HAND WRITTEN NOTES:

OF

CIVIL ENGINEERING

SUBJECT:

DESIGN OF STEEL STRUCTURE
Design of Steel Structures

Rivetted connection
Welded connection
Eccentric connection
Tension member
Compression members
Beams
Plate girders
Industrial building
Plastic Analysis - Most Imp

15:300 - 1981
15:300 - 2007
Based on limit state
Method of analysis
Although was not given in appendix
Is 875 - loading

By - Rekha Thakur
Rivetted Connection

Rivets are generally made of mild steel, and for normal construction purposes Fe 410 S grade of steel. The 410 N/mm² ultimate strength of steel. This grade is suitable for riveting for all thicknesses but for welding it is suitable only up to 20mm thickness of member.

For thicknesses more than 20 mm welding grade of steel is used for member, they are Fe 410 W4 and Fe 570 HT.

The other grades of steel are Fe 570 HT. HT stands high tension. High tension grade of steel has higher strength and they are more resistant to atmospheric corrosion; hence this grade is adopted when reduction in dead wt is desired and when steel is prone to air corrosion.

Specific gravity = 7.85

Oxidation density = 7850 Kg/m³

Unit wt = 785 Kgf/m³

\[ E = 2.15 \times 10^5 \text{ N/mm}^2 \]

\[ \alpha = 1.2 \times 10^{-6} /^\circ C \]
Advantage of steel structure over aluminium str.

Strength per unit wt. of aluminium is much greater than that of steel.

However \( E_{al} = \frac{1}{3} E_{st} \)

Hence all structure is more prone to buckling. To overcome this cross-sectional area of Al str. needs to be increased.

Even then a saving of 50% in wt. results by the use of Al str.

However Al is almost 8 times costlier than steel.
Thus greater economy is achieved by the use of steel str.

\[ \frac{\alpha_{st}}{\alpha_{al}} = 2 \alpha_{st} \]  

Hence it can not be encased in concrete as steel is normally done.

Note: Encasing is normally done for aesthetic & fire resistance.

Maintenance cost of Al structure is less as it is not prone to corrosion.

Method of Design:

Elastic method of design is used.

Simplified mild steel stress strain curve.

The structure is designed in such away that stress in the member do not exceed permissible stress.
To design the structure we need to access the loading to which the structure will be subjected to. Hence there is uncertainty in loading. Similarly there is uncertainty in material property.

Also to simplify the analysis certain assumptions are made because of which error will be introduced in the result, to account for all these we use F.O.S.

\[
\begin{align*}
\text{F.O.S. for tension} &= 1.67 = \frac{1}{0.6} \quad f_s = 0.6 f_y \\
\text{F.O.S. for bending} &= 1.5 = \frac{1}{0.66} \quad f_s = 0.66 f_y 
\end{align*}
\]

Note: In case of bending there is a margin to resist additional loading beyond the point of first yield. However in tension there is no margin beyond the point of first yield. Hence F.S in tension is more as compared to bending.

4. Nominal Dia of rivet

It is the dia of shank of rivet in cold condition.

Cold and Hot rivetting

Cold and hot rivettings are methods of rivetting. Cold rivetting is not adopted for dia > 10 mm.

In hot rivetting rivets are heated to 550 - 1000°C. They are inserted in the hole made in the member, and hammering is done so as to make head on the hammer side. The diameter of hole in the member should be more than the nominal dia of rivet so that the rivets can be easily inserted in heated condition.
In comp. gross area is effective and in tension net area is effective in resisting loads.

Rivets in group subjected to direct load share the load equally (if they are of the same diameter).

Actually stress is assumed to be equally shared.

\[
\text{Force taken by one rivet} = \frac{P}{g}
\]

The outer rivet will be streamed more as compared to the inner rivet hence in one line we do not accept more than 5 rivets otherwise shearing effect will occur and rivets will start breaking one by one.

Bending stress in rivet is neglected under normal situation but if grip length is more the bending can not be neglected in that case additional precautions are taken as follows. If packing is of larger depth, additional rivets are provided on packing extension. This additional rivet absorbs the effects and reduces the effect of bending stress.

Recommendation: If the grip of rivet exceed 6 times the dia of rivet hole, no of rivets required by normal calculation should be increased by not less than 1% for each additional grip of 1 in.
providing on backing extension.

