/2) \]

4. \[ M_A = c.o.f > \frac{1}{2} \]

5. \[ M_A = c.o.f < \frac{1}{2} \]
To find the stiffness of a member at joint B and carry over the factor if far end A is fixed and beam contains internal hinge at C. Near end B is roller support as shown in Fig.

\[ MA - C.O.F. = \phi \]
\[ \frac{M}{M} = \phi \]
\[ R_B = \frac{M}{B} \]

\[ \sum MA = 0 \]
\[ R_B \times (a+b) - M = MA = 0 \]
\[ MA = R_B (a+b) - M \]
\[ MA = \frac{M (a+b) - M}{b} \]
\[ MA = \frac{M \cdot a + M - M}{b} \]
\[ MA = \frac{M a}{b} \]

\[ C.O.F. = \frac{MA - \phi}{M} \]
M/EI or curvature diagram, for given beam.

Slope at B in real beam = SF. at B. in Conjugate beam = -R_b

To find R_b take SM_c = 0

\[ R_b = \frac{M \times x}{EI} \times b \times \frac{x}{2} b = \frac{1}{2} x a x M \cdot \frac{a}{b} \times 2 \cdot a = 0 \]

\[ R_b = \frac{M b + M}{b^2} \cdot \frac{a}{3EI} \cdot \frac{a}{2EI} \]

\[ R_b = 1 \quad M = 1 \]

\[ k = \left( \frac{3EI \cdot b^2}{a^2 + b^2} \right) \cdot 0 \]
Distribution theorem:

\[ K_1 = \text{Stiffness of } OA = \frac{yEI_1}{L_1} \]

\[ K_2 = \text{Stiffness of } OB = \frac{3EI_2}{L_2} \]

\[ K_3 = \text{Stiffness of } OC = 0 \]

\[ K_4 = \text{Stiffness of } OD = \frac{yEI_4}{L_4} \]

Total stiffness of all members at joint 0

\[ \Sigma K = K_1 + K_2 + K_3 + K_4 \]

At joint 0

Distribution factor of OA is \( \frac{K_1}{\Sigma K} \)

Distribution factor of OB is \( \frac{K_2}{\Sigma K} \)
Distribution factor of oc = \( \frac{K_3}{3K} \)

Distribution factor of od = \( \frac{K_4}{3K} \)

Distribution factor = 1

D.F. (distribution) = \( \frac{\text{stiffness of a factor}}{\text{Total stiffness of all members meeting at that joint}} \)

D.F. \( \rightarrow \) B.S. of a member

T. R.S.

Moment distribution Theorem:

If at a rigid joint where more than 1 member on meeting, tension and external moment is applied \( M_0 \), then it gets distribution in all members in the direction of applied moment in proportion of stiffness of each member.
\[ M_1 = K_1 \times X \nu_0 \quad M_2 = K_2 \times X \nu_0 \quad M_3 = K_3 \times X \nu_3 \quad M_4 = K_4 \times X \nu_4 \]

Procedure to draw bending moment dia in stiffness method:

- If the no. of spans/stories are large then force method is complex & lengthy. Such cases use

  In the stiffness method, each span is considered to draw separate moment curve at both ends. Final moment at each end is found which are called end moments. To find these, one can use any of the stiffness method.

  For every member end moment dia is drawn in which sagging moment (stives) are plotted below the beam. Inside the frame, and hogging moment are plotted above the beam outside the frame.

  For each member considering free simply supported w/ given loads, free ARD.

  Simply supported dia. is drawn separately.

  In which sagging above the beam is positive (stives). Hogging is taken below the beam (negative) inside the frame.
To obtain final diagram, end moment dia. and free BMD are superimposed.

\[ M_x = R_A \cdot x - \frac{PL}{8} \]

\[ M_A = -\frac{PL}{8} \]

\[ M_C = \frac{P}{2} \cdot \frac{L}{2} + \frac{PL}{8} = \frac{3PL}{8} \]

The common are is cancelled. At net diagram is rig. resulted magnitude.
Total B.M. = \[ \frac{pL}{8} - \frac{PL}{8} + \frac{PL}{8} \]

Sign Convention for computation of end moments:
- All clockwise moment will be taken as positive and anti-clockwise moment will be negative.

Fixed end moment for standard loading condition:

1. \( \overline{MAA} \):
   \[ \overline{MAA} = \text{Fixed End Moment at A for AB.} \]
   \[ \Rightarrow -\frac{PL}{8} \]
   \[ \overline{MBA} = +\frac{PL}{8} \]

2. \( \overline{MBA} \):
   \[ \overline{MBA} = -\frac{PaL^2}{2} \]
   \[ \overline{Mba} = +\frac{PaL^2}{2} \]

3. \( \overline{MBA} \):
   \[ \overline{MBA} = -\frac{wL^2}{2} \]
   \[ \overline{MBA} = \frac{wL^2}{2} \]
To obtain final diagram, end moment dia. and free BMD curve superimposed.

The common are is cancelled.
At net diagram is rigid-rotated magnitude.

\[ M_y = RA \times \frac{-PL}{8} \]
\[ MA = -\frac{PL}{8} \]
\[ Mc = \frac{P \times L}{2} - \frac{PL}{8} = \frac{+PL}{8} \]

End force diagram. 55, 8 MB: force BMD.
Total B.M. = \[ \frac{PL}{4} + \frac{PL}{8} + \frac{Pb}{8} \]

Final B.M. = 145

Sign Convention for computation of end moment:
- All clockwise moment will be taken as (five) and anti-clockwise moment will be (five).

Fixed end moment for standard loading condition:

\[ \bar{M}_{AB} = \text{Fixed End Moment at } A \text{ for } AB. \]
\[ \Rightarrow -\frac{PL}{8} \]
\[ M_{BA} = +\frac{PL}{8} \]

\[ \bar{M}_{AB} = -\frac{Pab^2}{L^2} \]
\[ M_{Ba} = +\frac{Pa^2b}{L^2} \]

\[ \bar{M}_{AB} = -\frac{wl^2}{8} \]
\[ M_{BA} = +\frac{wl^2}{8} \]
\[ M_{AB} = -\frac{wL^2}{30} \quad M_{BA} = +\frac{wL^2}{30} \]

If arrow of loading points on the Reference face, then loading is taken the (+ve) and if arrow of loading point away from Reference face then loading is (-ive).

\[ M_{AB} = -\frac{5}{96} wL^2 \quad M_{BA} = +\frac{5}{96} wL^2 \]

\[ M_{AB} = -\frac{11}{192} wL^2 \quad M_{BA} = +\frac{5}{192} wL^2 \]
\[ \text{Note:} \]

\[ dP = \omega \, dx \]

\[ x \quad \begin{array}{c} \gamma \cr \sigma \end{array} \quad L \quad \frac{L}{2} \]

\[ \frac{dP}{dP} = \omega \, dx \]

\[ \frac{x}{2} \quad \frac{L}{2} \quad \frac{L}{3} \]

\[ \frac{dM_{ab}}{dM_{ab}} = - \frac{dP \cdot x \cdot (L-x)^2}{L^2} \]

\[ \Rightarrow - (\omega \cdot dx) \cdot x (L-x)^2 \]

\[ \frac{dM_{ac}}{dM_{ac}} = \int_{0}^{2L/3} \omega \cdot x (L-x)^2 \, dx \]

\[ \frac{dM_{ca}}{dM_{ca}} = \int_{0}^{2L/3} \omega \cdot x^2 (L-x) \, dx \]

\[ M_{ab} = \frac{+ M_0}{4} \quad M_{ca} = \frac{+ M_0}{4} \]

\[ M_{ac} = \frac{M_0}{3} (3a-L) \]

\[ M_{bc} = \frac{M_0 \cdot a \cdot (3b-L)}{L^2} \]

\[ \text{(only direction, not suggesting anything better than the value)} \]

\[ M_0 \rightarrow \text{tive if } \theta \]

\[ M_0 \rightarrow \text{ive if } \theta \]
Note that if \( a > \frac{1}{3} \), then \( M_{AB} > 0 \), \( M_{AB} \) is in the direction of \( M_0 \).

If \( b > \frac{1}{3} \), then \( M_{BA} \) is in the direction of \( M_0 \).

Hence, if \( M_0 \) acts in the middle third strip, then fixed end moment at \( A \) and \( B \) will be in the direction of \( M_0 \).

\[ M_{AB} = \frac{-6EIA}{L^2} \]

\[ M_{BA} = \frac{-6EIA}{L^2} \]

\[ \Sigma f_y = 0 \]

\[ R_A + R_B = 0 \]

\[ \Sigma M_B = 0 \]

\[ R_A \cdot L - 6EIA \cdot \frac{6EIA - 6EIA}{AL^2} = 0 \]

\[ R_A = \frac{12EIA}{L^2} \]

\[ \Sigma f_e \text{ in } AB = +RA = \frac{12EIA}{L^2} \]

Effect of settlement of support:

\[ \Sigma \]
Note: if settlement of support causes clockwise rotation, then fixing moment development developed at both ends will be anti-clockwise and vice-versa.

\[ M_{AB} = M_{BA} = \pm \frac{6EI\Delta}{L^2} \]

\[ M_{AB} = -\frac{3EI\Delta}{L^2} \]

\[ M_{BA} = 0 \]

\[ M_{AB} = M_{BA} = 0 \]
Procedure:

1. Find fixed end moments for each member for both ends, considering clockwise moment tive & anti clockwise moment tive.

2. Find distribution factor for each member at those joints whose more than one member meet & there are 2 joints.

3. Distribute the moment after balancing arcwise joint according to their distribution factor & transfer carrying over moment to the far ends if far ends are fixed.

4. Find final end moment at the ends of tabular calculation tive, arcwise, tive means anticlockwise.

5. Draw end moment diagram and super imposed S.S. dia. to obtain resultant moment diagram in B.M.D tive sagging tive Hosking.

8. For the beam shown in fig. draw B.M.D using Hardy's gross method.

\[ F = \text{Constant} \]

\[ M_{AB} = -\frac{P L}{8} = -24 \times 8 = 1 - 24 \]

\[ M_{BA} = +\frac{P L}{8} = +24 \]
\[ \overline{M_{bc}} = -\frac{wL^2}{2} = -\frac{12 \times 4^2}{12} = -16 \]

\[ \overline{M_{Bb}} = \frac{twL^2}{12} = +16 \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Member</th>
<th>Stiffness</th>
<th>Total Stiffness</th>
<th>D.F. = \frac{K}{E_k}</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>bA</td>
<td>(4E_1I/8)</td>
<td>(\frac{3}{2}EI)</td>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td></td>
<td>bC</td>
<td>(4E_1I/4)</td>
<td>(\frac{3}{2}EI)</td>
<td>(\frac{2}{3})</td>
</tr>
</tbody>
</table>

\[ \text{Final} \]

\[ \begin{array}{ccc}
B & Y_3 & Z/3 \\
A & +24 & -24 & \text{+16} \\
\text{Top} & -2.67 & 5.33 & -2.67 \\
\text{Final} & -25.33 & +21.33 & +21.33 & = 18.33 \\
\end{array} \]
End moment dia

Resultant B.M. at centre of AB = \( 48 - \frac{(25.33 + 21.33)}{2} \)

\[ \Rightarrow 24.67 \]

\[
g_{b} = g_{c} = 24 - \left( \frac{21.33 + 13.33}{2} \right) = 11.67
\]

\[ 24 \text{ KN} \]

\[ \frac{48 \text{ KN-m}}{B} \]

\[ \frac{12 \text{ KN-m}}{C} \]

\[ \frac{24 \text{ KN-m}}{C} \]

Computation of Reactions:

\[ \Rightarrow \text{To find B} \text{ under free body equilibrium just to left of B.} \]
\[ RA = 12.5 \text{ kN} \]

To find \( R_c \), consider free body equilibrium of span \( BC \):

\[ EM_8 = 0 \]

\[ RA \times 8 = 25.33 - 24 \times 4 - 12 \times 4 \times 2 - 21.33 = 0 \]

\[ EM_8 = 0 \]

\[ -R_c \times 4 + 13.33 + 12 \times 4 \times 2 - 21.33 = 0 \]

\[ R_c = 22 \text{ kN} \]

\[ R_E = 0 \]

\[ \sum F_y = 0 \]

\[ RA + R_c - 24 - 12 \times 4 = 0 \]

\[ R_E = 87.5 \text{ kN} \]
Draw R.M.D. for beam ABC away in Fig.
Support B settled down by 1m
--- constant

\[ E_I = 48000 \, \text{kN-m}^2 \]

\[ \Lambda B = 1 \text{mm} \cdot \text{k} \]

\[ M_{AB} = +M_0 - \frac{6EI\Delta}{L^2} \]

\[ \Rightarrow \frac{+100}{4} - \frac{6 \times 48000 \times 1 \times 10^{-3}}{2} \]

\[ \Rightarrow 25 - 18 = +7 \]

\[ M_{BA} = +M_0 - \frac{6EI}{L^2} \]

\[ M_{BC} = - \frac{PL}{8} + \frac{6EI\Delta}{L^2} \Rightarrow \frac{-100 \times 4}{8} + 18 \]

\[ = -82 \]

\[ M_{CB} = + PL + \frac{6EI\Delta}{L^2} \Rightarrow +50 + 18 = +68 \]
<table>
<thead>
<tr>
<th>Plant</th>
<th>Member</th>
<th>R. stiff</th>
<th>T.R.S</th>
<th>B.P. = R.S</th>
<th>T.R.S</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0.4</td>
<td>1.4</td>
<td>2.5</td>
<td>1.4</td>
<td>0.2</td>
</tr>
</tbody>
</table>

\[ \text{F.E.M.} \]

\[ \text{B.M.} \]

\[ \begin{align*}
A & \quad 13.25 \\
C & \quad 3 \times 19.5
\end{align*} \]

\[ \begin{align*}
\text{Total} & \quad 13.25 + 19.5 - 19.5 + 34.25 \\
\text{Net B.M. at Centre of AB} & \quad 74.25
\end{align*} \]
<table>
<thead>
<tr>
<th>Joint</th>
<th>Member</th>
<th>R.S.</th>
<th>T.R.C</th>
<th>D.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B A</td>
<td>1/4</td>
<td>25/4</td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>B C</td>
<td>3/4 x 1/3</td>
<td></td>
<td></td>
<td>1/2</td>
</tr>
</tbody>
</table>

Diagram:

- A to B:
  - 0 - 3 - 3
  - 1.5 ≤ -3
  - End correction
  - 0 - 4.5 - 0
  - +2.25 +2.25
  - 1.125
  - Final 1.125 +2.25 -2.25 0

- Formula:
  \[ \frac{wL^2}{8} = \frac{4 \times 3^2}{8} \]

- Centre of B C:
  \[ 45 - \frac{2.25}{2} - 3.375 \]
To find the consider force body equilibrium of AA

\[ SB = 0 \]

\[-HAX + 4(1.125 + 2.25) = 0 \]

\[ HA = \frac{4(1.125 + 2.25)}{4} = 10.84 \text{ kN} \]

To find RA Consider F.B. equil of B.C.

\[ 2.25 - 4 \text{ kN/m} \]

\[ R_A + R_B + R_C = 0 \]

\[ R_B - 2.25 - 4 \times 3 \times 1.5 = 0 \]

\[ R_B = R_A = 6.75 \text{ kN} \]

\[ R_A + R_C - 4 \times 3 = 0 \]

\[ R_C = 12 - 6.75 = 5.25 \text{ kN} \]
Analysis frame shown in fig. using M.O.M.
Draw B.M.D. and elastic curve.

\[ M_{BA} = 0 = M_{BA} \]
\[ M_{BC} = 0 = M_{CB} \]
\[ M_{BD} = -15 \]
\[ M_{DB} = 0 \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Member</th>
<th>R.S.</th>
<th>T.R.S.</th>
<th>( D_d = R_S - T_R.S. )</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>BA</td>
<td>5/14</td>
<td>31/4</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>B</td>
<td>BC</td>
<td>25/14</td>
<td>31/4</td>
<td>21/5</td>
</tr>
<tr>
<td>A</td>
<td>BD</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \text{Sum} = 0 \]
\[ 2.25 \]
Horizontal member AB:
15 kN-m.

B.M.D.
5 kN
S.F.D.
Analysis given frame shown in Fig. using moment distribution method and draw bending moment diagram.

\[ \begin{align*}
\bar{M}_{BC} &= M_0 = \left( -\frac{100}{4} \right) = -25 \\
\bar{M}_{CB} &= -25
\end{align*} \]

All members \( EI = \text{constant} \)

Other members are zero

<table>
<thead>
<tr>
<th>Joint</th>
<th>Member</th>
<th>R.S.</th>
<th>T.R.S</th>
<th>D.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>BA</td>
<td>I/4</td>
<td></td>
<td>( V_2 )</td>
</tr>
<tr>
<td>C</td>
<td>BC</td>
<td>I/4</td>
<td></td>
<td>( V_2 )</td>
</tr>
<tr>
<td>C</td>
<td>CB</td>
<td>I/4</td>
<td></td>
<td>( V_3 )</td>
</tr>
<tr>
<td>C</td>
<td>CD</td>
<td>I/4</td>
<td></td>
<td>( V_3 )</td>
</tr>
<tr>
<td>C</td>
<td>CE</td>
<td>I/4</td>
<td></td>
<td>( V_3 )</td>
</tr>
</tbody>
</table>
SWAY Analysis (Joint sway)

\[ BB' = CC' = D \]

\[ \Delta \rightarrow \text{Transverse displacement of member or shear displacement} \]

\[ \bar{M}_{AB} = \bar{M}_{BA} = -\frac{6EIa}{L^2} \]

\[ \bar{M}_{CD} = \bar{M}_{DC} = -\frac{6EIa}{L^2} \]

\[ \bar{M}_{BC} = \bar{M}_{CB} = 0 \]

Thumb rule to find direction of sway.

\[ \frac{w/m}{L} \]

\[ \text{Sway towards right}\]

Because stiffness of \( CD \) is less than stiffness of \( AB \).

Loading is symmetrical but stiffness is not symmetrical.
If loading, stiffness and support condition are symmetrical about the centre of frame, there will be no sway.

\[ \frac{L}{2} \downarrow \frac{L}{2} \]

(No sway)

\[ a > b \] sway towards left

due to stiffness of AB is less than stiffness of CD

\[ a < b \] sway towards right

\[ (R_i < p) \]

Left side sway
Toward Right.

Resistant Sway Towards Right: (→)

Sway may occur, which can be found by computation.
Procedures to Analyse Sway Cases when $A$ is not known

\[ B \quad A_2 \quad A \]

\[ 166 \]

\[ 0_2 - 0_1 = 0 \]

that is effect of Axial force are neglected

implying length of member do not change.

1. initially analysis of frame is done neglected sway.

It's called non sway analysis. In non sway analysis find

fixed end moment due to loading,

distribution factor at joints and compute the final

end moment at the end of table. called non sway

moments.

2. if frame is vertically loaded then horizontal

reaction at the support using non sway moment

given loading and check

\[ \Sigma f_x = HA + H_0 + P_1 \]

if \( \Sigma f_x = 0 \) there will be no sway but

\[ \Sigma f_x \neq 0 \] than sway force will be present.

and sway force is equal to \( \Sigma f_x \).

\[ s = \Sigma f_x \]
(3) To find the effect of sway force remove loading and apply sway force at the Panel.

Under sway force strut will go sway as shown in Fig. due to a fixing moment will be produced in those member for which sway displacement is produced.

If a cause clockwise rotational to the member than fixing moment produced will anti-clockwise and vice-versa.

\[ M_{AB} = M_{BA} = 6 - 6EIA \frac{A^2}{L^2} \]

\[ M_{BC} = M_{CD} = 0 \]

\[ M_{CD} = M_{DC} = -6EIA \frac{A^2}{6L^2} \]

Since \( a \) is not known, hence following procedure is adopted. Find ratio of fixing moments produce it's ratio.

\[ \frac{M_{AB}}{M_{BA}} : \frac{M_{BC}}{M_{CD}} : \frac{M_{CD}}{M_{DC}} = \frac{-6EIA}{6L^2} : \frac{-6EIA}{6EIA} : \frac{-6EIA}{6EIA} \]

\[ \frac{1}{1} : \frac{1}{0} : \frac{1}{0} : \frac{1}{1} : \frac{1}{0} : \frac{1}{1} \]
Adopted any fixing moments in above ratio say -10 -10, 0 0, -10 -10

Entre above, assume fixing moment in this table. & distribute them according to the distribution factor, at the end of table. There are

![Graph with columns and values](image)

The moment given in column @ are not actual sway moment, which are due to same sway force 's'.

![Diagram with forces](image)
\[ E f_x = 0 \]
\[ S' + H_A + H_B = 0 \] —— (1)

\( S' \) is known from eqn (1). \( H_A \) and \( H_B \) can be computed by considering free body equilibrium of AB & CD with moments given in column A.

The moment obtain in column (a), are due to same sway forces (5) as found above. Hence actual sway moment actual sway force (5) will be \[ \text{Column (a) \times S'} \]

Final end moment will be algebraic sum of non-sway moment (Step -1) & actual sway moment obtained from previous (Step 5).

Analysis the frame shown in fig. the use Hardy cross method of moment distribution.

Since support C can carry moment hence stiffness of BC is equal to YES.
Joint | Member | Stiffness | Relative | Total Stiffness | Distribution Factor
--- | --- | --- | --- | --- | ---
B | BA | I/4 | | 2I/4 | 1/2
| | BC | I/4 | | | 1/2

\[ \bar{M}_{AB} = -\frac{PL}{8} = -\frac{8 \times 4}{8} = -4 \]

\[ M_{BA} = +4 \]

\[ \bar{M}_{BC} = -\omega L^2 = - \frac{4 \omega L^2}{12} \]

\[ \bar{M}_{BC} = -\frac{4 \times 4 \times L}{12} \]

\[ \bar{M}_{BC} = -4 \]

\[ \overline{M}_{BA} = 46 \]

\[ \begin{array}{c|c|c|c|c|c}
\hline
 & 1/2 & & & & \\
\hline
1 & -4 & 4 & -16 & & \\
2 & -6 & 6 & & & \\
3 & & & & & \\
\hline
\end{array} \]

Free body equilibrium of AB

\[ 8 \text{KN} \]

\[ \overline{F}_{BA} = 10 \]

\[ \overline{F}_{AC} = 1 \]

\[ \overline{F}_{RA} = 1 \]

\[ \overline{M}_{AC} = 46 \]
\( \Sigma M_B = 0 \)

\[-HA x y - 1 - 8 x 2 + 10 = 0 \]

\[ HA = -1.75 \]

\[ H_c = 0 \]

\[ 3F_x = 0 \]

\[ 8 + HA = 8 - 1.75 = 6.25 \] (→)

Since \( 3F_x \) is not equal to zero, hence sway will occur. Sway force:

\[ S = 3F_x = 6.25 \text{ kN} \] (→)

Post of question: Sway analysis

\[ S \quad \beta + \Delta \rightarrow \]

\[ C_k \rightarrow A \rightarrow y_x \]

\[ \overline{MAB} = \overline{MBA} = -6F_x \Delta \]

\[ \overline{MBC} = \overline{MCB} = 0 \]

Ratio of fixing moment produced due to A:

\[ \frac{\overline{MBA}}{\overline{MAB}} = \frac{\overline{MBC}}{\overline{MCB}} = \frac{-6F_x \Delta}{-6F_x \Delta} = 1 : 1 : 0 : 0 \]

\[ -1 : -1 : 0 : 0 \]

\[ -8 : -8 : 0 : 0 \]
the moment in given column A are due to some sway force s' such that
s' + H A = 0

MB = 0

\[ \text{Column 3} \times 4 \text{ kN} = 8 \times 20 \]

\[ -H A \times 4 - 6 - 4 = 0 \]

H A = 2.5

s' - 25 = 0 \[ s' = 25 \]

the moment given in column A are due to some sway force s' hence actual sway moment + Column 3 x s

\[ \text{MB} = - \frac{86 \times 6.25}{2.5} = -15 \]

\[ \text{MBA} = \frac{-4 \times 6.25}{2.5} = -10 \]

<table>
<thead>
<tr>
<th>( \text{tab.1} )</th>
<th>( \text{WSN} )</th>
<th>Net Sway moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-16</td>
<td>0</td>
</tr>
</tbody>
</table>
\[
M_{Ed} = \frac{+4 \times 6.25}{2.5} = +10
\]
\[
M_{Ed} = \frac{+2 \times 6.25}{2.5} = +5
\]

\[E_{MB} = 0\]
\[-H_A x y - 16 - 8x^2 = 0\]
\[H_A = 8\]
\[H_C = 0\]

\[E_{fx} = 0\]
\[8 + H_A + H_C = 0\]
\[8 + 8 = 0 (0.1c)\]

End Moment Diagram.

16 kN-m

\[
M_e \frac{y}{I} = \frac{F}{R}
\]

Convex

Elastic curve.
Q. Analyze the frame shown in Fig. and draw the BMD using moment distribution method.

<table>
<thead>
<tr>
<th>Joint</th>
<th>Member</th>
<th>Relative Stiffness</th>
<th>Total Stiffness</th>
<th>D.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>BA</td>
<td>1/4</td>
<td>2/3</td>
<td>1/2</td>
</tr>
<tr>
<td>B</td>
<td>BC</td>
<td>3/8 x 1/3</td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>CB</td>
<td>3/8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>CD</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Moment distribution factor not required. Moment will be zero.

\[
\begin{align*}
M_{BA} &= 0 \\
M_{BA} &= 0 \\
M_{BC} &= 0 \\
M_{PC} &= 0 \\
M_{BC} &= -\frac{wL^2}{12} - \frac{12\times 3\times 3}{12} = -9 \text{ kN/m} \\
M_{CB} &= \frac{wL^2}{12} + \frac{12\times 3\times 3}{12} = 9 \text{ kN/m}.
\end{align*}
\]
Free body dia. AB

$H_A = 6.75 \text{ kN}$

$EM_c = 0$

$HD = 0$

$EF_x = 0$

$HA + HD = 2.53 + 0 = 2.53 \text{ kN} \quad (\text{sway})$

$s = \Sigma F_x = 2.53 \text{ kN} \quad (-\) \\

Remove the loading and apply sway force.

Right = 2.25 kN
Fixing moment due to say

$$\bar{M}_{AB} = M_{BA} = -6EEl^2$$

$$\bar{M}_{BC} = M_{CB} = 0$$

$$\bar{M}_{CD} = M_{DC} = 0 \text{ Both are ends hinged.}$$

Fixing moments:

- $$M_{AB} = M_{BA} = 0$$
- $$M_{BC} = M_{CB} = 0$$
- $$M_{CD} = M_{DC} = 0$$

$$\Rightarrow -10000000000$$

$$\Rightarrow -10000000000$$

A

B

C

D

-10
-10
0
0

-\theta

+5

-5

-5

+5

0

0

0
Moment obtain in column A are due to some sway force \( D' = \alpha x' \).

\[ H_a \rightarrow (0) \quad 7.5 \quad A \]

\[ H_a' \rightarrow \]

\[ H_{01}' \rightarrow \]

\[ H_{01}'' \rightarrow R_a \]

\[ R_{01}'' \]

for AB

\[ H_a + H_{01} + t_s' = 0 \quad (1) \]

Taking moment about A

\[ \sum M_a = 0 \]

\[ -H_a \times 4 - 7.5 \times 5 = 0 \]

\[ H_a = -3.125 \quad H_{01} = 0 \]

From eqn(1)\[ H_a + H_{01} + t_s' = 0 \]

\[ -3.125 \times 0 + t_s' = 0 \]

\[ t_s' = 3.125 \text{ kN} \]

Actual sway moment

Column(0) \( X \times t_s' \)

| Column(0) \( X \times t_s' \) | NSM \( \text{Table I} \) | Final Moment
|-----------------------------|--------------------------|------------------|
| MAB \( = \frac{-7.5 \times 2.53}{3.125} \) | 1 \( -6.04 \) | 13.375 \( -2.75 \)
| MBE \( = \frac{-5 \times 2.53}{3.125} \) | \( -4.05 \) | 6.75 \( +2.75 \)
| MCE \( = \frac{5 \times 2.53}{3.125} \) | \( +4.05 \) | \( -6.75 \) \( -2.75 \)
| MEB | \( = 0 \) | 0 | 0 | 0
| MCB | \( = 0 \) | 0 | 0 | 0
| MBC | \( = 0 \) | 0 | 0 | 0

Red Swear

Final Combined

\[ s' = 3.125 \text{ kN} \]
\[ \sum M_B = 0 \]

\[ -H_A \times 4 - 2.7 + 2.7 = 0 \]

\[ H_A = 0 \]

\[ \varepsilon M_r = 0 \]

\[ H_D = 0 \]

\[ \varepsilon f_c = H_A + H_D \]

\[ = 0 \]

\[ 12.5 \text{kN} \]

\[ 13.7 - 9.25 = 12.75 \]

\[ 2.75 \text{kN} \]
Analysis the beam shown in Fig.

\[ M_{AB} = \frac{-PL}{8} = \frac{-16 \times 4}{8} = -8 \]

\[ M_{BA} = +8 \]

\[ M_{BC} = -\frac{W L^2}{12} = -\frac{12 \times 3^2}{12} = -9 \]

\[ M_{CB} = +9 \]

member join R.S T.R.S g.p.

B A 21/4 21/4 3/2

B C \[ \frac{3}{4} \times \frac{1}{2} \]

-8 +8 -9 +9

-8 +8 -13.5 -9

+1.88 +8.66 +1.88

-0.77 -11.57 -11.67 -0

End Corrected

Corrected Value

Non sway moment
Consider free body diagram AB

\[ F_y = 10kN \]
\[ -6.17 + 11.67 = 0 \]

\[ \text{EMB} = 0 \]
\[ RA \times 4 - 6.17 - (16 \times 2 + 11.67) = 0 \]

\[ RA = 6.625 \ (\uparrow) \]

\[ 12kN/\text{m} \]
\[ 11.67 \times 3m \]

\[ \text{EMB} = 0 \]
\[ RC \times 8 - 11.67 + 12 \times 3 \times 1.5 = 0 \]

\[ RC = 14.41 \ (\uparrow) \]

\[ \Sigma F_y = 16 + 12 \times 3 - 6.625 - 14.11 \]

\[ S = \Sigma F_y = 31.625 \ (\uparrow) \]

**Sway Analysis:**

\[ 31.625 \]

**Due to D fixing moment:**

\[ \overrightarrow{MA_B} = \frac{3 \times \text{load} \times \Delta}{2} \Rightarrow \overrightarrow{MA_B} = \frac{2 \times 6.625 \times \Delta}{2} \]

\[ \overrightarrow{MB_C} = +6.3 \times \Delta \Rightarrow \overrightarrow{MB_C} \]

\[ M_{CB} = 0 \]
Ratio of fixing moment

\[ \frac{M_{AR}}{M_{EA}} : \frac{M_{EC}}{M_{ECB}} = \frac{-12EID}{-12EID} : +3EIA : 0 \]

\[ \frac{16}{16} = \frac{9}{9} \]

\[ \Rightarrow \frac{-3}{4} : \frac{-3}{4} : \frac{1}{3} : 0 \]

\[ \Rightarrow -9 : -9 : +4 : 0 \]

\[ \Rightarrow -18 : -18 : +8 : 0 \]

Columns:

\[ -14.67 \]
\[ -11.23 \]
\[ +11.23 \]

Due to some sway force $S'$

Obtain moments from a column after...
\[ \begin{align*}
\text{Actual sway moment} & = -14.67 \times 31.625 - 44.65 \\
\text{NSM} & = -6.17, 50 \text{ kgf} \\
\text{Find} & = 82.82
\end{align*} \]
\[ M_{bc} = \frac{113.3 \times 31.625}{10.27} \Rightarrow 34.45 \quad 11.67 \quad 92.82 \]

Let us check for:

\[ Ra \times 4 - 16 \times 2 - 22.82 - 50.83 = 0 \]

\[ Ra' = 26.44 \text{ KN} \]

\[ 82 \text{ KN} \times 3 \quad 22.82 \quad 12 \times 3 \times 15 + 22.82 = 0 \]

\[ Rc' = +25.68 \]

\[ Ef_y = Ra + Rc' = 16 + 12 \times 3 \]

\[ = 16 + 36 \]

\[ 26.4 + 25.6 \Rightarrow 52 \]

\[ 52 \Rightarrow 82 \quad \text{to check save.} \]
Centre ordinate and moment \( y = \frac{-50.8 \times 22.82}{2} \)
\( y = -25.41 \)

\[ \frac{\text{MAC} \times \text{PL}}{9} = \frac{16 \times 8}{8} = 8 \]

Proof \( \text{MAC} = \frac{2 \times 12^2}{9} = 12 \times 3^2 = 18.5 \)

**SE:** Using moment distribution method. Analysis of beam and draw the bending moment diagram.

- **Resultant:**
  - **Hnc:** \( -\frac{PL}{8} \) \( \rightarrow \frac{16 \times 8}{8} \) \( = -16 \) kN
  - **Hca:** \( -16 \) kN
  - **Mca = Mca = 0**

**Distributed Factor:**
- **Joint:**
  - **R:** \( \frac{2 \times 5}{4} \) \( \rightarrow \frac{3}{2} \)
  - **R:** \( \frac{2 \times 5}{4} \) \( \rightarrow \frac{3}{2} \)
  - **R:** \( \frac{2 \times 5}{4} \) \( \rightarrow \frac{3}{2} \)
Distribution factor \( \theta \) is known.

\[
\begin{array}{cccc}
-16 & +16 & 0 & 0 \\
-8 & -16 & 0 & 0 \\
NS&-24 & 0 & 0 \\
\end{array}
\]

Consider \( \theta = B \)

\[A_4 = 14 \quad B_4 = \frac{1}{2}\]

\[E_{MB} = 0\]

\[R_A x 8 - 24 + 16 x 4 = 0\]

\[R_A = 64 + 24 = 11 \, kN\]

Consider \( E = C \)

\[R_C = 0\]

\[\varepsilon_{fy} = 15 - 11 - 8 = 0\]

\[-5^\circ(4)\]

\[M_{AC} = -3EIA = \text{MNN}\]

\[M_{NC} = 0 - M_{CD} \quad M_{CR} = 3EIA \quad L^2\]
Fixed Moment

\[ MA = MA' = MB = MC = MC' \]

\[ c = -\frac{3 E I A}{4} \left( \frac{L}{2} \right)^2 = 0 \quad 0 + \frac{3 E I A}{4} \]

\[ \Rightarrow -1 : 0 : 0 : 1 \]

\[ \frac{-16}{64} = 0 \quad 0 \quad 0 \quad +1 \]

Set 1

\[ x = -10 : 0 : 0 : +40 \]

Consider AB

\[ R_{A'} = +1.125 \]

\[ R_{A} = 10 \times 4 \quad EMS = 0 \]

\[ -R_{c} \times 4 + 40 = 0 \]

\[ R_{c} = 10 \]
\[ \delta' = Ra' + Rc' \]
\[ = 1.25 + 10 \]
\[ \delta' = 11.25 \]

<table>
<thead>
<tr>
<th>Actual Sway Moment</th>
<th>NSM</th>
<th>Net Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>MA_B = -10 \times S</td>
<td>-24</td>
<td>28.44</td>
</tr>
<tr>
<td>MB_A = 11.25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>MB_C =</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>MC_B = \frac{40 \times S}{11.25}</td>
<td>0</td>
<td>17.78</td>
</tr>
</tbody>
</table>

Let us check:

\[ M_{AB} = 28.44 \]
\[ M_{BC} = \frac{32 - 28.44}{2} = 17.78 \]
\[ M_{CB} = 17.78 \]

\[ M_{φ_0} = \frac{4P.L}{4} = 16 \times 8 = 32 \]
Analyze the frame shown in Fig. using moment distribution method.

EI = constant
8. For the frame shown in Fig., draw the EM dia using Hard-Goss method. E IDia.

\[ M_{AB} = M_{BA} = 0 \]
\[ M_{BC} = \frac{-wL^2}{12} = \frac{-24 \times 4^2}{12} = -32 \]
\[ M_{CD} = +32 \]
\[ M_{BD} = M_{BC} = 0 \]

**Distribution Factor:**

<table>
<thead>
<tr>
<th>Joint</th>
<th>Member</th>
<th>Relative Stiffness</th>
<th>Total Relative Stiffness</th>
<th>SIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>BA</td>
<td>5/5</td>
<td>5/5</td>
<td>4/9</td>
</tr>
<tr>
<td></td>
<td>BC</td>
<td>1/4</td>
<td>2/5</td>
<td>5/3</td>
</tr>
<tr>
<td>C</td>
<td>CE</td>
<td>1/4</td>
<td>1/4</td>
<td>1/2</td>
</tr>
<tr>
<td></td>
<td>CD</td>
<td>1/4</td>
<td>1/2</td>
<td>1/2</td>
</tr>
</tbody>
</table>

\[ \text{NSM} = 19.5 \]
\[ 112 - 19 = 123 \]
\[ 121 - 21.8 = 99.2 \]
\[ 10.93 \]
To find HA in inclined member AB, RA is required since there is no vertical loading in AB. Hence RA = RB. Hence, RC can be completed by taking free body diagram of BC.

\[ \text{EMc} = 0 \]
\[ REX4 - 19 - 24 \times 4 \times 2 + 21.86 = 0 \]
\[ RA = RB = 47.285 \text{ kN} \]

To find HA, consider free body equilibrium of AB.

\[ \text{EMb} = 0 \]
\[ -HA \times 4 - 19 \times 8 + 9.5 + 13 = 0 \]

\[ HA = \frac{3 \times 47.289 + 9.5 + 13}{4} \]

\[ HA = 42.58 \text{ kN} \]
To find $H_d$, consider free-body equilibrium of $dc$

$$\Sigma M_c = 0$$

$$-H_d\times 4 - 10.93 - 21.86 = 0$$

$$H_d = -81.97 \text{ (→)}$$

Net Horizontal force:

$$\Sigma F_x = HA + H_d$$

$$\Rightarrow 42.598 - 8.197$$

$$\Rightarrow 34.398 \text{ KN (→)}$$

Sway analysis:

![Diagram of sway analysis](image)

Let $BB' = 0$

$$BB'' = \cos \theta = \frac{4}{5} \text{ A}$$

$$BB'' = \sin \theta = \frac{3}{5} \text{ A}$$

Horizontal comp. of sway at $BB, C$ are equal

$$BB' = CC'$$

$$CC' = \frac{4}{5} \text{ A}$$

$\text{Hd}$
due to sway fixing moment are developed.

\[ \bar{M}_{AB} = \bar{M}_{BC} = -6 \frac{EIa}{5} \frac{1}{25} \]

\[ \bar{M}_{BC} = \bar{M}_{CD} = \frac{6E(2/5d)}{4^2} \times \frac{18 \frac{EIa}{80}}{80} \]

\[ M_{CD} = M_{CD} = -6 \frac{EIa}{4^2} \frac{18}{80} : -24 \frac{EIa}{80} \]

Ratio of fixing moment due to sway

\[ \bar{M}_{AB} : \bar{M}_{AB} : \bar{M}_{BC} : \bar{M}_{CD} : \bar{M}_{CD} = \bar{M}_{DC} \]

\[ \Rightarrow -\frac{6}{25} : -\frac{6}{25} : 18/80 : 18/80 : -24/80 : -24/80 \]

\[ \Rightarrow -\frac{1}{25} : -\frac{1}{25} : \frac{3}{80} : \frac{3}{80} : -\frac{1}{20} : -\frac{1}{20} \]

\[ \Rightarrow -16 : -16 : +15 : +15 : -20 : -20 \]

Let us distribute the above moment.