Dia of hole = 20 mm
grip length = 39.52 mm
load = 100 KN

Strength of 1 rivet = 20 KN
Calculate the no. of rivets required

\[ 6 \times \text{Dia of hole} = 6 \times 20 = 120 \text{ mm} \]

\[ \text{Grip over } 120 \text{ mm} = 152 - 120 = 32 \text{ mm} \]

So additional no. of rivets (over and above from normal calc.)

\[ = 1\% \text{ of the no. of rivets obtained from normal calculation per 1.6 mm} \]

\[ \text{No. of rivet req. by normal cal} = \frac{100}{20} = 5 \]

\[ \text{Add. no. of rivet required} = \frac{32.0 - 2.0}{1.6} \times 5 = 1 \]

Total no. of rivet = 5 + 1 = 6

Rivet fills the hole completely i.e. to calculate strength of rivet we use the gross dia of rivet i.e. the hole dia.

1. Pitch & Gauge

gauge length

\[ p - p \]

Distance b/w centre line of two rivets in the dia of force is called pitch of this b/w centre of two rivets at right angle to the dia of force is called gauge.
Dia of hole = Nominal dia of rivet + 1.5 mm for dia ≤ 25 mm
Dia of hole = Nominal dia of rivet + 2 mm for dia > 25 mm
Dia of hole is also called as gross dia of rivet. (This is under the assumption that rivet fills the hole completely)

Unwin's formula

Dia of rivet to suit the thickness of member

\[ d_{mm} = 6.05 \sqrt{t_{mm}} \]  
\[ d_{mm} \text{ in (mm)} \]

Normally adopted dia of rivets are 10, 12, 16, 18, 20, 22 in (mm)

Assumptions in rivetted connection

Friction b/w the plates is neglected.
In hot rivetting because of gripping friction develops b/w the plates. So long as the external force overcomes the friction force, rivets will not be subjected to any stress. Once the friction is overcome, the rivets will deform and hence will start sharing loads, but it is difficult to quantify the amount of friction hence it is neglected and all the forces are assumed to be resisted by rivets only.

Shear stress is uniform over the X-section of rivet.

Actual variation of shear stress

Assumed shear stress
Max S.F. resisted by 1 rivet = \( \sigma_{beam} \times \frac{T \cdot d_{hole}}{4} \)

Note: Shear failure is a sudden failure hence for shear it is large.

Above assumption is a simplifying assumption. No.

Distribution of direct stress on the portion of plate by the rivet hole is uniform.

\[ p \]

Actual stress variation

\[ p \]

Assumed stress variation

\[ P_{\text{max}} = (B - 3d') \cdot t \cdot \sigma_t \]

Note: Because of stress concentration stress is normally 2-3 times that of avg. stress.
Transmission of load through rivets

Max bearing force resisted by top portion of rivet

\[ P = (d' \times t) \sigma_{br} \]

by bottom

\[ P = (d' \times t) \sigma_{br} \]

Max force resisted by rivet in bearing = \( d'^{2} t \sigma_{br} \)

\[ L = \text{min thickness} \]

Max force resisted by rivet in shearing = \( \frac{4d^{2} t}{l} \sigma_{s} \)

Permissible shearing stress

The strength of rivet is min of shearing and bearing strength, this is called rivet value of the rivet.

\[ S = \text{strength of rivet} \]

If the rivet fails in shear by shearing on one plane the rivet is said to be in single shear.
$d'$ = Die of hole

Bearing strength of rivet = $d' \times t \times \sigma_{bt}$

$t = \min\left( t_1, \left(t_2 + t_3\right) \right)$

Shearing strength of rivet = $2 \times \frac{\pi a^2}{4} \times \sigma_S$

In this case rivet is said to be in double shear.

**Note:** Max. strength of rivet is taken as that corresponding to double shear.

**Type of Joints**

- **Lap Joint**

- **One pitch length**

- Single Rivetted Lap Joint
Double rivetted lap joint

Similarly when in one pitch length there are 3 rivets
Triple rivetted lap joint

Double rivetted double cover butt joint

Double rivetted single cover butt joint

Note: whenever we concentrate on butt joint we concentrate on one side of butt joint of connection.