<table>
<thead>
<tr>
<th>419</th>
<th>56</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>1/2</td>
</tr>
</tbody>
</table>

-16 -16 15 15 -20 -20

-22 -44 56 -2.5 -2.5 -1.25

-16.04 -16.08 +16.08 +17.48 -17.48 -18.28
Let us start given in code some way force both.

\[ R_A = R_B \]

To find the codenden slip \( \theta \):

\[ R_A \times 14 + 16.08 + 17.45 = 0 \]

\[ R_B = R_A = -8.785 \text{ kN} \]

To find the codenden slip \( \theta \) from \( R_A \rightarrow 2m > 0 \):

\[ R_A \times 3 - H_A \times 4 - 16.04 - 16.08 = 0 \]

\[ H_A = -14.32 \]
To find $H^b$ consider open $c^b$

$E_Mc = 0$

$H^b x 4 - 18.75 - 19.46 = 0$

\[ s' = 3.9A^b + H^b = 0 \]

\[ s' - 14.32 - 5.05 = 0 \]

\[ s' = 28.37 + kJ \]

Moment given in open area due to $s'$, hence actual tray moment.

\[
\begin{array}{c|c|c|c|c|c}
\text{equally spaced} & -25.58 & -23.64 & +23.64 & +25.58 & -25.67 & -24.20 \\
\text{Column x S} & s' & +19 & +19 & +21.86 & -21.86 & -10.93 \\
\text{NSM} & +11.08 & -4.64 & +4.67 & +47.53 & -47.53 & -38.00 \\
\text{final} & & & & & & \\
\text{moments} & & & & & & \\
\end{array}
\]

\[
M_{bc} = \frac{-48 \times 12}{8}
\]

\[
= \frac{24 \times 2 \times 4}{8}
\]

\[
= 48
\]

$M_{bc} = -48$

$\text{At centre } = -48 - \left( \frac{-47.53 - 47.53}{2} \right) = 26.55$
Analysis the frame shown in Fig. using E.M.M.

\[ \text{Distribution factor} \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Member</th>
<th>R.S.</th>
<th>T.R.S.</th>
<th>D.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>BA</td>
<td>1/6.5</td>
<td>23/1</td>
<td>10/23</td>
</tr>
<tr>
<td></td>
<td>BC</td>
<td>1/5</td>
<td></td>
<td>18/23</td>
</tr>
<tr>
<td></td>
<td>CB</td>
<td>1/5</td>
<td>8/5</td>
<td>1/2</td>
</tr>
<tr>
<td>C</td>
<td>CD</td>
<td>1/5</td>
<td>8/5</td>
<td>1/2</td>
</tr>
</tbody>
</table>

Fix. end moment all members

Hence Non sway moment not present

\[ S = 120 \]
Let $BB' = A$

$BB' = A \cos \theta_1 = 60 \Rightarrow \frac{120 \cos \theta}{65} = \frac{12}{13}$

$B'c'' = a \sin \theta_1 \Rightarrow \frac{2.5 A}{6.5} = \frac{5}{13} A$

Horizontal displacement $B = \text{Horizontal displacement}$

$c''c' = \frac{12}{13} A$

$c'c'' = \cos \theta_2$

$c'c' = \frac{12/13 A}{\cos \theta_2} = \frac{4.5}{13} A$

$c'' = c' \sin \theta_2$

$\Rightarrow 15 A \times \frac{2}{5} = \frac{9}{13} A$

Sway of $AB = BB' = A$

Sway of $BC = Bc'' + c''c'$

$= \frac{15 A + 9}{13}$

$= \frac{14 A}{13}$

Sway of $CD = cc' = 15 A$
fixing moment due to sway:

\[
\bar{M}_{ab} = -6EIa / 6.5^2
\]

\[
\bar{M}_{bc} = \bar{M}_{cb} = 6EIa / 5^2 = 6EI(14/13)a / 325
\]

\[
\bar{M}_{cd} = \bar{M}_{dc} = -6EI(15/12a) / 5^2 = 90a / 325
\]

ratio of fixing moment:

\[
\begin{align*}
&\frac{1}{6.5^2} : \frac{1}{6.5^2} : \frac{14}{13 \times 5^2} : \frac{14}{13 \times 5^2} : \frac{-15}{13 \times 5^2} : \frac{15}{13 \times 5^2} \\
&\Rightarrow 3 : 3 : 14 : 14 : 15 : 15 \\
&\Rightarrow \frac{3}{169} : \frac{3}{169} : \frac{14}{325} : \frac{14}{325} : \frac{15}{325} : \frac{15}{325} \\
&\Rightarrow -100 : -100 : +140 : +140 : -150 : -150 \\
&\Rightarrow \frac{-100}{13} : \frac{-100}{13} : \frac{140}{13} : \frac{140}{13} : \frac{-150}{13} : \frac{-150}{13} \\
\end{align*}
\]
Distribute above movement according to f. and obtain c' = let us movement given in c' = 0 are due to some sway force s', such that 
\[ c' + H_d + H_a = 0 \]

Find \( H_a \) and \( H_d \), \( R_a \) and \( R_d \) are required. \( R_a', R_d' \) can be found by using free body equation at span [c] with the given in column \( c' \) hence \( c' \) as known.

Final sway moment = \( s' \)

If moment sway moment are not given then final sway moment are final moment.

And moment dia will be final B.m.o. be there is no simply support dia.
Apex moment method.

Change in slope from A to B = 0

\[ \Delta \theta_{A-B} = 0 \]

\[
\delta = -\left[ \frac{M \times L}{EI} + \frac{M \times L}{2EI} + \frac{M \times L}{4} \right] + \\
\left[ \frac{1}{2} \times \frac{L}{g} \times \frac{P \times L}{8EI} \right] \times 2 + \frac{1}{2} \times \frac{P \times L}{g} \times \frac{L}{2} + \frac{P \times L}{g} \times \frac{L}{2} \]

\[ S_B = 0 \]

\[ \delta_{A-B} = 0 \]

\[ \theta_{A-B} = \Delta \theta_{A-B} \]
\[ Ma = -\frac{Pab^2}{L^2} \]
\[ Mb = \frac{Pa^2b}{L^2} \]
\[ \frac{Pab^2}{L^2} + \frac{Pa^2b}{L^2} = \frac{Pab(a+b)}{L^2} \Rightarrow \frac{pa}{b} = \frac{M}{L} \]

\[ AD + AO + OC + OD \]
\[ \frac{AC}{L} + \frac{CE}{L} + \frac{3EI}{L_0} \]
\[ \Rightarrow \frac{11EI}{L} \]

\[ (d) \]
so virtual work done by unit load = virtual work done by P1,

\[ 1. z = px + by \]

\[ z = px + by' \] (b)

**Diagram**

\[ \begin{align*}
\text{SM}_B &= 0 \quad \text{in Fig. 9 of AB}\n
Lx - HA + 6EI_1 - 6EI_2 &= 0 \\
HA &= - \frac{6EI_1}{L^2} \\
\text{SM}_C &= 0 \quad \text{in CP}\n
- H_0xL - \frac{3EI_2}{L^3} &= 0 \\
H_0 &= - \frac{3EI_2}{L^3} \\
F_A + H_0 &= 0 \\
P - 12EI_0 - \frac{3EI_2}{L^2} &= 0 \\
P &= \frac{15 EI_2}{L^3}
\end{align*} \]
Stiffness of BA = \frac{AE}{2}

Stiffness of CD = AE(2I) = \frac{8EI}{3}

Beam will sway towards left.

\overrightarrow{M_{BA}} = M_{BA} = -AEED
\frac{L_c^2}{2}

\overrightarrow{M_{CD}} = M_{CD} = \frac{-AE(2I)B}{3}

\frac{M_{BA}}{M_{CD}} = \frac{5}{2/9} = 9

\overrightarrow{M} = \frac{WL^2}{12}

\frac{P_1}{g} = 860

\frac{P_2}{g} = 480

\frac{f}{g} = 740 \text{ kNm.}
\[ \theta_{A_1} = \frac{ML}{3EI} \]

\[ \theta_{A_2} = \frac{ML}{6EI} \]

\[ N \theta = \theta_{A_1} - \theta_{A_2} = \frac{ML - ML}{3EI} - \frac{ML}{6EI} = 0 \]

\[(d)\]

\begin{align*}
\gamma & = (c) \\
\psi & = (c) \\
\beta & = (c) \\
\gamma & = (c) \\
\delta & = (c) \\
\end{align*}

**Stiffness of BC = \psi \left( \frac{EI}{L} \right) **

\[ \beta_n = 2\delta \left( \frac{EI}{L} \right) \]

**Total \( f \)**

**D.F. = \psi \left( \frac{1}{L} \right) \]

**Total \( f \)**
\[
\begin{align*}
\bar{M}_{AB} &= +\frac{wt^2}{12EI} - \frac{6EI}{1} \frac{wL}{2} \\
&= \frac{wt^2}{12EI} - \frac{6EI}{2} \frac{wL}{2} \\
&= 0 \\
\end{align*}
\]

(d.)

Shear Eqn.

\[\Sigma F_c = 0\]  \[\bar{H}_A + \bar{H}_D + F_P = 0\]

\[\Sigma \bar{M}_B = 0\] for AB

\[-H_A \bar{X}_L + \bar{M}_A + \bar{M}_{BA} = 0\]

\[\bar{H}_A = \frac{\bar{M}_{AB} + \bar{M}_{BA}}{L_1}\]

for BC

\[\Sigma \bar{M}_C = 0\]

\[\bar{H}_D = \frac{\bar{M}_{CD} + \bar{M}_{DC}}{L_2}\]

\[\frac{\bar{M}_{AB} + \bar{M}_{BA}}{L_1} + \frac{\bar{M}_{CD} + \bar{M}_{DC}}{L_2} + F_P = 0\]
\[ \theta = \frac{M}{2EI} \]

if \( \theta = 1 \)

\[ M = k \]

\[ K = \frac{2EI}{L} \]

\[ fEI = 81380 \text{ KN-m}^2 \]

Stiffness of all member \( B = \frac{4EI}{L} + \frac{4EI}{L} + \frac{3EI}{L} \]

\[ K = \frac{11EI}{L} \]

\[ M = K \cdot \theta \]

\[ M = \frac{11EI \cdot \theta}{11EI} \]
Based on stiffness concept, equilibrium concept, displacement concept.

**Given by G. A. Money**

Basic unit unknown are joint displacement (Ω & Δ).

Joint displacement are found using joint equilibrium condition & shear equation.

Joint displacement are relate to the member force i.e. moments, such equation are called slope-deflection equation.

Joint are considered rigid i.e. angle h.w member meeting at rigid joint do not change after loading.

Joint rotates a whole.

![Diagram of before and after loading](attachment:image.png)
**Derivation of Slope Reflection Eqn:**

1. **Sign Convention:**
   - (+) Clockwise
   - (−) Anticlockwise

2. **End Moments:**
   - (+) Clockwise
   - (−) Anticlockwise

3. **Slope (Rotation of Joints):**
   - (+) Clockwise
   - (−) Anticlockwise

4. **Slope Linear Displacement:**
   - Those ±S which will produce clockwise rotation to the member will be treated as +ive.

   Those ±S which produce anticlockwise rotation to the member will be treated as −ive.

![Diagram](image-url)
Derivation of S.D.E. Deflection 783

Step 1: Consider A & B to be fixed support and due to given loading fixed end moment will be M_{AB} and M_{BA}

\[ M_{AB} = -\frac{Pab^2}{L^2} \]

Step 2: If point A rotated rotates by OA then moment produced at A will be \[ \frac{uEIx \theta_A}{L} \]

Carry over moment of \[ \theta_A = \frac{2EIx \theta_A}{L} \]

3) If supports B by rotates by \[ \theta_B \] than moment produce B will be \[ uEI \theta_B \] than carry over moment of \[ \theta_B \] will be \[ \frac{2EIx \theta_B}{L} \]

4) If one of the support settles (say) support B settled by \[ \theta \] then fixed and produced to settlement

\[ M_{AB} = \frac{6EI \theta}{L^2} \quad M_{BA} = -\frac{6EI \theta}{L^2} \]

Where \[ \theta \] is treated (sign) if it produces clockwise rotation to the member.
Final effect of all the above 4 cases.

Final moment in AB at A

\[ M_{AB} = M_{AB} + \frac{4EI \cdot \theta A + 2EI \cdot \theta B - 6EI \Delta}{L} \]

\[ M_{AB} = M_{AB} + \frac{2EI}{L} \left[ 2\theta A + \theta B - \frac{8A}{L} \right] \quad \text{force-disp. relatio} \]

Final moment at A in AB

\[ M_{BA} = M_{BA} + \frac{2EI}{L} \left[ 2\theta A + \theta B - \frac{8A}{L} \right] \]

\[ M \Rightarrow \text{tive} \quad 2 \]

\[ \theta \Rightarrow \text{tive} \quad 2 \]

\[ A \Rightarrow \text{tive if produce Q not to the member} \]

The joints displacements (\( \theta, A \)) are basic unknowns which are to be determine using joint equilibrium conditions and shear equn. The no. of joint equilibrium condition will be equal to no. of rotational displacement(\( \theta \)).

No. of shear equation will be equal to no. of shear displacement(\( A \)).
In this analysis members are considered inextensible (axially rigid).

\( \theta_B + \theta_C + \Delta \)  \( (\Delta B = \Delta C = \Delta) \)

Eqn of joint B (joint equilibrium condition)

\[ \Sigma M_B = 0 \]
\[ -M_B A - M_B C = 0 \]
\[ M_B A + M_B C = 0 \]

Joint equil at C \( \Sigma M_C = 0 \)

\[ -M_C B - M_C D \Sigma M_B + M_D = 0 \]
\[ M_C B + M_C D = M_B \]
Shear Equation: \( \Sigma F_x = 0 \)

\[ M_A + M_B + M_C = 0 \quad \text{(iii)} \]

Solving eqn (i) \& (iii), \( M_A \), \( M_B \), and \( M_C \) will be known and hence final end moment in each member can be found.

Bending moment dia is similar to that of moment distribution method.

For analysis the beam by Slope Deflection method

\[ F \text{EI} = \text{Constant} \]

Fixed end moment

\[ \bar{M}_{AB} = -\frac{PL}{8} = \frac{60 \times 6}{8} = -45 \text{ KN.m} \]

\[ \bar{M}_{EA} = +\frac{PL}{8} = +45 \text{ KN.m} \]

\[ \bar{M}_{BC} = -\frac{wL^2}{24} = -\frac{10 \times 6^2}{24} = -30 \text{ KN.m} \]

\[ \bar{M}_{CE} = +30 \text{ KN.m} \]
slope deflection equation:

\[ MA = \overline{MA} + \frac{2EI}{L} \left( \theta_A + \theta_R - \frac{3\theta}{L} \right) \]

\[ MA = -45 + 2EI \left( \theta_B \right) \]

\[ MBA = 45 + 2EI \left( \theta_B \right) \]

\[ MBc = -30 + 2EI \left( \theta_B + \phi_c - \frac{3\theta}{B} \right) \]

\[ MBc = -30 + 2EI \left( \theta_B \right) \]

\[ MCB = 30 + 2EI \left( \theta_B \right) \]

Joint equilibrium equation at \( B \):

\[ MBA + MBc = 0 \]

\[ 45 - 30 + 2EI \left( \theta_B \right) + 30 + 2EI \left( \theta_B \right) = 0 \]

\[ 45 + 2EI \left( \theta_B \right) = 30 \]

\[ \theta_B = -15 \]

\[ OR = -15 \times 3 = -45 \]

\[ \theta_c = 45 + 2EI \]

\[ \theta_c = \frac{45}{2EI} \]
substituting \( a_b \) in eqn (a)(ii) (a iii) (iv) \\
\[
M_{AB} = -45 + \frac{\beta E I x - 45}{\beta} \\
M_{AB} = -45 + 0.37 = -45 + 3.75 \\
M_{BA} =
\]
\[
M_{AB} = -48.75 \\
M_{BA} = +37.50 \\
M_{AC} = -27.50 \\
M_{CB} = +26.25
\]

Simply support at Centre of \( AB \) = \( \frac{PL}{4} = \frac{60 \times 6}{4} = 30 \)

Net B.M. at Centre of \( AB \) = \[
90 - \left(\frac{48.75 + 37.5}{2}\right) = 46.875
\]

Simply supported, moment at \( BC = \frac{WL^2}{8} = \frac{10 \times 6^2}{8} = 45 \text{ KN-m} \)

Net B.M. at Centre of \( BC = 45 - \left(\frac{37.5 + 26.25}{2}\right) = 13.125 \)
a. Analysis the beam shown in Fig. using slope deflection method.

Note: Above Simply supported beam is determinate which can be analysed using equilibrium conditions alone. However if method of indeterminate analysis are required to use than any of the stiffness method may be applied but computations will be lengthy.

Those slope deflection method consider above structure is made of two member AB and BC hence B is a assumed rigid joint which is unsupported.

Therefore will be 4 unknowns:

\( \theta_A, \theta_B, \theta_C, \theta_D \)

Fixed end moment:

\[ M_{AB} = M_{BA} = M_{AC} = M_{CB} = 0 \]

Slope deflection equation:

\[ M_{AB} = \frac{M_{AB}}{L} + 2EI \left( \frac{2A + \theta_B - \frac{3A}{L}}{L} \right) \]

\[ -2EI \left( \frac{2A + \theta_B - \frac{3A}{L}}{L} \right) \]

\[ M_{AB} = \frac{4EI \theta_A}{L} \]

of \( \frac{4EI}{L} \left( 2A + \theta_B - \frac{3A}{L} \right) \) - (i)
\[ \text{MBA} = 0 + \frac{4\varepsilon}{L} (20b + OA + 30) - (ii) \]
\[ \text{MBA} = 0 + \frac{4\varepsilon}{L} (20b + OA - 6A) \]
\[ \text{MBA} = 0 + \frac{4\varepsilon}{L} (20b + Oa + 6A) - (iii) \]
\[ \text{MBA} = 0 + \frac{4\varepsilon}{L} (20b + 0a + 6A) - (iv) \]
\[ \text{Joint Eqm Equation.} \]
\[ \text{MAB} = 0 \quad - (i) \]
\[ \text{MBA} + \text{MBC} = 0 \quad - (ii) \]
\[ \text{Mc} = 0 \quad - (iii) \]
\[ \text{Shear Eqn. RA + RC} + x = 0 \quad - (iv) \]

for RA & RB

\[ \text{RA} = \text{MBA} \]

\[ \text{RA} = \frac{MBA}{2} \]
\(-RC \frac{L}{2} + M_{BC} + M_{CR} = 0\)

\[RC + \frac{M_{BC}}{L/2}\]

put in B

\[-\frac{M_{BA}}{L/2} + \frac{M_{BC}}{L/2} = -P, \quad -D\]

\[-M_{BA} + M_{BC} = \frac{PL}{2}\]

Solving eqns (A), (B), (C) & (D)

\[\theta_A = \frac{PLa^2}{16EI}\]

\[\theta_B = 0\]

\[\theta_C = -\frac{PL^2}{16EI}\]

on substituting values in eqn (D) confirms (6)

\[M_{BA} = 0\]

\[M_{BA} = -\frac{PL}{4}\]

\[M_{AC} = +\frac{PL}{4}\]

\[M_{CR} = 0\]
Analyzed the frame shown in fig. using slope deflection method.

\[ \begin{align*}
15 \text{kN/m} & \quad 2.4 \text{kN/m} \\
B & \quad C \\
15 \text{m} & \quad 12 \text{m} \\
10 \text{kN} & \quad 15 \text{m}
\end{align*} \]

Sol. Fixed End moment

\[ M_{AB} = 1 - \frac{M_1}{b} = -\frac{12 \times 3}{8} = -4.5 \text{ KN}\cdot\text{m} \]

\[ M_{EA} = +4.5 \text{ KN}\cdot\text{m} \]

\[ M_{EC} = -\frac{w L^2}{12} = -\frac{9}{8} \text{ KN}\cdot\text{m} = -\frac{209}{32} \text{ KN}\cdot\text{m} \]

\[ M_{EB} = +\frac{209}{32} \text{ KN}\cdot\text{m} \]
solve reflection eqn

\[ M_{AB} = M_{AB} + 2EI \left( \alpha A + \alpha B - \frac{3\alpha}{1} \right) \]

\[ M_{AB} = -4.5 + 2EI \left( 0B \right) \quad \text{(1)} \]

\[ M_{BA} = 4.5 + 2EI \left( 2OB \right) \quad \text{(2)} \]

\[ M_{BC} = M_{AC} + 2EI \left( 2OB + p \alpha - \frac{3\alpha}{1} \right) \]

\[ M_{BC} = -32 + 2EI \left( 0B \right) \quad \text{(3)} \]

\[ M_{CB} = 32 + 2EI \left( 0B \right) \quad \text{(4)} \]

Joint force equation

\[ M_{BA} + M_{BC} = 0 \quad -15 \]

\[ 4.5 + 2EI \left( 2OB \right) - 32 + EI0B = -15 \]

\[ 4.5 + 1.33EI0B - 32 + EI0B = -15 \]

\[ 2.33EI0B = -15 + 32 - 4.5 \]

\[ EI0B = 1.5 \quad \text{or} \quad 1.25 \]

\[ \text{OR} = \frac{18.29}{5} \quad \text{EI} \]
\[ \Sigma M_B = 0 \]

\[ -M_{BA} - M_{BC} - 15 = 0 \]

\[ M_{BA} + M_{BC} = 15 \]

\[ M_{BA} = -0.93 \]

\[ M_{BA} = +11.64 \]

\[ M_{BC} = -26.64 \]

\[ M_{GB} = +34.68 \]

\[ \frac{4}{3} \times 3 = 4 \]

\[ h_c = \frac{4h^2}{8} = 4h^2 - 4h - \left( \frac{26.64 + 34.68}{2} \right) \]

\[ = 17.34 \]
Consider B is an unsupported joint, unknown.

\( \theta_A, \Delta_B \)

\( \theta_C \) (\( \theta_A = \theta \)) are unknown

**Fixed End Moments**

\[
\begin{align*}
\bar{M}_{AB} &= \bar{M}_{EA} = \bar{M}_{EC} = \bar{M}_{CD} = 0

\text{Moment Slope Deflection Equations}

\bar{M}_{AB} &= \bar{M}_{AB} + \frac{2EI}{L} (2\theta_A - \theta_B - 3\theta_B) = 0

= 0 + \frac{2EI}{2} (\theta_B - \frac{3\theta_B}{2})

\bar{M}_{AB} &= EI \left( \theta_B - \frac{3\theta_B}{2} \right) \quad (1)

\bar{M}_{EA} &= 0 + \frac{2EI}{2} (2\theta_B - \frac{3\theta_B}{2}) \quad (2)

\bar{M}_{EC} &= \bar{M}_{EC} + \frac{2EI}{L} (2\theta_B + \theta_C - \frac{3\theta_B}{2})

\bar{M}_{BC} &= 0 + \frac{2EI}{2} (2\theta_B + A) \quad (3)

\bar{M}_{CD} &= 0 + \frac{2EI}{2} (\theta_C + B) \quad (4)
\end{align*}
\]
Joint eqn.

\[ M_{BA} + M_{BC} = 0 \]

\[ R_A \times 2 + M_{BC} + M_{BA} = 0 \]

\[ 2R_A - 1.5A \]

\[ M_{BA} + M_{BC} = 24 \]

\[ EI (2\theta - 1.5\delta) + \frac{2}{3} EI (2\theta + \theta) = 24 \]

\[ 2\theta EI - 1.5\delta EI + 1.32\theta EI + 0.66\theta EI = 24 \]

\[ 3.22\theta EI - 0.84\theta = 24 \]

\[ 3.22\theta - 0.84\theta = \frac{24}{EI} \]

Shear eqn. \( \delta_x = 0 \)

\[ R_C = M_{BC} + M_{CB} = 108 \]

\[ \Rightarrow 0.66 EI (2\theta + \delta) + 0.66 EI (\theta + \delta) \]

\[ \Rightarrow 1.32\theta EI + 0.66\theta EI + 0.66\theta EI + 0.66\theta EI \]

\[ \Rightarrow 0.44\theta EI + 0.22\theta EI + 0.22\theta EI \]

\[ \Rightarrow 0.66\theta EI + 0.44\theta EI \]

\[ b = \sum_{i=1}^{n} \left[ \frac{M_{BA} + M_{BC}}{2} \right] \]
\[-\frac{1}{2} \left[ E_1 (0.8 - 1.5 \Delta) + E_1 (2.0\Delta - 1.5\Delta) \right] \]

\[\Rightarrow -\frac{1}{2} \left[ E_1\Delta - 1.5 E_1 + 2 E_1 \Delta - 1.5 E_1 \Delta \right] \]

\[\Rightarrow -1.5 E_1 \Delta + 1.5 E_1 \Delta = 0 \]

\[0.66 E_1 \Delta + 0.44 E_1 = -1.5 E_1 \Delta + 1.5 E_1 \]

\[2.16 E_1 \Delta = 1.06 E_1 \]

\[\Delta = \frac{1.06 E_1}{2.16} \approx 0.50 E_1 \]

\[P = \frac{24}{E_1} \]

\[\theta_2 = \frac{9.06}{E_1} \]

\[\Delta = 3.456 \]

\[M_{AB} = M_0 \frac{b(3a - l)}{12} \]

\[= \frac{24 \times 1.3 \times (3 \times 2 - 5)}{52} \]

\[= \frac{24 \times 1.3 \times 1}{5} \Rightarrow + 2.88 \]

\[M_{AB} = 2.88, \quad M_{AB} = 10.94, \quad M_{AB} = 13.06, \quad M_{AB} = 7.44 \]

\[M = 7.44 \]
Analysis the beam shown in fig. Wtih no deflection method, joint C is supported over a spring with stiffness \( K = \frac{E I}{g} \).

\[ 26\text{KN} \]
\[ 26\text{KN} \]

\[ \begin{align*}
A &\quad 4m \quad 2m \quad B \\
\end{align*} \]

\[ \begin{align*}
\text{EA} &\quad \text{E constant} \\
RC &\quad RC \\
\end{align*} \]

\[ \boxed{224} \]

**Fixed end moment**

\[ \text{MA} = \frac{-wab^2}{2} \quad \text{Unknown are} \quad OB, OA, AC \]

\[ RC = K \cdot \Delta \text{spring} \]

\[ RC = K \cdot AC = \frac{E I \cdot AC}{g} \]

\[ \Delta \text{spring} = AC \]

**Fixed end moment**

\[ \text{MA} = \frac{-wab^2}{2} = \frac{-26 \times 4 \times 2 \times 2}{6 \times 6} = -16\text{KN}\cdot\text{m} \]

\[ \text{ MBA} = 16\text{KN}\cdot\text{m} \]

\[ \text{MBC} = 0 \]

\[ \text{RCB} = 0 \]
Slop Deflection Eqn

\[ M_{AB} = -10 + \frac{2EI}{6} (20A + 0A - \frac{L}{2}) \]

\[ = -16 + \frac{EI}{3} \]

\[ M_{AB} = -16 + \frac{EI\theta_B}{3} \]

\[ (1) \]

\[ M_{BA} = +32 + \frac{EI}{3} \frac{20B}{3} \]

\[ (11) \]

\[ M_{MC} = 0 + \frac{2EI}{12} (20B + 0C - 3\Delta C) \]

\[ = 0 + \frac{9EI}{4} (20B + 0C - \frac{10\Delta C}{8}) \]

\[ (11) \]

\[ M_{MC} = 0 + \frac{2EI}{3} (20B + 0C - \Delta C) \]

\[ (11) \]

\[ M_{CB} = \frac{2EI}{3} (20C + 0B - \Delta C) \]

\[ (11) \]

Joint equation at B

\[ M_{BA} + M_{MC} = 0 \]

\[ 82 + EI (0.66)0B + 0.66EI [20B + 0C - \Delta C] = 0 \]

\[ 82 + 0.66EI0B + 1.22EI0B + 0.66EI0C - 0.66\Delta C = 0 \]

\[ 82 + 1.98EI0B + 0.66EI0C - 0.66\Delta C = 0 \]

\[ 1.98EI0B + 0.66EI0C - 0.66\Delta C = -82 \]
Joint eqn
\[ A + C = 0 \]
\[ M_{CB} = 0 \]  \( \textbf{8} \)

Shear eqn for Joint C
\[ R_c - 26 - R_c_1 = 0 \]
\[ \frac{EI \Delta_c}{g} - 26 - R_c_1 = 0 \]
\[ EM_B = 0 \]
\[ -R_c_1 \times 2 + M_{BC} = 0 \]
\[ R_c_1 = \frac{M_{BC}}{3} \]

Shear eqn will be
\[ \frac{EI \Delta_c}{g} - 26 - \frac{M_{BC}}{3} = 0 \]  \( \textbf{6} \)

\[ M_{CB} = 0 \]
\[ 0.66EI (20c + 0.8 - A_c) = 0 \]
\[ 1.32EI \Delta_c + 0.66EI \theta_B - A_c = 0 \]

Sliding eqn \( \theta_B = \frac{6}{EI} \quad \theta_c = \frac{60}{EI} \)
\[ A_c = \frac{126}{EI} \]
Analysis of the continuous beam shown in figure using the slope reflection method. Support and settle down 5 mm.

\[ E = 2 \times 10^5 \text{ N/mm}^2 \quad I = 36 \times 10^6 \text{ mm}^4 \]
\[ I = 36 \times 10^{-5} \text{ mm}^4 \]
\[ \Delta_B = 5 \text{ mm} \]

\[ M_{AB} = M_{AB} + 2EI(2\theta_1) \left( \frac{1}{3} \theta_A + \theta_B - 3 \times 5 \times 10^{-3} \right) \]

\[ M_{BA} = \frac{4EI}{3} (\theta_B - 5 \times 10^{-3}) \quad \text{(i)} \]

\[ M_{BC} = \frac{4EI}{4} (2\theta_B - 5 \times 10^{-3}) \quad \text{(ii)} \]

\[ M_{c} = 6 + 2E(2.5\pi) \left( \frac{2\theta_B + \theta_c + 3 \times 5 \times 10^{-3} \pi}{4} \right) \]

\[ M_{BC} = \frac{5EI}{4} \left( 2\theta_B + \theta_c + 15 \times 10^{-3} \right) \quad \text{(iii)} \]

\[ M_{CB} = \frac{5EI}{4} \left( 2\theta_c + \theta_B + 15 \times 10^{-3} \right) \quad \text{(iv)} \]

\[ M_{CD} = \frac{5EI}{4} \left( 2\theta_c \right) \quad \text{(v)} \]

\[ M_{DC} = \frac{5EI}{4} \left( \theta_c \right) \quad \text{(vi)} \]

Joint EQU at B

\[ M_{BA} + M_{BC} = 0 \]

\[ 1.25EI(2\theta_B - 5 \times 10^{-3}) + 1.25EI(2\theta_B + \theta_c + 3 \times 5 \times 10^{-3}) \]

\[ 1.25 \times 2 \times 10^5 \times 36 \times 10^6 \]

\[ 0 \times 1.75 \times 2 \times 10^5 \times 36 \times 10^6 \]

\[ 1.5 \times 10^7 \]
\[ 2.66 \times 10^{-3} EI + 2.50 \times 10^{-3} EI = 0 \]
\[ 0.16 EI \phi + 1.25 EI \theta = 1.97 \times 10^{-3} EI = 0 \]
\[ 5.16 EI \phi + 1.25 EI \theta = -1.97 \times 10^{-3} EI \]

At joint C:
\[ M_{DA} + M_{CB} = 0 \]
\[ 1.25 EI (20\phi + \theta + 3.75 \times 10^{-3}) + 1.25 EI \theta = 0 \]
\[ 2.50 EI \phi + 1.25 EI \theta + 4.68 \times 10^{-3} EI = 0 \]
\[ 5.16 EI \phi + 1.25 EI \theta + 4.68 \times 10^{-3} EI = 0 \]

\[ 1.25 EI \theta + 5 EI \phi = -4.68 \times 10^{-3} EI \]

\[ \theta = +6.49 \times 10^{-4} \]
\[ \phi = -1.09 \times 10^{-3} \]

\[ M_{AB} = -41.77 \quad M_{CB} = +19.78 \]
\[ M_{BA} = -35.53 \quad M_{D} = -19.78 \]
\[ M_{BC} = +35.33 \quad M_{Dc} = -9.89 \]
For the frame shown in Fig. support A settled by 15 mm vertically down. Analyse the frame by slope-deflection method and draw the end dia. Given that EI = 48000 KN-m.

\[ \text{Vertical deflection at } A = 15 \text{ mm} \]

\[ \text{Horizontal deflection at } B = \Delta \]

\[ \text{Deflection at } C = \Delta \]

\[ \text{FEM} = \Delta MA_B \]

\[ MA_B = 0 + \frac{2EI}{L} \left( \frac{2\theta_B + \theta_A + \frac{3A}{L}}{2} \right) \]

\[ MA_B = \frac{2EI}{3} (\theta_B + A) \quad \text{(i)} \]

\[ MB_A = \frac{2EI}{3} (2\theta_B + A) \quad \text{(ii)} \]

\[ MA_C = 0 + \frac{2EI}{3} \left( \frac{2\theta_B + \theta_C + \frac{5}{3} \times 15 \times 10^{-3}}{3} \right) \]

\[ MC_C = \frac{1}{3} \frac{4EI}{3} \left( \frac{2\theta_B + \theta_C + 15 \times 10^{-3}}{3} \right) \quad \text{(iii)} \]

\[ MC_B = \frac{4EI}{2} \left( \frac{2\theta_C + \theta_B + 15 \times 10^{-3}}{3} \right) \quad \text{(iv)} \]
\[ M_{CD} = 0 + 2E(3I) \left[ 20c+0 \right] \left[ 20c+0 \right] \frac{d}{4} + \frac{3d}{4} \]

\[ M_{CD} = EI \left[ 20c + \frac{3d}{4} \right] \quad \text{(7)} \]

\[ M_{BC} = EI \left[ 20c + \frac{3d}{4} \right] \text{or} 750 \quad \text{(7)} \]

Joint eqn A+B

\[ M_{BA} + M_{BC} = 0 \]

\[ 0.67EI (20c + 10) + 1.33EI (20c + 0.75a + 15 \times 10^{-3}) = 0 \]

\[ 1.34EI 0c + 0.67EI A + 2.66EI 0c + 1.33EI 0c + 19.95 \times 10^{-3} = 0 \]

At joint C

\[ M_{CG} + M_{CD} = 0 \]

\[ 1.33EI (20c + 0c + 15 \times 10^{-3}) + EI (20c + 0.75a) = 0 \]

\[ 2.66EI 0c + 1.330c + 19.95 \times 10^{-3} + 2EI 0c + 0.75a = 0 \]

\[ 4.66EI 0c + 1.330c + 0.75a + 19.95 \times 10^{-3} = 0 \]

Shear eqn HA + HD = 0

To find HA, consider FB of AB

\[ S_{MB} = 0 \]

\[ HA \times 3 + M_{BA} + M_{BA} = 0 \]

\[ HA = \frac{M_{BA} + M_{BC}}{3} \]
To find \( H_0 \), consider \( FB = 8 + c_d \)

\[ EM_c = 0 \]

\[ H_0 x + M_{CD} + FM_{DC} = 0 \]

\[ H_0 = \frac{M_{CD} + FM_{DC}}{x} \]

\[ HA = \frac{MAB + MBA}{3} \]

\[ = \frac{1}{3} \left[ 0.67EI\theta + 0.67\Delta + 1.34EI\theta + 0.67\theta D \right] = 0 \]

\[ = \frac{1}{3} \left[ 2EI\theta + 1.34\Delta \right] = 0 \]

\[ = 0.67 EI\theta + 0.45\theta = 0 \] \( \text{(1)} \)

\[ HD = \frac{M_{CD} + M_{DC}}{41} \]

\[ = \frac{1}{4} \left[ 0.75 EI\theta c + 0.75 EI\theta c + 0.75 \theta \right] \]

\[ = 0.75 EI\theta c + 1.5 \theta = 0 \] \( \text{(2)} \)

\[ \theta EI\theta + 2EI\theta + 19.95 \times 10^{-3} = 0 \]

\[ MAB = +91.63 \text{ kN-m} \]

\[ MBA = -65.82 \]

\[ MBC = +65.82 \]

\[ MCB = +116.35 \]

\[ MCD = -116.35 \]

\[ MDC = +21.92 \]
Joint equation:

At joint B:

\[ M_{BA} + M_{BC} + M_{BE} = 0 \]  \( \text{(i)} \)

At joint C:

\[ M_{CR} + M_{CD} = 0 \]  \( \text{(ii)} \)

At joint D:

\[ M_{DF} + M_{DC} = 0 \]  \( \text{(iii)} \)

At joint F:

\[ M_{EF} + M_{ER} + M_{EF} = 0 \]  \( \text{(iv)} \)

Shear equation:

\[ 3F = 0 \]

\[ HA + HF + 10 + 20 = 0 \]  \( \text{(v)} \)

\[ \Delta F = 0 \]

\[ HA + HE = 10 = 0 \]  \( \text{(vi)} \)
\[ M_{BC} = M_{AB} + 2EI \left( 2\theta_B + \frac{3\theta_C}{4} \right) \]

\[ M_{AB} = \frac{M_{AB}^0}{4} + 2EI \left( \frac{20C + 2B - 3(C - A)}{4} \right) \]

Slope deflection equation for AB

At B sink 50 mm

50 kN/m

At A sink 30 mm

1600 kN/m

\[ \delta_C = 3 \frac{P_C x 10^{-3}}{1600} \]

\[ P_C \text{ unknown} \]
\[ E_{M_B} = 0 \]

\[ M_{RA} + M_{RB} = 0 \quad (1) \]

\[ M_{AB} = 0 \quad (2) \]

\[ M_{CB} = 0 \quad (3) \]

\[ E_{fy} = 0 \]

\[ P_{A} + P_{B} + P_{C} = 1600 \times 8 \quad (4) \]
Assumptions:

- Pin joint frames
- All joints are connected by frictionless hinges
- All members are straight
- Loading is applied only at joint
- Self weight of member is neglected
- Members of truss carry only axial forces (Caused by Tension) No S.F. or B.V.

Hooke's Law is valid; the material is isotropic, homogeneous, and linear elastic.

Method: Method of Joint
- Method of section
- Graphical method

Method of Force (method/strain energy method) (Compatability method)

Second Method: Displacement method
- Equilibrium method
- Stiffness method

Graphical Method:
Bridge Trusses. The main structural elements of a typical bridge truss are shown in Fig. 3–4. Here it is seen that a load on the deck is first transmitted to stringers, then to floor beams, and finally to the joints of the two supporting side trusses. The top and bottom cords of these side trusses are connected by top and bottom lateral bracing, which serves to resist the lateral forces caused by wind and the sideways caused by moving vehicles on the bridge. Additional stability is provided by the portal and sway bracing. As in the case of many long-span trusses, a roller is provided at one end of a bridge truss to allow for thermal expansion.

A few of the typical forms of bridge trusses currently used for single spans are shown in Fig. 3–5. In particular, the Pratt, Howe, and Warren trusses are normally used for spans up to 200 ft (61 m) in length. The most common form is the Warren truss with verticals, Fig. 3–5c. For larger spans, a truss with a polygonal upper cord, such as the Parker truss, Fig. 3–5d, is used for some savings in material. The Warren truss with verticals can also be fabricated in this manner for spans up to 300 ft (91 m). The greatest economy of material is obtained if the diagonals have a slope between 45° and 60° with the horizontal. If this rule is maintained, then for spans greater than 300 ft (91 m), the depth of the truss must increase and consequently the panels will get longer. This results in a heavy deck system and, to keep the weight of the deck within tolerable limits, subdivided trusses have been developed. Typical examples include the Baltimore and subdivided Warren trusses, Figs. 3–5e and 3–5f. Finally, the K-truss shown in Fig. 3–5g can also be used in place of a subdivided truss, since it accomplishes the same purpose.
BRIDGE TRUSSELS

- Pratt (a)
- Howe (b)
- Warren (with verticals) (c)
- Parker (d)
- Baltimore (e)
- Subdivided Warren (f)
- K-truss (g)

Fig. 3-5

Equilibrium method | Stiffness method

3 Graphical methods
Uses of Trusses:
- Roof Truss
- Bridge Truss

Type of Truss:
- Simple Truss:
  The Simple Truss consists of a triangular block. The simplest frame work is a triangular with three members.

Compound Truss:
- It is formed by 2 or more simple trusses. It is often used for longer spans and is proved cheaper than a single single truss for the same span.

Principal Simple Truss
2nd Secondary Simple Truss
Complex Truss: - If Truss is neither simple nor even then it will be complex truss. Generally a complex truss has polygonal structure.
Analysis of Truss:

Step 1: Check the degree of static indeterminacy.

\[ D_s = (m + p) - 25 \]

If \( D_s = 0 \) then Truss is called determinate.

Perfect frame:

For such Trusses, equilibrium equations are used for analysis. And method of joint and method of section can be used.

Step 2:

If \( D_s > 0 \) then Truss is called indeterminate.

Redundants may be internal and external. To analyze such Truss methods of force method or displacement method can be used.

Sign Convention for member force:

\[ \begin{align*}
& \uparrow \rightarrow \text{Tension} \\
& \downarrow \rightarrow \text{Compression}
\end{align*} \]

\[ \begin{align*}
& AC \rightarrow \text{Compression} \\
& RA \rightarrow \text{Tensile}
\end{align*} \]
Joint free body diagram.

Member Free Body

At joints, tensile force is represented by arrow moving away from joint.

Compression force points towards the joint.

In 2 dimension case (plane truss), for equilibrium of each joint following condition should be satisfied

1. $\sum F_x = 0$
2. $\sum F_y = 0$

Analysis of determinate frame plane truss is $E I = 0$ (Perfect P frame)

Step 1: Find support reactions using following 3 conditions of static equilibrium for truss as a whole:

1. $\sum F_x = 0$
2. $\sum F_y = 0$
3. $\sum M_z = 0$
To determine member forces use either method of joint or method of section.

**Method of Joint**

This method is suitable when force in all member of trusses are required.