Eccentricity is eliminated in the case of double cover butt joint.

\[
m = \frac{P}{2} \left( \frac{c}{2} - \frac{P}{2} \right) = 0
\]
Failure of Rivet Joint

1) by tearing of plate b/w rivet hole and edge of plate

1/4

Shear failure

edge distance

It is a type of shearing failure of plate at ends and occurs due to insufficient edge distance. It is prevented by keeping the edge distance to be twice the diameter of rivets hole.

2) by tearing of plate b/w rivet

$d'$ - dia of hole

$t$ - thickness of plate

NEA in tension = $(8 - 4d')t$

Strength of main plate in tearing = $(8 - 4d')\delta_{at}$

$\delta_{at}$ - permissible stress in axial tension

Strength of main plate / pitch length = $(p - d')t\delta_{at}$
In case of double rivetted butt joint strength of joint in pitch length will be

\[ (b-d) t \delta a t \]

Shearing failure of joint by shearing of rivets strength of joint in shearing is equal to summation of shearing strength of all rivets in the joint.

\[ \text{Strength of joint in shearing} = \frac{\pi \delta d'^2}{4} x \delta \]

Rivets are in single shear No. of rivets/pitch length = 3

Strength of joint in shearing per pitch length

\[ = \frac{\pi}{4} \left( d'^2 \delta \right) x 3 \]

Strength of triple rivetted double convex butt joint in shearing per pitch length

\[ = 2 \times \frac{\pi}{4} \left( d'^2 \delta \right) x 3 \]

strength of one rivet in double shear.
Bearing or crushing of rivet.

Strength of joint per pitch length in bearing = \( d' \times t \times \sigma_{br} \times \frac{L}{t} \)

\( d' \) = min thickness of plate

Strength of joint per pitch length in bearing

\( \sigma_{br} \) = Bearing strength of plate

\( d' \times t \times \sigma_{br} \times \frac{L}{t} \times B \)

\( B \) = Area of hole

\( t \) = Combined thickness of two cover plate or thickness of main plate whichever is min

Bearing failure of plate

Bearing failure of rivet
Bearing failure of plate

Strength of joint is the minimum of shearing, bearing and leaing strength of joint.

Efficiency of Joint

\[ \eta = \frac{\text{Strength of joint (Min of shearing, bearing, tearing)}}{\text{Strength of main plate without deduction for hole}} \]

Note: For efficient utilization of material, rivets and plates should fail simultaneously.

IS Code Recommendation.

1) Permissible stress in rivets

<table>
<thead>
<tr>
<th>Type</th>
<th>Axial Tension</th>
<th>Shearing</th>
<th>Bending</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shop rivet</td>
<td>100 MPa</td>
<td>100 MPa</td>
<td>300 MPa</td>
</tr>
<tr>
<td>Power-driven</td>
<td>80 MPa</td>
<td>80 MPa</td>
<td>250 MPa</td>
</tr>
<tr>
<td>Hand-driven</td>
<td>80 MPa</td>
<td>80 MPa</td>
<td>250 MPa</td>
</tr>
</tbody>
</table>

For field riveting, the above permissible values are reduced by 10%
Permissible stress values are increased by 25% in case of wind or earthquake loading.

Note: Wind and/or EQ forces act for very short duration for that short duration member force will increase and hence the no. of rivet requirement will increase significantly if permissible stress are kept constant but over there is a margin between permissible stress and yield stress and if that margin is utilized for that short duration no. of rivet requirement will not increase significantly hence economy will result thus for that short duration FOS is reduced and hence permissible stress is increased.

Min Pitch:

Min pitch = 2.5 times the nominal dia of rivet

Max pitch:

a) Dis b/w the centres of two consecutive rivets in the dia of stress should not exceed 16t or 250 mm whichever is less in tension member where t is the thickness of thinner flake.

b) In comp. it is 12t or 200 mm whichever is less
for tacking shits & max pitch is 325 or 300 mm which ever is less.

Tacking rivets are used to ensure that the two members being joined along the length behave as a single unit.

Packing.

If the difference of thickness of two plates being joined ($t_1 - t_2$) is greater than 6 mm, then additional rivets shall be provided on packing extension.