The free body diagram of each joint is considered and following conditions of static equilibrium are used to find member forces:

1. \( E_fx = 0 \)
2. \( E_fy = 0 \)

This method is applicable only when the number of unknown force at a joint is less than or equal to two. In this method, the judicious selection of joint is done in such a way that no. of unknown at joint are \( \leq 2 \).

**Method of Section**

It is suitable when forces are required only in two member i.e. 1, 2, 3.

After computation of support reaction two find member forces an imaginary section is cut such that not more than 3 member will be cut across the section i.e. unknown are not more than 3.

The section assume must cut those member in which forces are required. The section can be vertical, inclined, horizontal.
of \( 2 \text{g} - 2 \text{g} \). Free body equilibrium either from left portion or for right portion the section is considered. And only following condition are applied.

1. \( \Sigma F_x = 0 \)
2. \( \Sigma F_y = 0 \)
3. \( \Sigma M_z = 0 \)

It means at a time more than 3 unknown force can be found.

Thumb rule to find members which carry zero forces:

1) If at a joint only three members meet two of them are co-linear then there is no external force and reaction at that joint than 3rd member (no co-linear member) will carry zero force.

\[ F_3 = 0 \]
\[ \theta \neq 0 \]
\[ \theta \neq 180 \]

F3 = 0
\[ \Sigma F_y = 0 \]
\[ F_3 \sin \theta = 0 \]
\[ F_3 = 0 \]
Furthermore, notice that under such case both co-linear member will have equal and alike forces.

\[ F_1 = F_2 \]

( Either both will be tensile or compressive or both will be zero )

\[ \phi \neq 0 \]
\[ \phi = 180 \]

If at joint only two member are present which are not co-linear and their is no external load or reaction at the joint then both members will carry zero forces.

\[ \Sigma F_y = 0 \quad \phi \sin \theta = 0 \]
\[ F_2 = 0 \]

\[ \Sigma F_x = 0 \quad F_1 \cos \phi = 0 \]
\[ F_1 = 0 \]
If sum of the members in a truss carry zero force at a given loading then such member should not be removed. Because under change loading condition such member may carry dead. Further over a members are remove then may become less than 0. Hence geometry of the truss will not be preserved under general loading condition.

Q. In this truss shown in fig. Identify the no. of member which carry zero forces.

(a) 8  (b) 10  (c) 12  (d) 14
Identify the zero force member in the frame. (Shown in fig).

(A) 12  (B) 28  (C) 9  (D) None of these

Analyses the truss given below using method of joint and find forces in all member.
\[ x_0 = 4 \]
\[ D_{sc} = 8e - 3 = 4 - 3 = 1 \]

\[ D_{si} = m - (21 - 3) \]
\[ = 18 - (2 \times 11 - 3) \]
\[ = 18 - 19 \]
\[ = -1 \]

\[ D_s = D_{sc} + D_{si} \]
\[ = 1 - 1 = 0 \]

\[ \Sigma f_x = 0 \]
\[ \Sigma f_y = 0 \]
\[ \Sigma M_A = 0 \]
\[ \Sigma M_B = 0 \]

in this case all member concurrence at O.
Hence, two can be cut in 2 part and each part can be rotated about O.

\[ \Sigma f_x = 0 \]
\[ H_A + H_B = 0 \]

\[ \Sigma f_y = 0 \]
\[ R_A + R_B = 0 \]

\[ \Sigma M_A = 0 \]
\[ R_B \times 16 - 10 \times 8 = 0 \]

\[ R_B = 13.75 \]
\[ R_A = -3.75 \]
\[ 8 \text{ Me}_0 = 0 \]
\[ RB \times 8 = HB \times 6 = 0 \]
\[ HB = \frac{RB \times 8}{6} = \frac{3.75 \times 8}{6} = 5 \text{ kN} \]

\[ HA = 5 \text{ kN}, \quad HB = 5 \text{ kN} \]

<table>
<thead>
<tr>
<th>Member</th>
<th>Force</th>
<th>+ (↑)</th>
<th>- (↓)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AD</td>
<td>6.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CE</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DE</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DF</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DG</td>
<td>6.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EF</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FN</td>
<td>-10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GH</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HI</td>
<td>0</td>
<td></td>
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<tr>
<td>GJ</td>
<td>6.25</td>
<td></td>
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<tr>
<td>HJ</td>
<td>0</td>
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</tr>
<tr>
<td>IJ</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JK</td>
<td>0</td>
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<tr>
<td>JK</td>
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<tr>
<td>JB</td>
<td>-6.25</td>
<td></td>
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</tr>
<tr>
<td>KB</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Consider joint A

\[ \text{HA} = 5 \]

\[ \sin \theta = \frac{L - y}{k} \]

\[ F_{\alpha x} = 0 \]

\[ F_{\alpha y} = 0 \]

\[ F_{\alpha x} \sin \theta - 5 = 0 \]

\[ F_{\alpha y} = \frac{5}{\sin \theta} = \frac{5}{4} \]

\[ F_{\alpha y} = 6.25 \text{ (+) } \]

\[ F_{\alpha x} = \frac{3.75 - 25}{5} \]

\[ \text{consider joint D at D member AD } x \text{ phy one co-linear and other member have zero forces. Hence both co-linear force } AD \text{ phy will have equal forces} \]

Joint E

\[ \theta \]

\[ F_{\alpha y} \]

\[ F_{\alpha x} \]

\[ F_{\alpha z} \]
\[ \begin{align*}
\text{If } x &= 0 \\
F_{FF} + 10 + F_{CD} \cos \theta &= 0 \\
F_{FF} - 10 - f_0 &= 0 \\
F_{FF} &= 10
\end{align*} \]

\[ F_{PP} = 0 \]

A+ Joint A:

\[ F_{FF} = F_{F0} \]

Consider Joint B:

\[ F_{Ej} \]

\[ F_{FK} \]

\[ H = 5 \text{ kN} \]

3.75

\[ \text{If } x &= 0 \\
F_{CR} + F_{ER} \cos \theta &= 3.75 = 0 \\
-5 - F_{Bj} \sin \theta &= 0 \\
F_{Bj} &= -5 \frac{\cos \theta}{\sin \theta} = -6.15 \]

\[ \text{If } y &= 0 \\
3.75 + F_{BK} + F_{Bj} \cos \theta &= 0 \\
3.75 + F_{BK} + 5 \sqrt{2} = 0 \\
F_{BK} &= 0
\]
Find force in all member of frame.

\[
m = 13 \quad \delta e = 3
\]

\[
DS = m + \delta e - 2 \theta
\]

\[
= 13 + 3 - 2 \times 8
\]

\[
= 0
\]

<table>
<thead>
<tr>
<th>Serial</th>
<th>Member</th>
<th>Force (KN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AB</td>
<td>7.5</td>
</tr>
<tr>
<td>2</td>
<td>AC</td>
<td>+1.875</td>
</tr>
<tr>
<td>3</td>
<td>AD</td>
<td>+9.375</td>
</tr>
<tr>
<td>4</td>
<td>CD</td>
<td>-12.5</td>
</tr>
<tr>
<td>5</td>
<td>CG</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>CF</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>DE</td>
<td>2.5</td>
</tr>
<tr>
<td>8</td>
<td>BD</td>
<td>-9.375</td>
</tr>
<tr>
<td>9</td>
<td>BE</td>
<td>-6.875</td>
</tr>
<tr>
<td>10</td>
<td>Fm</td>
<td>-5</td>
</tr>
<tr>
<td>11</td>
<td>GH</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>EG</td>
<td>-8.125</td>
</tr>
<tr>
<td>13</td>
<td>HC</td>
<td>-5</td>
</tr>
</tbody>
</table>
\[ \Sigma F_x = 0 \]
\[ S + 10 - HA = 0 \]
\[ HA = 15 \]
\[ RA + RB = 5 \]
\[ EMA = 0 \]
\[ -RB \times 8 + 5 \times 8 + 10 \times 3 + 5 \times 6 = 0 \]
\[ RB = \frac{-40 + 20 + 20}{8} = \frac{0}{8} = -2.5 \]
\[ RA = -2.5 \]

At the support member B

![Diagram of forces](image)

Consider member F

\[ \Sigma F_y = 0 \]
\[ \Sigma M_c = 0 \]
\[ EF_y = 0 \]
\[ FF_c - 5 = 0 \]
\[ FF_c = -5 \]
\[ FF_c = 0 \]
Consider joint H

\[ \Sigma F_x = 0 \]
\[ F_{xH} = 0 \]
\[ \phi \]

\[ \Sigma F_y = 0 \]
\[ -F_{yH} - 5 = 0 \]
\[ F_{yH} = -5 \]

Consider joint O

\[ \Sigma F_x = 0 \]
\[ F_{xO} + 5 = 0 \]
\[ F_{xO} = -5 \]

\[ \Sigma F_y = 0 \]
\[ -F_{yO} - 5 = 0 \]
\[ F_{yO} = -5 \]

\[ F_{rO} = F_{rO} \]

\[ \iota \]

\[ F_{rO} \cos \phi = F_{rO} \]

\[ F_{rO} \sin \phi = F_{rO} \]

\[ F_{rO} \sin \phi = F_{rO} \]

\[ F_{rO} \cos \phi = F_{rO} \]

\[ F_{rO} = \frac{5 \times 5}{8} = 2.8125 \]

\[ 2F_{yO} + 5 = 5 \]

\[ F_{yO} = \frac{5 \times 5}{8} = 2.8125 \]
\[ \text{Joint C} \]

\[ E_{fy} = 0 \]
\[ 3.125 \sin \theta - F_{AC} = 0 \]
\[ F_{AC} = 3.125 \times 3 / 5 \]
\[ F_{AC} = +1.875 \]
\[ E_{fx} = 0 \]
\[ F_{CD} + 3.125 \cos \theta + 10 = 0 \]
\[ F_{CD} = -12.5 \]

\[ \text{Consider joint F} \]

\[ \sin \theta \quad 5 \text{KN} \]
\[ F_{DF} \quad F_{BE} \]

\[ E_{fy} = 0 \]
\[ F_{DF} - 3.125 \cos \theta = 0 \]
\[ F_{DF} = 3.125 \times 3 / 5 \]
\[ F_{DF} = 2.5 \]

\[ E_{fy} = 0 \]
\[ -F_{BE} - 3.125 \sin \theta = 0 \]
\[ F_{BE} = -6.875 \]
Consider joint A

\[ \begin{align*}
\ang & = 25.6 \\
1.875 \ FAD \sin \ang & = 7.5 \\
1.875 + 8 \ \frac{\ FAD \times 3}{5} & = 7.5 \\
\ FAD & = 9.375
\end{align*} \]

\[ \begin{align*}
EFX & = 0 \\
-15 + FAB + FAD & = 0 \\
FAB & = 15 - 9.375 \times \frac{4}{3} \\
FAB & = 7.5
\end{align*} \]
Analysis the truss shown in fig. & list forces in the table.

\[ \Sigma F_x = 0 \]
\[ H_c = 0 \]
\[ D_s = 0 \quad \text{(Perfect frame)} \]
\[ \Sigma F_y = 0 \]
\[ R_E + R_C = 3000 \text{ N} \]

\[ \Sigma M_c = 0 \]
\[ R_F \times 3.8 - 2000 \times 7.2 - 1000 \times 3.6 = 0 \]
\[ R_E = \frac{144000 + 3600}{4} \]
\[ R_E = 37000 \text{ N} \]

\[ R_C = -7000 \text{ N} \]
A square truss ABCD shown in fig. Carry load of 10 kN apply through a string of which is passing over a frictionless fulcrum as shown in fig. GF & AF are parallel. The truss is supported on roller at CD. A link BE is shown in fig. is part of truss. Find forces in all member of the truss.

Net Vertical force at C1 = 10 - 10

\[ \frac{10}{J_2} \]

\[ \Rightarrow 2.93 \text{kN} \]

Net Horizontal at C2 = 10 - \[ \frac{10}{J_2} \]

\[ \Rightarrow 7.07 \text{kN} \]
\[ Ds = m + 9e - 2f \]
\[ \Rightarrow 6x4 - 2x5 \]
\[ \Rightarrow 0 \]

Joint A + E

\[ \Sigma F_y = 0 \]
\[ RE = 0 \]

\[ \Sigma F_y = 0 \]
\[ Rd + RE - 2.93 = 0 \]
\[ Rd = 2.93 \text{ KN} \]

\[ \Sigma F_x = 0 \]
\[ 7.07 - Hc - HE = 0 \]
\[ Hc + HE = 7.07 - (1) \]

\[ \Sigma M_c = 0 \]
\[ \frac{Rd 	imes 4}{J_2} - 2.93 \times 4 J_2 - HE \times \frac{4}{J_2} = 0 \]
\[ HE = -2.93 \text{ KN} \]

Consider joint E's

\[ \Sigma F_x = 0 \]
\[ EBE = 2.93 \]

\[ \Sigma F_x = 0 \]
\[ EBE = 2.93 \]
Joint B

\[ \text{Ebf} = 0 \]

\[ 2.93 + Fbc \cdot \sin \theta - Fba \cdot \sin \theta = 0 \]

\[ 2.93 + \frac{Fbc}{\sqrt{2}} - \frac{Fba}{\sqrt{2}} = 0 \] \hspace{1cm} (i)

\[ \text{Efy} = 0 \]

\[ -\frac{Fba}{\sqrt{2}} - \frac{Fbc}{\sqrt{2}} = 0 \] \hspace{1cm} (ii)

From (i) & (ii) \hspace{1cm} Fba = -Fbc

\[ \text{FBA} = +2.07 \]
\[ \text{FBC} = -2.07 \]

(3) Consider joint C

\[ \text{FAC} = 10 \text{ KN} \]

\[ \text{FCD} = -2.07 \]
\[ \text{FAC} = -7.07 \]
Calculate forces in the member EF & DF.

\[ R_A = 1.25 \text{ KN} \]

\[ F = m + 3e - 2i \]

\[ \Rightarrow 18 + 4 - 2 \times 11 = 0 \]

\[ EFX = 0 \quad HA + H_B = 10 \]

\[ EFY = 0 \quad RA + R_B = 5 \]

\[ EMF = 0 \]

\[ R_I A 8 - H_I 6 = 0 \]

\[ H_I = \frac{375 \times 8}{6} \]

\[ H_F = 5 \]

\[ HA = 5 \]
\[ F_{x} = 0 \]
\[ F_{y} = 0 \]

\[ F_{EF} + F_{DF} \sin \theta + 10 - S = 0 \]
\[ F_{EF} = 5 - 10 - F_{DF} \sin \theta \]
\[ = -5 - 6.25 \times 4 \times \frac{4}{5} \]
\[ F_{EF} = -10 \]

Find forces in member HR and HT.
\[ DC = m + \sin e - 2J \]

\[ DS = 10 \]

\[ Efx = 0 \quad HA = 0 \]

\[ Efy = 0 \]

\[ RA + RB = 30 + 10 + 30 \]

\[ 2RA = 70 \]

\[ RA = 35 = RB \]

\[ \sin \theta_1 = \frac{1}{117} \quad \cos \theta_1 = \frac{y}{117} \]

\[ \sin \theta_2 = \frac{3}{5} \quad \cos \theta_2 = \frac{y}{5} \]

Taking \( EMH = 0 \)

\[ RA \times 4 = FCD \times 3 = 0 \]

\[ FCD = \frac{RA \times y}{3} = \frac{37.5 \times 4}{3} \]

\[ FCD = 1504 \]
\[ S_{fy} = 0 \]
\[ F_{cp} + F_{hd} \cos \theta_2 + F_{hj} \cos \theta_1 = 0 \]

\[ S_0 + F_{hd} \frac{y}{5} + F_{hj} \frac{y}{14} = 0 \quad (0.8) \quad (0.9701) \]

\[ E_{fy} = 0 \]
\[ R_A + F_{hj} \sin \theta_1 - F_{hd} \sin \theta_2 = 0 \]
\[ 37.5 + F_{hj} \times \frac{1}{517} - F_{hd} \times \frac{3}{5} = 0 \quad (0.2422) \quad 0.6 \]

From equa 8.43

\[ F_{hd} = 31.25 \]
\[ F_{hj} = -77.30 \]
Analysis of indeterminate truss

\[ D_5 > 0 \]

\[ = 1 \text{ or higher degree.} \]

- Force method
- Unit load method
- Strain energy method
- Use of Castigliano's theorem

![Diagram of a truss with labeled points W and P]

\[ D_5 = m + 9e^{-2j} \]

\[ \Rightarrow 6 + 3 - 2 \times 4 \Rightarrow 9 - 8 = 1 \]

\[ D_{5e} = 0 \]

\[ D_{5f} = m - (2f - 3) \]

\[ \Rightarrow 6 - (2 \times 4 - 3) \]

\[ \Rightarrow 1 \]

p-1

Find degree of static indeterminacy and check:

\[ D_5 \text{ & } D_{5f} \text{. If degree of indet. is } 1, \text{ it means there is one redundant reaction force.} \]

Identify the redundant is above. Case redundant is internal force force member member force.

Let Redundant force is force in member AB.
Step 2: Due to given load forces in truss member are $s_1, s_2, \ldots, s_n$ which are required to compute.

If redundant member is remove, the rest of the truss will be determinate and stable due to given loading in such determinate truss after removing one redundant, let force in members are $p_1, p_2, \ldots, p_m$ which can be found by method of joint, method of section.

\[ \text{force in AB} = 0 \]

Step 3: Remove loading and assume the value of redundant force is 'X' since force in AB is 'X', hence apply 'X' force in A & B in the direction of the truss.
let due to $x$ force in AB force developed in other member figure $a_1, a_2, \ldots, a_n$

$$a_{AB} = x$$

Step 4

if $x$ is taken common and unit load is applied at AB in the direction of AB, then forces in members are $k_1, k_2, k_3, \ldots, k_n$ then $\varphi = kx$

$$\varphi = kx$$

Step 5

The final force in tense member $s_1 = p_1 + a_1$

$$s_1 = p_1 + xk_1$$

$$s_2 = p_2 + xk_2$$

$$s_n = p_n + xk_n$$

the total strain energy stored

$$u = \sum_{2AE} \frac{(p + kx)^2 \cdot L}{2AE}$$
The true objective function will be such that for that the total strain energy stored in the system is minimum.

\[
\frac{du}{dx} = 0
\]

\[
\sum 2(p + kx) \cdot k \cdot L = 0
\]

\[
\frac{2AE}{2AE}
\]

\[
\sum \frac{PKL}{AE} + x \sum \frac{k^2L}{AE} = 0
\]

\[
\frac{X}{AE} = \frac{1}{AE}
\]

\[
\frac{1}{AE}
\]

\[
\sum \frac{k^2L}{AE}
\]

Summary:

<table>
<thead>
<tr>
<th>Member</th>
<th>P</th>
<th>K</th>
<th>L</th>
<th>AE</th>
<th>(\frac{PKL}{AE})</th>
<th>(\frac{k^2L}{AE})</th>
<th>S = R + kx</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
1. Check all Rs and identify Redundant
2. Remove the Redundant find P force system due to given loading
3. Remove the loading and applied unit load in the direction of redundant and find K force system in all the member.
4. Find value of Redundant force on

\[
x = \frac{\sum PKL}{AE} - \frac{\sum K^2}{AE}
\]

5. Find final force in the member

\[
S = P + K \cdot X
\]

Special case:
If \(D_r = 2\), means there are two redundant say Redundant force are \(X_1 \& X_2\). Remove both the redundants and find P force system in the determinate truss.

Apply unit load in the direction of first redundant and find K force system in all member.

Now apply unit load in the direction of second redundant and find K system of force in all members.

\[
X_1 = \frac{\sum PKL}{AE} - \frac{\sum K^2}{AE}
\]
\[
X_2 = \frac{\sum P^2 L}{AE}
\]
Analyse the truss shown in below and find force in all member if AE is axial.

Rigidity \( \frac{AE}{L} \) = Axial stiffness.

\[ D_5 = m + re^{-2} \]
\[ D_5e = 0 \]
\[ = 10 + 3 - 2 \times 6 \]
\[ D_5 = 13 - 12 = 1 \]

Let the redundant member is EC. If redundant is removed, then rest of the truss will be determinate.

\[ H_A = 0 \]
\[ Rn = w \]

\[ 3@am \]
<table>
<thead>
<tr>
<th></th>
<th>AB</th>
<th>B</th>
<th>D</th>
<th></th>
<th></th>
<th></th>
<th>L</th>
<th>PKL</th>
<th>K - L</th>
<th>S = P + K ⋅ X</th>
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<tbody>
<tr>
<td>1</td>
<td>-W</td>
<td>0</td>
<td>0</td>
<td>a</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-W</td>
<td>2Jω</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>J2ω</td>
<td>0</td>
<td>aJ2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-W + 20°7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-W</td>
<td>-YJ2</td>
<td>a</td>
<td>wa/12</td>
<td>aJ5</td>
<td>-W + 20°7</td>
<td></td>
<td></td>
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<tr>
<td>4</td>
<td>B</td>
<td>W</td>
<td>-YJ2</td>
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<td>-W</td>
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<td>1</td>
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<td>aJ5</td>
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<tr>
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<td>W</td>
<td>-YJ2</td>
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<td>wa/12</td>
<td>aJ5</td>
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<tr>
<td>9</td>
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<td>aJ2</td>
<td>0</td>
<td>0</td>
<td>J2ω</td>
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<td></td>
</tr>
<tr>
<td>10</td>
<td>BF</td>
<td>0</td>
<td>0</td>
<td>aB</td>
<td>0</td>
<td>aJ5</td>
<td>-0.293</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Consider joint A**

\[ F_{AB} \]

\[ Efy = 0 \]

\[ BAE = 0 \]

\[ FAB + FAE \cos 45° = 0 \]

\[ FAB = -\frac{\sqrt{2} \omega}{J_2} \]

\[ F_{FE} = J_2ω \]

\[ Efy = 0 \]

**Consider joint F**

\[ F_{EB} \]

\[ Efy = 0 \]

\[ FEB + J2ω \cos 45° = 0 \]

\[ FEB = -J2ω \times \frac{1}{J_2} \Rightarrow -W \]
\[ FFE = 0 \]

\[ FFE + B = T \sin 45^\circ = 0 \]

\[ FFE = W \]

Joint B

\[ \begin{align*}
\sum F_y &= 0 \\
F_{BD} + P_{BE} \cos 45^\circ - w + w &= 0 \\
P_{BE} &= 0 \\
F_{BC} &= -w \\
\text{See: Remove loading and apply unit load at } F_{BC} \\
\sum F_x &= 0 \\
-w + F_{BC} + F_{BE} \cos 45^\circ &= 0 \\
F_{BC} &= -w \\
\end{align*} \]

Consider joint E

\[ \begin{align*}
\sum F_y &= 0 \\
F_{BE} + F_{BE} - 2w &= 0 \\
F_{BE} &= 0 \\
\end{align*} \]
\[ E_{fy} = 0 \]
\[ F_{BE} + 16 \sin \theta = 0 \]
\[ F_{BE} = -\frac{1}{\sqrt{2}} \]
\[ F_{EF} = -\frac{1}{J_2} \]

Consider point C

\[ E_{fx} = 0 \]
\[ F_{CE} - P_1 \sin \theta = 0 \]
\[ F_{CE} = \pm \sqrt{2} \]

Consider point B

\[ E_{fx} = 0 \]
\[ F_{BC} - 16 \cos \theta = 0 \]
\[ F_{BC} = \sqrt{2} \]
\[ x = - \frac{\sum PKL}{AE} \]

\[ x = - \frac{550 \times w_1}{AE} \rightarrow - 0.52 \]

\[ \frac{520 (52 + 1)}{AE} \frac{85 (52 + 1)}{AE} \]

\[ x \rightarrow 0.293 \text{ W} \]

**A.** Analyze the structure shown in Fig. and find the force in each member. The diagonal member makes an angle of 45° with AE (constant).

- \( M_1 = 2 \text{ kN} \cdot \text{m} \)
- \( F = 4 \text{ kN} \)
- \( D_{ge} = 9 \times 10^{-3} \rightarrow 1 \)
- \( D_{si} = n - (2J - 3) \)
- \( = 17 - (2 \times 10 - 3) \)
- \( = 0 \)
- \( R_X + R_c + R_f = 9 + 4 + 2 \)
- \( R_A + R_c + R_f = 18 \)
- \( \sum M_A = 0 \)
- \( R_E 	imes A_4 - 23A - 0.2a - 0.2a = 4 \)
since RBE is External, hence external reaction is indeterminate (R.E indeterminate)

\[ RA = RE = 4 \]

Find force system, remove redundant (say re)

<table>
<thead>
<tr>
<th>S-No</th>
<th>Member</th>
<th>P</th>
<th>K</th>
<th>L</th>
<th>PlL</th>
<th>k²L</th>
<th>S = P + kX</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>AB</td>
<td>0</td>
<td>0</td>
<td>a</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>AF</td>
<td>-4</td>
<td>½</td>
<td>a</td>
<td>-2a</td>
<td>4/4a</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>BC</td>
<td>4</td>
<td>-½</td>
<td>a</td>
<td>-2a</td>
<td>a/4</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>RP</td>
<td>½2</td>
<td>½2</td>
<td>a</td>
<td>-2a</td>
<td>a/2</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>BC</td>
<td>-4</td>
<td>½2</td>
<td>a</td>
<td>-2a</td>
<td>a/2</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>CO</td>
<td>2½2</td>
<td>-½2</td>
<td>a</td>
<td>-2a</td>
<td>a/2</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>FO</td>
<td>-4</td>
<td>½2</td>
<td>a</td>
<td>-2a</td>
<td>a/2</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>CH</td>
<td>-6</td>
<td>0</td>
<td>a</td>
<td>6a</td>
<td>a</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>HC</td>
<td>-4</td>
<td>0</td>
<td>a</td>
<td>6a</td>
<td>a</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>IH</td>
<td>-6</td>
<td>1</td>
<td>a</td>
<td>-6a</td>
<td>a</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>JI</td>
<td>-4</td>
<td>½2</td>
<td>a</td>
<td>-2a</td>
<td>a/2</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>CI</td>
<td>½2</td>
<td>-½2</td>
<td>a</td>
<td>-2a</td>
<td>a/2</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>DI</td>
<td>-4</td>
<td>½2</td>
<td>a</td>
<td>-2a</td>
<td>a/2</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>DJ</td>
<td>-4½2</td>
<td>½2</td>
<td>a</td>
<td>-2a</td>
<td>a/2</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>DC</td>
<td>4</td>
<td>-½2</td>
<td>a</td>
<td>-2a</td>
<td>a/4</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>EJ</td>
<td>-4</td>
<td>½2</td>
<td>a</td>
<td>-2a</td>
<td>0.49</td>
<td>1</td>
</tr>
<tr>
<td>17</td>
<td>ED</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Consider joint A

\[ \begin{align*}
R_A &= 0 \\
F_{AF} &= 0 \\
F_{AB} &= 0
\end{align*} \]
Consider point F

\[
\begin{align*}
\text{EF}_x &= 0 \\
\text{EF}_y &= 0 \\
\text{FAF} &= 4 \\
\text{FAF} &= -4 \\
\text{277}
\end{align*}
\]

\[
\begin{align*}
\text{FF}_F &= 45^\circ \\
\text{FF}_F &= 4 \\
\text{FF}_F &= 4 \times \sqrt{2} \\
\text{FF}_F &= 4\sqrt{2}
\end{align*}
\]

Consider point B

\[
\begin{align*}
\text{EF}_x &= 0 \\
\text{EF}_y &= 0 \\
\text{FB}_B + \text{FB}_F \cos 45^\circ &= 0 \\
\text{FB}_B + 4\sqrt{2} \times \frac{1}{\sqrt{2}} &= 0 \\
\text{FB}_B &= -4
\end{align*}
\]

\[
\begin{align*}
\text{EF}_y &= 0 \\
\text{FB}_B + 4\sqrt{2} \sin 45^\circ &= 0 \\
\text{FB}_B &= -4
\end{align*}
\]
For a system of force, apply will load of \( C \) a unit

Consider joint \( A \):

\[ \mathbf{K}_{AB} = 0 \]

\[ \mathbf{KAF} = \frac{y_2}{2} \]

Consider joint \( F \):

\[ \mathbf{K}_{FF} = \frac{y_2}{2} \]

\[ \mathbf{K}_{BF} = -\frac{1}{\frac{y_2}{2}} \]

\[ \mathbf{\epsilon}_{Fy} = 0 \]

\[ \mathbf{K}_{BF} = \frac{y_2}{2} \]
Q: For the truss shown in Fig. all member have same axial rigidity. Calculate horizontal and vertical reaction at supports.

\[
X = \frac{\Sigma P \cdot L / \Sigma E \cdot A}{\Sigma K / \Sigma E \cdot A} \Rightarrow \left( \frac{-44.978}{6.8289} \right) \Rightarrow + 6.5852
\]

\[
\Sigma S_x = m + P - 25 = 11 + 4 - 2 \times 7 = 4 - 3
\]

\[
\Sigma S_y = 8E - 3 = 4 - 3
\]

\[
\Sigma \delta = 1 + 2
\]

\[
\Sigma F_x = 0 \Rightarrow HA = H_B = 4
\]

\[
\Sigma F_y = 0 \Rightarrow \quad R_A + R_B = 6
\]


$E A = 0$

$R_A \times 16 - 6 \times 8 = 0$

$R_B = 36 \times 8 = 3 \text{ kN}$

$162 -$

$R_A = R_B = 3 \text{ kN}$

$3t$

$\text{280}$

$\rightarrow$ Let the redundant be $H$. $(H_A = H_B = H)$

$H = 5.27 t$

Lack of fit problems

The truss members are fabricated in the factory and assembled in the field. If any one member is fabricated either too short or too long, if defective member is force fully fitted, then all members so may be sub to forces.

The square truss $ABCD$ is to be assembled but member $AC$ is fabricated too short. for force full fitting of $AC$ let us apply a force in $AC$. 
Hence joints ABC will be also under force in direction AC. First forces in all members due to force in AC.

<table>
<thead>
<tr>
<th>Member</th>
<th>Force (kN)</th>
<th>L</th>
<th>$a^2 - L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>$-x/\sqrt{2}$</td>
<td>$a$</td>
<td>$x^2a/2$</td>
</tr>
<tr>
<td>AD</td>
<td>$-x/\sqrt{2}$</td>
<td>$a$</td>
<td>$x^2a/2$</td>
</tr>
<tr>
<td>AC</td>
<td>$x$</td>
<td>$a\sqrt{2}$</td>
<td>$x^2a\sqrt{2}$</td>
</tr>
<tr>
<td>BC</td>
<td>$-x/\sqrt{2}$</td>
<td>$a$</td>
<td>$x^2a/2$</td>
</tr>
<tr>
<td>BD</td>
<td>$-\sqrt{2}x$</td>
<td>$a\sqrt{2}$</td>
<td>$x^2a\sqrt{2}$</td>
</tr>
<tr>
<td>CD</td>
<td>$-x/\sqrt{2}$</td>
<td>$a$</td>
<td>$x^2a/2$</td>
</tr>
</tbody>
</table>

\[\sum a^2L = 2x^2a + 2\sqrt{2}x^2a + 2x^2a(\sqrt{2}+1)\]

Consider joint A

\[F_{AD} + RA = 0\]
\[R_A = R_B = 0\]
\[N_A = 0\]
\[\varepsilon_f = 0\]

\[F_{AB} + x \cos \theta = 0\]
\[F_{AB} = \frac{-x}{\sqrt{2}}\]
\[\varepsilon_y = 0\]
\[F_{AP} + x \sin \theta = 0\]
\[F_{AP} = -\frac{x}{\sqrt{2}}\]
Total Strain Energy in all member of Truss

\[ U = \frac{\varepsilon^2 L}{2AE} \]

\[ \rightarrow \frac{2x^2a(J^2+1)}{2AE} \]

\[ U = \frac{x^2a(J^2+1)}{AE} \]

\[ \delta u = \frac{A}{L} \cdot \frac{2xa(J^2+1)}{AE} = \delta \]

\[ x = \frac{A\cdot AE}{2a(J^2+1)} \]

Note: The above forces shown in the table are due to lack of fit effect when there is no loading in the truss if loading effect is also to be considered, then separate analysis as per previous discussion will be required and combined effect algebraic sum of lack of fit effect and loading effect.

Q: in fig. Shows a pin joint a truss having constant axial stiffness for each member. Calculate the axial force in member GF.

Q: Calculate the force in same member GF if it is retrofitted with ten short columns structure carrying no loading.
so.

**Loading Effect**

\[ D_S = m + re - 2.5 \]

\[ = 10 + 5 - 2 \times 5 \]

\[ D_{se} = 0 \]

\[ D_5 = 1 \]

**Hence redundant is internal force. Let member AF is redundant.**

To find the force system, remove the member and find forces in all members by method of joint.

<table>
<thead>
<tr>
<th>S.no</th>
<th>Member</th>
<th>member</th>
<th>k</th>
<th>PK</th>
<th>k^2</th>
<th>( \delta = kx^1 )</th>
<th>( \delta^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(-3)</td>
<td>AB</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>(w, \bar{w})</td>
<td>AC</td>
<td>0</td>
<td>-2w</td>
<td>1</td>
<td>( \bar{x}_1^2 )</td>
<td>( x_1^2 )</td>
</tr>
<tr>
<td>3</td>
<td>(-2w)</td>
<td>BC</td>
<td>1</td>
<td>-2w</td>
<td>1</td>
<td>( \bar{x}_1^2 )</td>
<td>( x_1^2 )</td>
</tr>
<tr>
<td>4</td>
<td>(-2w)</td>
<td>BD</td>
<td>1</td>
<td>-2w</td>
<td>1</td>
<td>( \bar{x}_1^2 )</td>
<td>( x_1^2 )</td>
</tr>
<tr>
<td>5</td>
<td>(w, \bar{w})</td>
<td>BF</td>
<td>-2 ( \bar{w} )</td>
<td>( \bar{w}^2w )</td>
<td>2</td>
<td>(-\sqrt{2}x^1 )</td>
<td>( 2x^1 )</td>
</tr>
<tr>
<td>6</td>
<td>(-2w)</td>
<td>CD</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>(-2w)</td>
<td>CF</td>
<td>0</td>
<td>( \bar{w}^2w )</td>
<td>1</td>
<td>( \bar{x}_1^2 )</td>
<td>( x_1^2 )</td>
</tr>
<tr>
<td>8</td>
<td>(w, \bar{w})</td>
<td>CG</td>
<td>-2w</td>
<td>2w</td>
<td>2</td>
<td>(-\sqrt{2}x^1 )</td>
<td>( 2x^1 )</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>CF</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>(w, \bar{w})</td>
<td>CF</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
consider point A

\( \text{For} = w \)  
\( \text{For} = w \sqrt{2} \) 
\( \text{For} = w \sqrt{2} \cos 45^\circ \) 
\( \text{For} = -w \sqrt{2} \cos 45^\circ \) 
\( \text{For} = -w \) 

No reaction, because there are internal reaction

To find k force system, remove given loading and apply unit load at C and at D. Find k force method of joint.
Stiffness \( \frac{AE}{L} = \text{constant} \)

\( X = \text{Redundant force in CF will } \dot{x} \)

\[ X = \frac{-\sum P_{KL}}{AE} \Rightarrow \frac{-\sum P_{KL}}{\frac{X^2L}{AE}} = \frac{-3P_{KL}}{AE} \]

\[ = \frac{33}{1000} \Rightarrow 1.25w \]

\( w_4 \)

**Sect**

\( A = 0.1 x_0 \)

\( 180 \)

\( A = \frac{a}{1000} \)

Let due to lack of fit force in member

\( X' \)

Hence \( \frac{dU}{dX'} = \Delta = \frac{a}{1000} \)

\( U = \sum \frac{P_{KL}^2}{2AE} = \frac{L}{2AE} \sum P_{KL}^2 \)

\( \Delta_1 \Delta_2 = \text{an are forces in member due to } X' \text{ force in CF} \)

\( \dot{x}_2 = \frac{\sum P_{KL}^2}{2AE} \)
due to unit force in bar of \( K_1, K_2, K_3 \),
where \( K_1 \) are the force in truss member. Hence due to \( x' \) forces in bar forces in,
truss member will be

\[
\begin{align*}
\delta_1 &= K_1 x' \\
\delta_2 &= K_2 x' \\
\delta_n &= K_n x'
\end{align*}
\]

\[ x' = \frac{ac}{8000} \]

Deflection of Truss Joint's Method

- Force method
- Unit Load method
- Maxwell's method
- Strain energy method
- Stiffness method (Equilibrium method)
- Graphical method

- Williot-Mohr diagram applicable
- Box chain method for truss
Unit Load Method / Maxwell's Method:

1. Let vertical deflection of joint C is required.

Procedure:

Step 1: Due to given load system find P-force system in all members, unit method of joint say member forces are $P_1, P_2, \ldots, P_n$.

Step 2: Remove given loading and apply unit load at C in the direction of desired deflection (vertical) and find K-force system say due to unit load at C member forces are $K_1, K_2, \ldots, K_n$.

Step 3: The vertical deflection at C is given as

$$\delta C = \frac{\sum P K_i}{AE}$$
if \( S \) is (five) \( \rightarrow \) direction of unit load.
if \( S \) is (-ive) \( \rightarrow \) opposite direction of unit load.

**Special Case 3**

\[ S_c = \sum P (\frac{RL}{AE}) \]

- \( RL \): Axial deflection or change in length of any member due to force pin that member
- \( AE \): Axial change

\[ P \rightarrow (+) T \]
\[ P \rightarrow (-) c \]

\[ S_c = \sum_{i=1}^{n} K_i \Delta_i^a \]

If temperature of some of the member is change
\( \Delta_i^a \) will be due to temp effect

\[ \Delta_i^a = (L \Delta T)_i \]

\( T = \text{Temp. Change} \; ^\circ C \)
(+9 if \( T \) is +)
(-) if \( T \) is -

\( a = \text{Coeff. of thermal change} \)

\( \Delta \) in any member \( i \) may be due to defective manufacture also if member \( i \) manufacture.
\( \Delta \) to long \( \Delta_i^a \rightarrow \) five
\( \Delta \) to short \( \Delta_i^a \rightarrow \) -ive
Moment movement of truss joint is also occurring due to settlement of supports for such conditions virtual work method may be applied.

The combined effect of loading and temp. change will be algebraic sum of individual effect.

\[ \delta_c = \sum_{i=1}^{n} K_i \left( \frac{PL}{AE} + L_o \Delta T \right) \]

The computation may be performed in tabular form.

<table>
<thead>
<tr>
<th>member</th>
<th>P</th>
<th>k</th>
<th>L</th>
<th>AE</th>
<th>PKL/AE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

S member truss shown in the fig. below have been cut either too long or too short.

Calculate the vertical displacement of joint D due to discrepancy's length of these member by unit load method. All members have same cross-section.

\[ \Delta \text{constant} \]

\[ \Delta AB = -3 \text{mm} \]

\[ \Delta BC = +2 \text{mm} \]

\[ \Delta EF = +1 \text{mm} \]
Vertical deflection at joint A

\[ \Delta D = \sum_{i=1}^{n} E_i \Delta_i \]

[290]

\[ \Delta D = K_{AB} \Delta_{AB} + K_{BC} \Delta_{BC} + K_{EFDEF} \]

\[ \varphi_S = m + \frac{30e - 2\theta}{2} \]

\[ \Rightarrow \theta + \frac{1}{2} - 2x_1 \]

\[ \Rightarrow 0 \]

Find K force is due to unit load at D in vertical direction.

- RA = 0
- RX = 0
- RA = 0
- RA + RF = 1 = 0
- RF = 1
- RA

At joint A

\[ \varepsilon_{fy} = 0 \]
\( C_{Fx} = 0 \)

\[ HA + HF = 0 \]
\[ \Sigma \text{ME} = 0 \]
\[ HA \times 4 + 1 \times 6 = 0 \]
\[ HA = -1.5 \]
\[ HF = +1.5 \]

Consider joint A

\[ F_{PB} \]
\[ \Sigma F_x = 0 \]
\[ FA_B - 1.5 = 0 \]
\[ FA_B = 1.5 \]

Consider joint P

\[ \Sigma F_y = 0 \]

\[ K_{FB} \cdot \sin \theta + 1 = 0 \]
\[ K_{FB} = -1 \times 5 = -1.25 \]

\[ E_{Fx} = 0 \]
\[ 1.5 + K_{FF} + K_{FB} \cdot \cos \theta = 0 \]
\[ K_{FF} = -1.5 + 1.25 \times 3 \]
\[ K_{FF} = 0.75 \]

Consider joint B

\[ K_{BC} \]

\[ \Sigma F_x = 0 \]
\[ K_{BC} + 1.25 \cos \theta - 1.5 = 0 \]
\[ K_{BC} = -0.75 \]
\[ \Delta D = K_a a_a + K_a c \cdot a_c + K_F F \cdot a_F \] 

\[ \Rightarrow (5 + 5) \times (-3) + (0.75)(2) + (0.75)(4) \]

\[ \Delta D = -6 \text{ mm} \]

\( \Delta \) p is \((-\)) it means \( \Delta \) p is in opposite direction of unit load.