No. of additional rivets will be 2.5% of actual no. of rivet obtained from normal calculation per 2 mm thickness of packing.

Arrangement of rivets.
Force in the main plate before 1-1 is less than that before 1-1 because rivets at section 1-1 absorb some force and transmit it to the cover plate. The same thing holds for 2-2 and 3-3 therefore main plate required max. net area at section 1-1 than at other sections. Thus it is desirable to have minimum no. of rivets at section 1-1 providing max. N EA. No. of rivets can be increased in inner radii that is 2-2 & 3-3. Thus diamond rivetting is better as compared to chain rivetting.

Tearing strength of Diamond

1-1: \((B-d') t \sigma_{at} + \tau = P\)
2-2: \((B-2d') t \sigma_{at} + R_v = P\)
3-3: \((B-3d') t \sigma_{at} + 3R_v = P\)

Tearing strength of Chain

1-1: \((8-2d) t \sigma_{at} = P\)
2-2: \((8-2d) t \sigma_{at} + 2R_v = P\)
3-3: \((8-3d) t \sigma_{at} + 4R_v = P\)
\( R_v - d' \times t \sigma_{at} \leq 0 \)

Section 2-2 will fail in tearing

2:2 more critical as compared to 1-1 (2)

At

For chain riveting, most critical section is section 1-1. If plate does not fail at section 1-1, it will never fail at section 3-3.

For cover plate, section 3-3 is most critical.

**Problem:** A member of a truss consists of 2 angles 6 mm thick placed back to back. It carries a direct load of 100 kN and is connected to a gusset plate 8 mm thick. Determine the no. of power driven field rivet of 18 mm dia required for the joint:

\[
\begin{align*}
\text{Strength of rivet in shearing} & = \frac{2 \times d'^{12} \times 90}{4} \\
& = 2 \times 19.5^2 \times 90 = 5375.7 \text{ kN}
\end{align*}
\]

in bearing

\[
\frac{19.5 \times 8 \times 8 \times 300 \times 9}{42.12} = 422118 \text{ kN}
\]

Rivet value \( R_v \) = 5375.7 kN

\[
\text{No. of rivets} = \frac{100}{42.12} = 3.81 \text{ rivets}
\]

\[
\frac{5375.7}{42.12} = 2.57 \text{ rivets}
\]

\[
3 \text{ rivets}
\]
Note

Shearing strength of rivet $\propto d^2$

Bearing strength of rivet $\propto d$

Hence when shearing governs the design it is better to use small no. of large dia rivets. If however bearing governs the design it is better to use large no. of small dia rivets.

Q. The plate of a boiler 6 mm thick is connected by single rivetted lap joint with 16 mm dia power-driven shop driven at 50 mm pitch. Calculate the efficiency of joint.

\[ d' = 16 + 1.5 = 17.5 \]

\[ f_y = 50 \times 6 \times 150 = 45 \text{ KN} \]

Tearing strength of plate = \( (60 - 17.5) \times 6 \times 150 = 29.25 \text{ KN} \)

Strength of rivet in shearing = \( \frac{\pi d'^2}{4} \times 100 = 24.053 \text{ KN} \)

Bearing = \( 17.5 \times 6 \times 270 = 28.95 \text{ KN} \)

\[ \eta = \frac{24.053 \times 100}{45} = 53.45\% \]
A tie member has to translate a pull of 400 N. Design a butt joint to connect two no. of 12-mm thick plate and also find it's efficiency for power driven shop rivet.

No. of rivets and it's dia
Arrangement of rivets
Width of main and cover plate
Thickness of cover plate
Efficiency

Diameter of rivet = \(6 \cdot 05 \times \sqrt{12} = 20.95 \text{ mm} = \)

Let us adopt 20 mm dia rivet. \(d = 20 + 1.5 = 21.5 \text{ mm} \)

Rivet value = Min of \((\text{Shearing strength, Bearing strength})\)

Shearing strength = \(2 \times \frac{7 \times (2.5)^2}{4} \times 100 = 72.61 \text{ KN} \)

Bearing strength =

Assuming that combined thickness of two cover plate is more than main plate

\[ \text{Bearing strength} = d' \times t \times 300 \]

\[ = 77.41 \text{ KN} \]

Rivet value = 72.61 KN

No. of rivets = \(\frac{900}{72.61} = 5.5 \)

Adopt 6 no. of 20 mm dia rivet

Arrangement of rivets will decide the dimensions of main & cover plate

Let us adopt diamond rivetting pattern
Width of main plate will be provided such that the main plate does not tear off at any section 1, 2, 3.

Tearing strength of main plate at section 1-1

\[(B - d') \times 12 \times 150 > 480 \times 10^3\]
\[B - d' > 222.22\]
\[B > 222.22 + 21.5\]
\[B > 243.72\text{ mm}\]

Tearing strength at 2-2

\[(B - 2d') \times 12 \times 150 + k_v > 400 \times 10^3\]
\[B > 224.88\text{ mm}\]

at 3-3

\[(B - 3d') \times 12 \times 150 + 3k_v > 400 \times 10^3\]
\[B > 165.7\text{ mm}\]

Let us adopt width of main plate = 250 mm

Adopting the width of cover plate same that of main plate

Tearing strength of cover plate at section 3-3

\[(250 - 3d') \times 2t o_4 \sigma_t > 900\]
Recmmendation: Min thickness of plate in steel structure should be adopted 6mm when it is not exposed to atm. and 8mm if it is exposed to atm.

Let us adopt \( t = 8\) mm

Arrangement — Diamond

No of rivet — 6

Width of main plate — 250mm

Width of cover — 250mm

Thickness of cover plate — 8mm

\[
C = 2 \times 21.5 = 43
\]

\[
P_{mn} = 2.5 \times 20 = 50
\]

\[
4e + 4h + 5\text{mm} = 4 \times 43 + 4 \times 50 + 5 = 377\text{mm}
\]

Efficiency = Min of Shearing bearing, tearing st of joint

\[
\text{Strength of main plate without deduction for hole}
\]

\[
\text{Min} \left\{ \begin{align*}
\text{Shearing st. of joint} & = 6R_v = 6 \times 72.61 \\
\text{Bearing st. of joint} & = 435.66 \text{KN}
\end{align*} \right.
\]

Tearing strength of main plate = \( 250 - 21.5 \times 12 \times 150 = 411.3 \text{KN} \)

Cover plate = \( 250 - 3 \times 21.5 \times 16 \times 150 = 445.2 \text{KN} \)
hence strength of joint = 411.3 kN

\[ \eta = \frac{411.3 \times 10^3 \times 100}{250 \times 12 \times 150} = 91.4\% \]

8. Design a rivetted splice for a tie of a steel bridge 20 cm wide 20 mm thick carrying an axial load of 5000 kN.

Use 12 mm thick cover plate and 22 mm dia rivet.

\[ \delta_{al} = 150 \text{ MPa}, \quad \delta_{c} = \text{permissible shear in shear} = 100 \text{ MPa} \]

\[ \delta_{br} = 300 \text{ MPa} \]

Sol. \( d' = 22 + 7.5 = 29.5 \text{ cm} \)

Shearing

Rivet value = \( \min \left( \frac{2 \times \eta \cdot d'^2}{100}, \frac{d' \times 20 \times 300}{100} \right) \)

= \( 2 \times 93.37 \text{ kN}, \quad 141 \text{ kN} \)

Rivet va = \( \frac{86.74 \text{ kN}}{5.50} = 5.76 \)

No. of rivets = \( \frac{5.60}{86.74} = 5.76 \)

Adopt 6 rivets of 22 mm dia.

Arrangement

\[
\begin{align*}
\text{tensile strength of section 1-1} &= (B - d') \times 20 \times 150 \\
&= 529.5 \text{ kN} > 500 \text{ safe} \\
\text{tensile strength of section 2-2} &= (B - 2d) \times 20 \times 150 + 86.74
\end{align*}
\]
3 - 3 = (8 - 3d') x 20 x 150 + 3 x 8.674
= 648.72 kN

For cover plate (Adopting width of cover plate = width of main plate)
= (200 - 3d') x 24 x 150
= 466.2 kN ≤ 500 kN
So cover plate is not safe in tension.