Q. Compute vertical displacement of the displacement at joint C due to decrease in temp. by 50 °C in bottom chord member only given that

\[ \alpha = \frac{1}{150000}, -\frac{1}{\circ}C = 6.6 \times 10^{-5} \]

\[ D_5 = m + 97e - 2i \]

\[ = 12 + 2 - 2 \times 8 = 0 \]

Vertical deflection at joint C

\[ \Delta_c = \sum K_i \Delta_i \] 

\( \Delta_i = \text{ Change in length of member due to temp change.} \)
Since since temp change is only bottom chord member hence for other member $d_1 = 0$

$$Sc = K_{AB} A_{AB} + K_{BC} A_{BC} + K_{CD} A_{CD} + K_{DE} A_{DE}$$

$$A_{AB} = A_{BC} = A_{CD} = A_{DE} = 1 \times 1$$

$$\Rightarrow 4 \times 6.67 \times 10^{-6} \times (-50) \times 10^3$$

$$-20 = 1.33 \text{ mm}$$

$$\Rightarrow -1.33 \text{ mm}$$

Consider joint A a force system.

$$\begin{align*}
5 \rightarrow FAF \\
\frac{10}{13} \rightarrow FAB \\
\frac{4}{5} \rightarrow FBA \\
BA = VB
\end{align*}$$

$$\begin{align*}
E_{Fx} = 0 \\
E_{Fy} = 0 \\
F_{AF} \sin \theta + \frac{1}{2} = 0 \\
F_{AF} = -\frac{1}{2} x 5 = -5 = -0.833
\end{align*}$$

$$\begin{align*}
E_{Fx} = 0 \\
F_{AB} + F_{AF} \cos \theta \\
F_{AB} = -0.833 x 4 \\
F_{AB} = \frac{5 x 4}{6} = \frac{5 x 4}{6}
\end{align*}$$

$$\begin{align*}
F_{AB} = F_{CD} = F_{DE}
\end{align*}$$

$$\begin{align*}
Sc = \sum 10
\end{align*}$$

$$\begin{align*}
-4 x 2^2 x -1.33 = -3.33 \text{ mm} (L1)
\end{align*}$$
A small truss as shown in Fig. has all pin jointed member having area of cross section 20 cm² each.

- the plane of rollar is inclined and is parallel to AD determine movement of joint C when
- vertical dead of utron is applied at B given that $F = 2 \times 10^6$ Kg/cm².

\[
\begin{align*}
V_C &= R_c \cos \theta \\
H_C &= R_c \sin \theta \\
H_C &= \tan \theta = 1 \\
V_C &= 2t \\
H_C &= \frac{V_C}{2} \\
E_M &= 0 \\
V_C \times 4 - 4 \times 2 &= 0 \\
V_C &= 2t \\
H_C &= 1t \\
E_F &= 0 \\
V_A + V_C - 4 &= 0 \\
V_A &= 2t
\end{align*}
\]
<table>
<thead>
<tr>
<th>Member</th>
<th>$P$</th>
<th>$K$</th>
<th>$L$</th>
<th>$PKL$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AB$</td>
<td>3</td>
<td>0.5</td>
<td>2</td>
<td>$6.708 \times 1000 = 6708$</td>
</tr>
<tr>
<td>$AD$</td>
<td>-4.47</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$BC$</td>
<td>3</td>
<td>0.5</td>
<td>2</td>
<td>$6.708 = 6708$</td>
</tr>
<tr>
<td>$BD$</td>
<td>4</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>$CD$</td>
<td>-4.47</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$EPKL = 13146$  
$19.47$

Diagram:
- Joint $A$
- $F_{APD} = 295$ N

$\epsilon_{x} = 0$

$F_{AB} + F_{AD} \cos \theta = 1 - 1 = 0$

$F_{AB} = \frac{2}{55} = 0.0364$ N

$\epsilon_{y} = 0$

$F_{AD} \sin \theta - 2 = 0$

$F_{AD} = \frac{2 \times 55}{1} = 110$ N

$F_{APD} = -4.472$ N

To find deflection in the direction of plane, apply unit load $11$ to the plane. Unit load when $\theta$ with the horizontal.
\[ EC = 0 \]
\[ V_{AX} = 0 \]
\[ V_A = 0 \]
\[ V = 0 \]
\[ V_{AC} = -\frac{1}{2} \frac{1}{2} \frac{1}{2} \]
\[ V_{CE} = -\frac{1}{2} \frac{1}{2} \frac{1}{2} \]
\[ H_c = V_c = -\frac{1}{2} \frac{1}{2} \frac{1}{2} \]
\[ \sum F_x = 0 \]
\[ H_A + 1 \cdot \cos \theta + H_c = 0 \]
\[ H_A + 2 -\frac{2}{2} = -\frac{1}{2} \frac{1}{2} \frac{1}{2} \]
\[ A_c = \sum P \cdot L \]
\[ P \rightarrow \text{kg} \]
\[ \rho \rightarrow \text{cm}^2 \]
\[ E = \text{kg} \cdot \text{cm}^2 \]
\[ L = \text{m} \]
\[ A_c \text{ will be in m} \]
\[ A = 12416 \frac{m}{20 \times 2 \times 10^6} \]
\[ 18287 \times 10^5 \]
\[ A_c = 0.32865 \]
Q. 2. Straight bars AC & BC 8 mm India made at joint C. Calculate horizontal and vertical component of deflection if all joints are hinge on vertical load of 500 kg is apply at joint C. Young modulus 2 x 10^6 kg/cm².

Ans:

ΔH = 0.823 mm

ΔV = 0.4929 mm
stiffness method to solve

**Force-displacement Relation**

\[ \Delta \text{Ax} \]
\[ \Delta \text{Ay} \]
\[ \Delta \text{Bx} \]
\[ \Delta \text{By} \]

Consider AB is a truss member and A and B are truss joints.

Let \( \Delta \text{Ax} \) and \( \Delta \text{Ay} \) are displacement of joint A in x and y direction respectively. Similarly, \( \Delta \text{Bx} \) and \( \Delta \text{By} \) are displacement of joint B.

The displacement of joint A in the direction of member AB is:

\[ \Delta A = \Delta \text{Ax} \cos \theta + \Delta \text{Ay} \sin \theta \]

Displacement of joint B in the direction of member AB is:

\[ \Delta B = \Delta \text{Bx} \cos \theta + \Delta \text{By} \sin \theta \]

\( \theta \) is angle of member AB with horizontal.

Axial displacement of member AB is:

\[ \Delta \text{AA} = \Delta \text{AB} \]

\[ = \Delta \text{Ax} \cos \theta + \Delta \text{Ay} \sin \theta - \Delta \text{Bx} \cos \theta - \Delta \text{By} \sin \theta \]

\[ = (\Delta \text{Ax} - \Delta \text{Bx}) \cos \theta + (\Delta \text{Ay} - \Delta \text{By}) \sin \theta \]
\[ \Delta \bar{e} = \Delta \bar{a} - \Delta \bar{A} \]
\[ \Delta \bar{e} = (\Delta \bar{x} - \Delta \bar{a} \bar{x}) \cos \theta + (\Delta \bar{y} - \Delta \bar{a} \bar{y}) \sin \theta \]
\[ \frac{\Delta \bar{a} \cdot \bar{L}}{\bar{AE}} = \frac{(\Delta \bar{x} - \Delta \bar{a} \bar{x}) \cos \theta + (\Delta \bar{y} - \Delta \bar{a} \bar{y}) \sin \theta}{\bar{AE}} \]
\[ \Rightarrow K \left[ (\Delta \bar{x} - \Delta \bar{a} \bar{x}) \cos \theta + (\Delta \bar{y} - \Delta \bar{a} \bar{y}) \sin \theta \right] \]

\[ K = \frac{\Delta \bar{a} \cdot \bar{L}}{\bar{AE}} \Rightarrow \text{Axial displacement} \]

A BAR A, B of a Pin-jointed Truss line in the x-y plane has the coordinates of A (0, 0) and coordinates of B (3, 3) in m. Joint A & B are displaced and the displacement in x-y direction are \( \Delta \bar{x} = 1 \) \( \Delta \bar{y} = 5 \) \( \Delta \bar{a} \bar{x} = 2 \) \( \Delta \bar{a} \bar{y} = 2.5 \) mm. The axial stiffness of BAR is 100 kN/m. The force induced in the bar due to above displacement is 100 kN.

(a) 100 kN  
(b) 200 kN  
(c) 173 kN  
(d) 273 kN

\[ (x, y) = (3\sqrt{3}, 3) \]
\[ \tan \theta = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}} \]
\[ \theta = 30^\circ \]
\[ \cos \theta = \sin 30 = \frac{\sqrt{3}}{2} \]
\[ \sin \theta = \sin 30 = \frac{1}{2} \]
\[ P_{AB} = 100 \left[ \frac{(2-1) \cdot 5}{2} + (2 \cdot 5 - 13) \cdot 1 \right] \]

\[ \Rightarrow 100 \left[ \frac{5}{2} + \frac{5}{2} \right] = 178 \text{ kN} \]

8. For three bar truss shown in fig compute vertical displacement of node 1 using stiffness method.

\[ \Delta f = \text{const.} \]
\[ L_{12} = L \]
\[ L_{13} = 2L \]
\[ L_{14} = \frac{L}{2} \]

\[ D_5 = m + \delta e - 2\delta \]
\[ \Rightarrow 3 + 6 - 2 \delta 4 \Rightarrow 9 - 8 = 1 \]

\[ \delta x = 2\delta - \delta e \]
\[ \Rightarrow 2 \times 4 - 6 = 9 \]

For displacement relations for member 1,2

\[ P_{12} = \frac{AE}{L} \left[ (\Delta x - \Delta x) \cos \theta + (\Delta y - \Delta y) \sin \theta \right] \]

\[ P_{12} = \frac{AE}{L} \left[ (-\Delta x) \right] \Rightarrow \]
\[ P_{13} = \frac{AE}{2\sqrt{5}} \left[ (\Delta x - \Delta x) \cos 30^\circ + (\Delta y - \Delta y) \sin 30^\circ \right] \]

\[ \Rightarrow \frac{\sqrt{3}}{2} \frac{AE}{L} \left[ -\Delta x \cdot \frac{\sqrt{3}}{2} - \Delta y \cdot \frac{1}{2} \right] \]

\[ = \frac{\sqrt{3}}{4} \frac{AE}{L} \left[ \frac{\sqrt{3}}{2} \Delta x + \Delta y \right] \]  
(2)

For member P14

\[ P_{14} = \frac{AE}{\sqrt{2}L} \left[ (\Delta x - \Delta x) \cos 45^\circ + (\Delta y - \Delta y) \sin 45^\circ \right] \]

\[ = -\frac{AE}{2L} \left[ -\Delta x + \Delta y \right] \]  
(3)

To find joint displacement nodal consider free body equilibrium of node 1.

\[ \Sigma F_x = 0 \]

\[ P_{13} + P_{14} \cos 30^\circ + P_{14} \cos 45^\circ = 0 \quad \text{(1)} \]

\[ \Sigma F_y = 0 \]

\[ P_{14} \sin 45^\circ + P_{14} \sin 45^\circ = P = 0 \quad \text{(2)} \]

Solving equation (2)

\[ \Delta y = 0.297L \]

\[ \frac{\Delta y}{AE} \]
MATRIX METHOD

Flexibility matrix method
(stiffness method
(force method)
(displacement method)

Properties of matrix
If a matrix is \([A]_{mxn}\), then:
- It means \(m\) rows and \(n\) columns.
- Flexibility matrix \(A\)
- Stiffness matrix will be always a square matrix.

2. A unit matrix is that which has unit as element along the diagonal.

3. A diagonal matrix is that which has all members zero other than diagonal members.

4. The magnitude of a matrix represented by its determinate.

5. A determinate will have zero value if any two rows and any two columns identical.

6. The sign of determinate will change if any two rows or any two columns are interchanged.
The inverse of a matrix $A$ is

$$A^{-1} = \text{Adj}(A)$$

$\text{Adj}(A)$ matrix is obtained by transpose of cofactor of matrix $A$.

Flexibility and Stiffness:

$$\Delta = \frac{P}{K} \quad \text{or} \quad f = \frac{P}{\delta}$$

$$K = \frac{P}{\delta} \quad k - \text{stiffness} \quad f - \text{flexibility}$$

Stiffness is equal to force required to produce unit displacement. & Flexibility is displacement produced by unit force.

Properties of flexibility and stiffness matrix:

A $f \times k$ matrix always square matrix $[m \times n]$.

$$[A]_{m \times n} = \left[ \begin{array}{cccc}
 a_{11} & a_{12} & \cdots & a_{1n} \\
 a_{21} & a_{22} & \cdots & a_{2n} \\
 \vdots & \vdots & \ddots & \vdots \\
 a_{m1} & a_{m2} & \cdots & a_{mn}
\end{array} \right]$$
- The diagonal element will be always non-zero and non-negative i.e. always positive.

- The matrix will be always symmetrical about the diagonal. It means the transpose of matrix will be unchanged it is because of applicability of Maxwell's Reciprocal Theorem.

\[
\begin{align*}
& f_{12} = f_{21} \quad \text{or} \quad k_{12} = k_{21} \\
& f_{xy} = f_{yx} \\
& k_{xy} = k_{yx} \\
& a_{xy} = a_{yx}
\end{align*}
\]

- Procedure to develop flexibility matrix.

\[
\begin{bmatrix}
 f_{11} & f_{12} & \cdots & f_{1n} \\
 f_{21} & f_{22} & \cdots & f_{2n} \\
 \vdots & \vdots & \ddots & \vdots \\
 f_{m1} & f_{m2} & \cdots & f_{mn}
\end{bmatrix}_{n \times n}
\]

- \( f_{11} = \) displacement in the direction of \( 1 \) due to \( f_{11} = \) unit force in the direction of \( 1 \).

- \( f_{xy} = \) displacement in the direction \( x \) due to \( f_{xy} = \) unit force applied in the direction of \( y \).
To generate first column of flexibility matrix give apply unit force in dir. 1 only and measure the displacement in directions 1, 2, -- (8)

To obtain second column apply unit force in dir. 3 only and repeat the procedure

1. A, E, I Constant

To develop first column of flexibility matrix apply unit force in direction 1 only and measure the displacement in all the coordinate directions.

\[ f_{11} = \text{displacement in dir. 1 due to unit force in the dir. 1} \]
\[ \frac{1}{AE} \]

\[ f_{12} = \text{displacement in dir. 2 due to unit force in dir. 1} \]

\[ 0 \]

\[ f_{13} = 0 \]

To generate second column of flexibility matrix apply unit force in dir. of 3 only and measure the displacement in all the coordinate directions.
To generate 3 columns of flexibility using unit force in the column.

\[ f_{11} = 0 \quad f_{22} = \frac{L^3}{3EI} \quad f_{32} = -\frac{L^2}{2EI} \]

\[ f_{12} = 0 \quad f_{23} = \frac{L^3}{2EI} \quad f_{33} = \frac{L}{EI} \]

\[ \begin{bmatrix} \frac{L}{AE} & 0 & 0 \\ 0 & \frac{L^3}{3EI} & -\frac{L^2}{2EI} \\ 0 & \frac{L^3}{2EI} & \frac{L}{EI} \end{bmatrix} \]

diagonal members are non-zero and non-negative.

Develop flexibility matrix for column shown in fig.

\[ AB \quad 0 \quad 0 \quad 0 \]

\[ AJ \quad 0 \quad 0 \quad 0 \]

\[ AB = 2m \quad BC = 2m \quad CD = 2m \]
0. First column.

\[ f_{11} = \frac{1 \cdot (2)^3}{3EI} \Rightarrow \frac{8}{3EI} \]

\[ f_{21} = AB + OB - LBC \]

\[ \Rightarrow \frac{8}{3EI} + \frac{1 \cdot (2)^2 \times 2}{2EI} \Rightarrow \frac{40}{3EI} \]

\[ f_{21} = DD = AB + OB \times LBD \]

\[ \Rightarrow \frac{8}{3EI} + \frac{1 \cdot (2)^2 \times 4}{2EI} \Rightarrow \frac{48}{3EI} \]

0. Second column.

\[ f_{x1} = \frac{DY \cdot 3}{8EI} \]

\[ f_{x1} = \frac{20}{3EI} = f_{21} \]

\[ f_{22} = \frac{1 \cdot (4)^3}{3EI} \Rightarrow \frac{64}{3EI} \]

\[ f_{22} = \frac{64}{3EI} + \frac{48}{3EI} \Rightarrow \frac{112}{3EI} \]

0. Third column.

\[ f_{13} = \frac{32}{3EI} \]

\[ f_{23} = \frac{112}{3EI} \]

\[ f_{23} = \frac{16 \times 3}{3EI} \]
A developed flexibility matrix for the beam coordinates shown in fig.

\[
\begin{bmatrix}
8 & 20 & 32 \\
20 & -64 & 112 \\
32 & 112 & 216
\end{bmatrix}
\]

\[
\text{EI = constant}
\]

\[
\phi_{B1} = \frac{L}{3EI} \quad \phi_{B2} = \frac{L}{3EI} \quad \phi_{C} = \frac{L}{6EI}
\]

\[
\delta_{11} = \frac{L}{3EI} + \frac{L}{3EI} = \frac{2L}{3EI}
\]

\[
\delta_{21} = \frac{6L}{6EI}
\]

\[
\theta = \frac{L}{3EI} \quad \delta_{12} = \frac{L}{6EI} \quad \delta_{22} = \frac{L}{6EI}
\]
Developed the flexibility matrix for the beam with coordinate shown in Fig.

\[
\begin{bmatrix}
\frac{2L}{3EI} & \frac{L}{6EI} \\ \\
\frac{L}{6EI} & \frac{L}{3EI}
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 1 \\ \\
-1 & 0
\end{bmatrix}
\]

\[
E I = \text{constant}
\]

\[
A = \frac{M L}{3EI}
\]

\[
O B = \frac{M L}{3EI}
\]

\[
f_{11} = \frac{L}{3EI}
\]

\[
f_{21} = \frac{L}{6EI}
\]

\[
f_{31} = \frac{L^2}{16EI}
\]

\[
f_{12} = \frac{L}{6EI}
\]

\[
f_{22} = \frac{L^2}{16EI}
\]

\[
M = 1 \text{ kN-m}
\]

\[
\theta A = \frac{M L}{6EI}
\]

\[
O B = \frac{M L}{6EI}
\]

\[
M = 1 \text{ kN-m}
\]
Develop flexibility matrix method for the spring system shown in fig.

\[ \theta_A = \frac{L^3}{16EI} \]
\[ \theta_B = \frac{L^3}{16EI} \]
\[ S_C = \frac{L^3}{48EI} \]

\[ f_{12} = \frac{L^2}{16EI} \]
\[ f_{23} = \frac{L^2}{16EI} \]
\[ f_3 = \frac{L^3}{48EI} \]

\[ f_A = 0.05 \text{ cm/kN} \]
\[ f_B = 0.10 \]  \[ \text{“} \]  
\[ f_C = 0.20 \]  \[ \text{“} \]  

\[ f_{11} = f_A \times 1 = 0.05 \text{ cm} \]
\[ f_{21} = f_2 = f_1 = 0.05 \text{ cm} \]
\[ f_{31} = f_3 = f_1 = f_2 = 0.05 \text{ cm} \]

\[ f_{12} = 0.05 \text{ cm} \]
\[ f_{22} = 0.05 + 0.10 = 0.15 \text{ cm} \]
\[ f_{22} = 0.15 \text{ cm} \]

\[ \phi = \frac{f}{p} \Rightarrow \Delta = 5 \times \rho \]
\[ f_{12} = 0.05 \]
\[ f_{23} = 0.05 + 0.10 = 0.15 \]
\[ f_{32} = 0.05 + 0.10 + 0.20 = 0.35 \]

\[ \begin{bmatrix}
 0.05 & 0.05 & 0.05 \\
 0.05 & 0.15 & 0.15 \\
 0.05 & 0.15 & 0.35
\end{bmatrix} \]

A develop flexibility matrix for cantilever beam with coordinate marked in fig.

\[ f_{i1} = -\frac{L^2}{3EI} \]
\[ f_{i1} = -\frac{L^2}{2EI} \]
\[ f_{31} = -\frac{L^2}{2EI} \]
\[ f_{12} = -\frac{L^2}{2EI} \]
\[ f_{032} = \frac{L}{ES} \]
\[ f_{22} = \frac{1}{kE} \]
\[ \sigma = \frac{du}{dm} \]
\[ u = \frac{M^2L}{2EI}, \frac{M^2L}{2EI} \]
\[ F_3 = -\frac{L^2}{2EI} \]
\[ F_{23} = \frac{2L}{E} \]
\[ \sigma = \frac{M^2L}{2EF} \]
\[ \frac{du}{dm} = 2ML \rightarrow \frac{2L}{EI} \]
\[ [f] = \begin{bmatrix} \frac{L^3}{3} & -\frac{L^2}{2} & -\frac{L^2}{2} \\ \frac{L^2}{2} & 1 & 1 \\ -\frac{L^2}{2} & 1 & 2 \end{bmatrix} \]
Stiffness matrix

Procedure to develop stiffness matrix:

\[ K = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{bmatrix} \]

- \( k_{11} \) = Force dev. req. at 1 due to unit disp. at 1 only
- \( k_{21} \) = " " " " due to unit disp. at 1 only
- \( k_{xy} \) = Force dev. req. at 2 due to unit disp. at 1 only
- \( k_{xy} \) = Direction of force measured
- \( k_{x1} \) = Direction of unit disp.