After adopting diamond arrangement connection will

See 3 - 3

\[
\begin{align*}
&55 \\
\frac{e}{d} = 2.5d = 55 \\
2x23.5 = 47
\end{align*}
\]

So take

\[
\begin{align*}
&45 \\
&55 \\
&55 \\
&45
\end{align*}
\]

\[
e = 1.6 \text{ to } 1.7 \text{ time } d'
\]

So 3 rivets can be accommodated

Hence we can apply 3 rivets at secn 3 - 3 b/c cover plate is not safe

Let us adopt chain rivetting

Check tearing strength of main plate at secn 1 - 1

\[
(8 - 2d') x 20 x 150 = 459 kN \leq 500
\]

Not safe
Hence let us adopt
(i) or (ii)

(1)

or

(11)
determine the no. and pattern of 20 mm dia rivet used for
connection

permissible stress in shearing in rivet: \( 102.5 \, \text{N/mm}^2 \)

\[
\sigma_{br} = 2.36 \, \text{N/mm}^2
\]

for connection of angle I & II rivet is in single shear but
for connect of angle III with gusset rivet is in double shear.
Hence rivet value for I, II: \( \text{min} (\text{shearing strength, Benningst}) \)

\[
= \min \left( \frac{x \cdot 21.5^2}{4} \times 102.5, \ 21.5 \times 8 \times 2.26 \right)
\]

\[
= \min \left( \frac{37.213 \, \text{KN}}{1}, \frac{100.59 \, \text{KN}}{1} \right)
\]

Rivet value = 37.2 \, \text{KN}

No. of rivets = \( \frac{50}{37.2} = 2 \) rivet adapt

5

3 rivet adapt
Location of connection for angle

Double shear

First value for angle III = \( \min \left( \text{shearing, bearing} \right) \)

\[ = \min \left( \frac{2 \pi d^2}{4} \times 102, \ d \times t \times 2.36 \right) \]

\[ = \min (74.92, 21.5 \times 12 \times 2.36) \]

\[ = 60.89 \text{ kN} \]

Net force needs to be transferred to the gusset

\[ = 412 \times 300 = 112 \text{ kN} \]

No. of rivet = \( \frac{112}{60.89} = \text{Adopt 2 rivet} \)

For thin shells

For thin shells the connection will be done per pitch

Length, force in one pitch length will be stress \( f \times \text{pitch length} \times \text{thickness} \)

Stress will be for cylindrical shell

1) \( \text{hoop stress } = \frac{pd}{2t} \)

2) \( \text{long stress } = \frac{pd}{4t} \)

The force for which connection will be designed

\[ f = \text{stress } \times 5 \times \text{thickness} \]

...
To start with efficiency will be assumed and finally it will be shown that actual efficiency is more than the assumed value.

Note: If the rivet is subjected to combined shear and tension

\[
\frac{\text{(Shear stress)}_{\text{calc.}}}{\text{(Permissible shear stress) in rivet}} + \frac{\text{Tensile stress calc.}}{\text{Per.}} \leq 1.4
\]

Eccentric Connection

\[F_{di} = \text{force on the } i\text{th rivet due to dead loading} = \frac{P \cdot A_i}{n \sum A_i}\]
Under the assumption that direct shear stress is equally shared.

Direction of this force will be parallel to the line of action of force.

We know that torsional shear stress \( \tau_i = \frac{T_i}{J} \)

Torsional shear force \( T_i = \frac{I_i\tau_i}{J} \)

\( F_{ti} = \frac{(pe)A_i\tau_i}{J} \)

For discrete area \( J = \sum A_i\tau_i^2 \)

\( F_{ti} = \frac{(pe)\sum A_i\tau_i^2}{J} \)

\((pe) \rightarrow \text{Torsional Moment}\)

\( A_i \rightarrow \text{Area of } i^{th} \text{ rivet}\)

\( r_i \rightarrow \text{Distance of } i^{th} \text{ rivet from the C.G. of rivet group}\)

The direction of this S.F. is \( 1 \) to line joining the C.G. and the rivet under consideration and it will be in the same sense as that of the torsional moment.

Resultant force on \( i^{th} \) rivet

\( F_i = \sqrt{F_{ti}^2 + F_{ti}^2 + 2 F_{ti} F_{ti}\cos\theta} \)

For safety of connection resultant force in all rivets should be less than their rivet value.

Note: To design the connection no. of rivets are chosen as follows.
If all the rivets are of same dia then most critical rivit is the one which is farthest from the C.G. If there are more than one rivets equally distant from C.G. then the rivet in which angle b/w $F_0 + F_t$ is the most critical rivet.

If dia of all the rivets are same

\[ F_{di} = \frac{F_0}{n} \]

\[ F_{F_t} = \frac{(PE)Y_1}{\sum d_i} \]

1, 2, 3 & 4 have same value of $F_0$ and $F_t$ but angle $b/w F_0 + F_t$ is smaller in $\theta < \theta$ hence they are the most critical rivet.

1 & 2 are most critical rivet
A bracket connected to the flange of a column through a group of rivets to support a load of 90 kN as shown in figure below. Thickness of bracket plate is 10 mm and that of flange of column is 8 mm. Determine the max force developed in the rivet and design a suitable riveted joint.

Allowable stress in single shear = 100 MPa
in double shear = 200 MPa
σbr = 180 MPa

Assume size of rivets to be 14.2 mm. Hence, rivet 1 is the most critical.

Sol. \( f_{D_i} = \frac{90}{9} = 10 \text{ kN} \)

Hence let us calculate force in rivet 1 which is max
\( f_{D_1} = \frac{90}{9} = 10 \text{ kN} \)

\[
F_T_1 = \frac{PE \cdot X_1}{\Sigma b_i^2} = \frac{90 \times 150 \times 100}{(4 \times 100^2 + 2 \times 80^2 + 2 \times 60^2)} \approx 22.5 \text{ kN}
\]

\[
\cos \theta = \frac{3}{5}
\]

\[
F_{r_1} = \sqrt{F_{r_1}^2 + F_{d_1}^2 + 2 F_{r_1} F_{d_1} \cos \theta} = 29.6 \text{ kN}
\]

\[
\frac{F}{A} \left( d + 1.5 \right)^2 \times 100 > 29.6 \times 10^{-3}
\]

\[
d \geq 19.4 + 17.9 = 37.3 \text{ mm}
\]
Bearing strength of rivet: \( d + 1.5 \times 8 \times 180 > 29.5 \times 10^3 \)
\[ d > 19.05 \text{ mm} \]

hence adopt rivet dia. = 20 mm

\[ \text{(35)} \]

Rivets 1-6 are of dia. \( d' \)

Rivet 7 is of dia. \( 1.2d' \)

Find force in 7th rivet

\[ \cos \theta = \frac{2}{5}, \quad \sin \theta = \frac{1}{5} \]

\[ \tan \phi = \frac{1}{x+y} \]

\[ \phi = \arctan \left( \frac{1}{x+y} \right) = 60.255^\circ \]

\[ P = 10 \times 10^3 = 100 \text{ KN} \]

\[ \tan \alpha = \frac{1}{d} \]

\[ \alpha = 3.761^\circ \]

\[ a = 26.565 \]

\[ \phi = \arctan \left( \frac{1}{x+y} \right) = 60.255^\circ \]

\[ \bar{X} = \sum A_1 X_1 + A_2 X_2 + A_3 X_3 + \ldots + A_n X_n \]

\[ \bar{Y} = \frac{0 + 0 + 3kA \times 80 + 1.44A \times 160}{7.44A} \]

\[ = 63.226 \text{ mm} \]

\[ \bar{Y} = \frac{0 + 0 + 2A \times 80 + 2A \times 160}{7.44A} \]

\[ = 64.516 \text{ mm} \]

\[ F_{D7} = \frac{100 \times 1.44A}{7.44A} = 19.355 \text{ KN} \]

\( (p_c) = \frac{100 \times 0.0 \times (80 + 80 - 63.226) + 100}{\sqrt{5}} \)

\[ q = 115.91 \text{ mm} \]

\[ pt = \frac{17540 \text{ KNmm}}{57.70 \text{ KNmm}} \]

\[ pe = 57.70 \text{ KNmm} = 5.77 \text{ KNm} \]
\[ \begin{align*}
\gamma_1 &= 114.51 \text{ mm} \\
\gamma_2 &= 96.94 \text{ mm} \\
\gamma_3 &= 68.08 \text{ mm} \\
\gamma_4 &= 22.83 \text{ mm} \\
\gamma_5 &= 90.33 \text{ mm} \\
\gamma_6 &= 26.67 \text{ mm} \\
\gamma_7 &= 110.31 \text{ mm} \\