To generate 1st column of stiffness matrix give unit displacement at 1 only it means all other coordinate are locked and measure force develop required in other coordinate direction.

To generate 2nd column of stiffness matrix give unit displacement in dir. 2 only and measure the forces in all other coordinate direction. When all other coordinate are locked.
Develop stiffness matrix for beam AB with end rotations shown in Fig.

\[ \Delta = \frac{\delta}{4} \]

\[ \delta = \text{constant} \]

\[ k_{11} = R_B = \frac{12EI}{L^3} \]

\[ k_{21} = -\frac{6EI}{L^2} \]

\[ \sum M_a = 0 \]

\[ -R_B x_1 L + (6EI + 6EI) \theta B = 0 \]

\[ R_B = \frac{12EI}{L^3} \]

\[ k_{12} = R_B = -\frac{6EI}{L} \]

\[ k_{33} = -\frac{4EI}{L} \]

\[ k = \begin{bmatrix} \frac{6EI}{L} & -6EI & \vdots \\ -6EI & \frac{12}{L} & \vdots \\ \vdots & \vdots & \ddots \end{bmatrix} \]
A: Jomard stiffness matrix beam with coordinate shown in fig.

\[
\begin{bmatrix}
-1 & 0 \\
0 & 4
\end{bmatrix}
\]

\[
\begin{bmatrix}
2E & -E \\
-E & 2E
\end{bmatrix}
\]

Develop stiffness matrix for a beam shown in fig.
\[ K_{12} = \frac{2EI}{L} \]
\[ K_{22} = \frac{8EI}{L} + \frac{4EI}{L} = \frac{12EI}{L} \]

\[
K = \begin{bmatrix}
\frac{2EI}{L} & \frac{4EI}{L} \\
\frac{4EI}{L} & \frac{12EI}{L}
\end{bmatrix}
\]

\[ K = \left( \frac{2EI}{L} \right) \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \]

*Note: Stiffness matrix for beam shown in Fig.*

\[
\frac{u_{2}}{l_{1}} = \frac{8}{l_{1}} \begin{bmatrix} 2 \end{bmatrix} \]

\[
K_{11} = 2EI + EI = 3EI \\
K_{12} = EI - 0.5EI = 0.5EI \\
K_{22} = 2EI
\]
Develop stiffness matrix for the beam with coordinate shown in fig.

\[ K = \begin{bmatrix} 3 & 0.5 \\ 0.5 & 2 \end{bmatrix} \]

\[ \delta_{11} = \frac{4EI}{L} \quad \delta_{12} = \frac{-2EI}{L} \quad \delta_{31} = \frac{-6EI}{L^2} \quad \delta_{32} = \frac{4EI}{L} \frac{6EI}{L^2} \]

\[ R_A = \frac{6EI}{L^2} \quad R_A + R_B = 0 \quad R_B = -\frac{6EI}{L^2} \]

\[ K_{11} = \frac{4EI}{L} \quad K_{21} = \frac{-2EI}{L} \quad K_{31} = \frac{-6EI}{L^2} \quad K_{32} = \frac{4EI}{L} \frac{6EI}{L^2} \]

\[ K_{12} = \frac{-2EI}{L} \quad K_{22} = \frac{4EI}{L} \]

\[ K_{31} = K_{32} = \frac{6EI}{L^2} \quad K_{33} = \frac{-6EI}{L^2} \]

\[ K = \begin{bmatrix} 4 & -2 & -6 & 6 \\ -2 & 4 & 6 & -6 \end{bmatrix} \]

\[ K_{11} = K_{22} = \frac{6EI}{L^2} \]

\[ K_{12} = K_{21} = \frac{4EI}{L} \]

\[ K_{33} = K_{34} = \frac{6EI}{L^2} \]

\[ K_{43} = K_{44} = \frac{6EI}{L^2} \]
\begin{align*}
\sigma_{RB} &= 0 \\
\frac{RA}{L^3} &= 12EI \\
\frac{RA}{L^3} &= -12EI \\
K_{12} &= -6EI \\
K_{23} &= 6EI \\
K_{33} &= 12EI \\
K_{343} &= -12EI \\
K_{344} &= 12EI \\
R_{A} &= -12EI/L^3 \\
R_{B} &= 12EI/L^3 \\
K_{11y} &= 6EI/L^2 \\
K_{22y} &= -6EI/L^2 \\
K_{33y} &= -12EI/L^3 \\
K_{44y} &= 12EI \\
K &= \begin{bmatrix}
2 & -1 & -3 & 3 \\
-1 & 2 & 3 & -3 \\
-3 & -1 & 6 & -6 \\
3 & -\frac{3}{L} & 6 & \frac{6}{L^2} \\
\end{bmatrix}
\end{align*}
Develop stiffness matrix for the frame shown in Fig.

with coordinate indicated.

\[
\begin{align*}
Ea & > \quad \frac{6EI_1}{L_1^2} \\
H_A + H_B &= 0 \\
\Sigma M_B &= 0 \\
&= -H_A x L_1 - 6EI_1 - 6EI_1 \\
&= -12EI_1 \\
&= H_B = \frac{12EI_1}{L_1^3} \\
K_{11} &= \frac{12EI_1}{L_1^3} \\
K_{22} &= -\frac{6EI_1}{L_1^2}
\end{align*}
\]
to generate 2nd column:

\[ k_{22} = \frac{4EI_1 + 3EI_2}{L_1} \]

\[ k_{12} = \frac{-6EI_1}{L_1^2} \]

\[ k_{21} = \frac{4EI_1 + 3EI_2}{L_2} \]

\[ K = \begin{bmatrix} \frac{12EI_1}{L_1^3} & \frac{-6EI_1}{L_1^2} \\ \frac{-6EI_1}{L_1^2} & \frac{4EI_1 + 3EI_2}{L_2} \end{bmatrix} \]

Flexibility matrix method.

\[ A \quad \text{w}lm \quad B \quad \text{M}_C \quad H_C = 0 \]

\[ RA = R_1 \quad R_B = R_2 \quad R_C \]

**Step 1**: Determine degree of static indeterminacy in beam (neglect axial forces).

\[ DS = D_e + DS_1 \]

\[ = 2e^{-3} \]

\[ = 5 - 3 = 2 \]

In above structure \( DS = 2 \) hence identify redundant. Say redundant are \( RA \) & \( RB \) (Re).

(Handwritten notes)
Step 2. Remove the redundant and assign one co-ordinate in the direction of each redundant the no. of co-ordinate will be equal to 3x i.e. no. of redundant.

\[ \text{Diagram} \]

Develop flexibility matrix for the above assign co-ordinates.

Step 3. Remove the redundant and find the deflections in the direction of assign co-ordinate due to given load system. When the redundant are release, the structure will be determinate co-ordinates called Relative structure basic determinate structure. Let due to given loading the deflections in the direction of co-ordinate are \( A_1 \), \( A_2 \), \( A_3 \).

\[ \text{Diagram} \]

Step 4. Remove the given loading and apply the redundant reaction in the direction of assigned co-ordinate.

\[ \text{Diagram} \]
And find displacement in the direction of co-ordinate due to Redundant let $\Delta 1R, \Delta 2R$ 

$\Delta 1R = f_{11} R_1 + f_{12} R_2$

$\Delta 2R = f_{21} R_1 + f_{22} R_2$

$$\begin{bmatrix} \Delta 1R \\ \Delta 2R \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

the final displacement in the direction of co-ordinate will be $\Delta 1 = \Delta 1L + \Delta 1R$

$\Delta 2 = \Delta 2L + \Delta 2R$

but final displacement are

$\Delta 1R = \beta_{1} - \Delta 1L$

$\Delta 2R = \beta_{2} - \Delta 2L$

$$\begin{bmatrix} \Delta 1R \\ \Delta 2R \end{bmatrix} = \begin{bmatrix} \Delta 1 - \Delta 1L \\ \Delta 2 - \Delta 2L \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} \Delta 1 - \Delta 1L \\ \Delta 2 - \Delta 2L \end{bmatrix}$$
\[ \frac{R_1}{R_0} = \left[ f \right]^{-1} \left[ n - A_1 \right] \]

\[ f_{11} = \text{displacement at 0 due to unit force at O} \]
\[ f_{12} = \text{displacement at O due to unit force at 2} \]

\[ A_{1R} = f_{11} \cdot R_1 + f_{12} \cdot R_2 \]
\[ A_{2R} = f_{21} \cdot R_1 + f_{22} \cdot R_2 \]

**Ex.** Analysis the beam shown in fig. Find the reaction using flexibility matrix method.

\[ \begin{array}{c}
| & \text{10KN} & \text{1.5m} & 40KNm & \text{1m} \\
\hline
A & 1.5m & \text{1.5m} & \text{40KNm} & \text{1m} \\
\hline
R_1 = RA & R_2 = RB & \text{EI constant} & \text{EI constant} \\
\end{array} \]

\[ D_1 = 5 \times 10^{-3} = 5 \times 3 = 2 \]

Let Redundent are, RA & RB.

Assign one coordinate in dim of each Redundent.

1st column of flexibility matrix

\[ f_{11} = \frac{-73}{3EI} \Rightarrow \frac{7^3}{3EI} = \frac{343}{3EI} \]

\[ f_{21} = \frac{136}{3EI} \]
1st column of flexibility matrix

\[ f_{11} = \frac{1 + 8}{3EI} \to \frac{34\beta}{3EI} \]

2nd column of flexibility matrix

\[ f_{12} = \frac{4\beta^3}{3EI} + \frac{1 + 8}{2EI} \times 3 \]
\[ \Rightarrow \frac{6\beta}{3EI} + \frac{2\beta}{EI} \to \]

\[ f_{22} = \frac{1 \cdot 4\beta}{3EI} = \frac{6\beta}{3EI} \]

\[ \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} \frac{34\beta}{3EI} & 136 \\ \frac{136}{3EI} & \frac{6\beta}{3EI} \end{bmatrix} \]

\[ \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{6\beta}{3EI} & \frac{-136}{3EI} \\ \frac{-136}{3EI} & \frac{34\beta}{3EI} \end{bmatrix} \]

\[ \Rightarrow \frac{34\beta}{3EI} \times \frac{6\beta}{3EI} - \left( \frac{136}{3EI} \right)^2 \]
in Basic Slab Structure find $D_1$ and $D_{21}$ due to given loading in the direction of assigned co-ordinate.

\[
\begin{align*}
A & = 1.5m \\
B & = 8.4m \\
C & = \frac{60}{EI}
\end{align*}
\]

\[
\Delta_{11} = -\frac{11654.58}{EI}
\]

\[
\Delta_{21} = -\frac{0253.33}{EI}
\]

\[
\begin{bmatrix}
\frac{R_1}{R_2}
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}^{-1} \begin{bmatrix}
\Delta_{11} \\
\Delta_{21}
\end{bmatrix}
\]

\[
\begin{bmatrix}
R_1 \\
R_2
\end{bmatrix} = \begin{bmatrix}
0.055EI & -0.118EI \\
-0.118EI & 0.297EI
\end{bmatrix} \begin{bmatrix}
0.00165428/\text{EI} \\
0.005253/\text{EI}
\end{bmatrix}
\]
\[ R_1 = 0.055 \times 11654.58 \times (0.118 \text{ in}) \times 5253.33 \text{ in} \]

\[ R_1 = 21.11 \text{ kN} \]

\[ R_2 = 185 \text{ kN} \]

**Stiffness matrix method**

**Step-1:** Neglecting Axial deformation, find degree of kinematic indeterminacy

\[ DK = 3J - 9 \varepsilon - m - 2 = 9 - 9 = 2 \]

(Axial rigid member)

**Step-2**

Find stiffness matrix for the assigned co-ordinates and determine \([K]^{-1}\)
Let $P_1'$ and $P_2'$ are forces of the moment required develop at coordinate $O$ & $O$ to lock the structure due to given loading.

$$M_{AB} = P_1' = \frac{-P_1}{8}$$

$$M_{BA} + M_{Bx} = P_2' = \frac{P_1}{8} - \frac{wL_2}{12}$$

Let $P_1, P_2$ are final forces/moment available in the direction of assign coordinate. If no external load or moment acts in direction of origin co-ordinate, then final value $P_1$ and $P_2 = 0$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} K_1 \end{bmatrix} \begin{bmatrix} P_1' \\ P_2' \end{bmatrix}$$

$P_1'$ and $P_2'$ Nothing that fixed end moment acts co-ordinate $O$ and $O$ when structure is locked due to given loading.

If co-ordinates are linear then $P_1'$ and $P_2'$ will be reactions due to given loading in the direction of co-ordinate.
\[ K_{11} = \frac{4EJ}{3} \]
\[ K_{21} = \frac{2EJ}{3} \]
\[ K_{22} = \frac{7EJ}{3} \]

\[ [K] = \begin{bmatrix} \frac{4EJ}{3} & \frac{2EJ}{3} \\ \frac{2EJ}{3} & \frac{7EJ}{3} \end{bmatrix} \]

\[ K^{-1} = \begin{bmatrix} \frac{7EJ}{18EJ} & -\frac{1}{4EJ} \\ -\frac{1}{4EJ} & \frac{1}{2EJ} \end{bmatrix} \]

\[ P' = M_{AB} = -10 \times 3 = -30 \]

\[ P_2' = M_{BC} = +3.75 - \frac{6 \times 4^2}{12} = 3.75 - 8 = 4.25 \]
In simply supported trusses when unit load moves from one end to the other ends then top chord members are in compression. Bottom chord are in tension and in inclined member force change to tension for compression to tension.

When unit load is left of member then force is inclined member is compression and when unit load is right of the member then force is tension. In vertical members force changes tension to compression.

If load moves over top chord, then truss is called deck type truss and if load moves over bottom chord then truss is called through type.
Q. Draw E.M. for \( u_2, u_3, l_2 l_3 \) and \( l_2 \) and \( l_3 \)

\[
\begin{align*}
L_1, L_2, L_3, L_4, L_5, L_6, h, S @ a, (1-x) = 5a-x
\end{align*}
\]

Consider unit load placed at a distance \( x \)

\[
1 = 5a
\]

\[
3M_6 = 0
\]

\[
R_1 x L - 1 (1-x) = 0
\]

\[
\frac{R_1}{L} = \frac{L-x}{5a}
\]

\[
R_6 = \frac{x}{L} \cdot \frac{x}{5a}
\]

\[
I.D. \text{ for } l_2 l_3
\]

\[
\text{Case 1: Unit load is to the left of } l_2 l_3 \text{. Consider free body dia. to the left of } x
\]
To find $F_1$ in $L_2$ take moment about $y_3$.

$\Sigma M_{y_3} = 0$

$R_1 \cdot a - 1 \cdot (a-x) - F_{L_2} \cdot L_2 = 0$

$F_{L_2} = \frac{R_1 \cdot a - (a-x)}{h}$

$F_{L_2} = \frac{5a-x \cdot a}{5a} - (a-x)$

$F_{L_2} = \frac{5a-x-5a+5x}{5a}$

$F_{L_2}$

$F_{L_2} = \frac{ya}{5h}$

When $x=0$, $F_{L_2} = 0$

When $x=a$, $F_{L_2} = \frac{ya}{5h} \quad (+)$ Tension.
Case I: When unit load is in $L_2$ and $L_3$

\[ \Sigma M_{L_2} = 0 \]

\[ R_{1a} = F_L L_2 x h \]

\[ F_{L_2 L_3} = \frac{R_{1a} h}{h} \]

\[ F_{L_2 L_3} = \frac{(5a-x)g}{h} \]

\[ \text{If } x = a \]

\[ F_{L_2 L_3} = \frac{u a}{5 h} \]

\[ \frac{h}{h} x = 2a \]

\[ F_{L_2 L_3} = \frac{3a}{5h} \]

\[ g x = 0 \]

\[ x = 5a \]

\[ F_{L_2 L_3} = 0 \]

\[ \Sigma U \] for $U_2 U_3$

\[ F_{U_2 U_3} < U_3 \]

\[ F_{L_2 L_3} \]

\[ F_{L_2 L_3} < (x-2a) \]
\[
\begin{align*}
D_1 &= \frac{\theta A}{EI} = 2.22 \\
D_2 &= \frac{\theta B}{EI} = 1.187
\end{align*}
\]

To find final end moment write slope deflection equations.

\[
M_{AB} = \bar{M}_{AB} + \frac{2EI}{3} \left[ 2\theta A + \theta B - 3\theta \right] = 0
\]

\[
= -3.75 + \frac{2EI}{3} \left[ \frac{2\times2.22}{EI} + \frac{1.187}{EI} \right]
\]

\[
\bar{M}_{AB} = -3.75 + 3.75 = 0
\]

\[
M_{BA} = \bar{M}_{BA} + \frac{2EI}{3} \left[ 2\theta B + \theta A - 3\theta \right] = 0
\]

\[
= +13.95 + \frac{2EI}{3} \left[ \frac{2\times1.187}{EI} + \frac{2.22}{EI} \right]
\]

\[
= +6.81 + m
\]

\[
M_{BC} = \bar{M}_{BC} + \frac{2EI}{3} \left[ 2\theta B + \theta c - 3\theta \right] = 0
\]

\[
= -8.81 + \frac{6.157}{EI}
\]

\[
M_{CB} = -8.55
\]
Chapter 6

0. d
1. b
2. A
3. a
4. c
5. a, d, c

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Case 1: when unit load is in L₁, L₃

\[ \Sigma M_{L_3} = 0 \]
\[ F_{u_2}u_3xh + R_6x3a = 0. \]
\[ F_{u_2}u_3 = -R_6 \cdot \frac{3a}{h} = -\frac{2}{5a} \cdot \frac{3a}{h} \]
\[ \Rightarrow -\frac{3}{5} \cdot \frac{2}{h} \]

Case 2: when load is in L₂, L₆

\[ \Sigma M_{L_2} = 0 \]

\[ R_6 \times 3a + F_{u_2}u_3xh - 1 \times (x - 2a) = 0 \]
\[ F_{u_2}u_3 = -R_6 \times 3a + (x - 2a) \]
\[ F_{u_2}u_3 = -\frac{x}{5a} \times 3a + x - 2a \]
\[ F_{u_2}u_3 = \frac{a}{5} \left( x - \frac{5a}{h} \right) \]

When \( x = 2a \)
\[ F_{u_2}u_2 = -\frac{6a}{5h} \]

When \( x = 5a \)
\[ F_{u_2}u_3 = 0 \]
Inclined member makes angle $\theta$ with the vertical.

Case 1: When unit load is 1 kN in L1.

$$Efy = 0$$

$$F_{u3} \cos \theta + R_6 = 0$$

$$F_{u3} = \frac{-R_6}{\cos \theta} = \frac{-x}{5a} \frac{1}{\cos \theta}$$

$$\cos \theta = \text{when } 1kN 18 \text{ in } L1$$

$$F_{u1} \cos \theta + R_6 - 1 = 0$$

$$F_{u1} = \frac{1 - x/5a}{\cos \theta}$$

If $x = 2a$

$$F_{u1} = \frac{3}{5 \cos \theta}$$

If $x = 5a$

$$F_{u2} L_3 = 0$$
Problem 5

1. b
2. c
3. d
4. e
5. b (by method of sections)
6. d
7. d
8. c
9. a
10. b
11. None (C)
12. b
13. C
14. d
15. C
16. a
17. a
18. b
19. a
20. a
21. a

1 3 9

For max tension live load should be only in tension from Zone.

\[ d = D \times L \times (\text{Net Area}) + L \times L \times (\text{Area}) \]

\[ = 20 \times [-(10+20) + 10 \times 20] \]

\[ = 400 \text{ kN} \]