\frac{(Pe) \ A \gamma_7}{\sum A \gamma_i^2} &= \frac{(Pe) \ A \left(\gamma_1 + \ldots + \gamma_6 \right) + 1.44 \ A \times \gamma_7}{A \left(\gamma_1^2 + \ldots + \gamma_6^2 \right) + 1.44 \ A \times \gamma_7^2} \\

F_{d_1} &= 16.28 \text{ KN} \\

F_{d_2} &= \sqrt{F_{d_1}^2 + F_{c_2}^2 + 2 \ F_{d_1} \ F_{c_2} \ \cos(60.25^\circ)} \\
&= 30.86 \text{ KN} \\
\end{align*} \]
Welded Connection

Fillet weld

Butt welding

Butt weld

Side fillet weld

End fillet weld

Slot weld

Types of weld

1) Butt weld or groove & weld
2) Fillet weld
3) Plug weld
4) Slot weld

Welding is done by electric arc welding
Butt weld

The various types of butt welds are square butt weld adopted up to $t \leq 8\text{mm}$.

Other types of weld are

- single U - butt weld
  Normally up to 40\text{mm}

- double U - butt weld
  $> 40\text{mm}$

Other types of butt weld are single V, single J, single double V, etc.

Butt weld is normally done in the workshop. Partial penetration and complete penetration when weld metal does not penetrate to the complete depth of the plate.

Weld metal penetrates completely to the full length is known as complete penetration.

- \[ P_{\text{max}} = B \times t \times \frac{t}{2} \] \( \text{complete penetration} \)

- \[ P_{\text{max}} = B \times t' \times \frac{\Delta t}{2} \] \( \text{partial or incomplete penetration} \)

If the thickness of penetration is not given, $t'$ can be taken as $\frac{5}{8}$ times of $t\text{min}$, but it is not IS code recommended.
Fillet weld can be done in the field as lesser precision is required in it. It is cheaper as compared to butt weld. A fillet weld is almost always assumed to fail in shear.

Permissible Stresses

Tension and compression on section through the throat of butt weld

\[ \sigma_{tkm} = 0.6 f_y \]

Shear of section through the throat of butt or fillet weld

\[ = 0.4 f_y \] (Normally it is 100 to 110 N/mm²)

Then values are for welding done in workshop they are reduced to 80%. In case of field welding, in case of wind or EQ the permissible values are increased by 25%.

Design of butt weld

- Butt weld or groove weld is more suitable for alternating stresses provided that full penetration of weld is achieved.
- Reinforcement is good for static load condition. It is not suitable for alternating stress condition due to huge stress conc. at this location leading to onset of crack.

However even in this case reinforcement of 0.6 to 0.75t may be kept.

The strength of weld in tension and compression is given by

\[ \sigma_{\text{eff}} = \frac{L_{\text{eff}} \times A_{\text{eff}}}{L_{\text{actual}} \times \text{thickness of thinner part being joined (mean of complete penetration)}} \times 0.6 f_y \]
For incomplete penetration \( t_{eff} \) is \( t_{min} \) thickness of weld metal common to the parts being joined excluding reinforcement.

When two plates are of different thickness

\[
\begin{align*}
\frac{t_1}{t_2} & > \frac{t_2}{4} \\
& \text{or} \quad 3 \text{mm whichever is greater}
\end{align*}
\]

then we provide tapering in the thickness. The tapering should not be greater than 1 in 5.

If the thickness of weld common by \( t_{eff} \) the plates is not given in incomplete penetration weld \( t_{eff} \) can be taken as \( \frac{5}{8} \) times \( t_{min} \).

**Design of Fillet weld.**

**Size of the weld.**

Size of the weld is decided on the basis of largest right angle that can be inscribed in the weld for normal fillet weld size is taken as the minimum weld leg.

\[
\begin{align*}
\text{Size of weld} = s_1 & \quad \text{Size of weld} = s
\end{align*}
\]
For right angle fillet weld minimum throat size will be 
\( \frac{s_1 s_2}{s_1^2 + s_2^2} \). However, as per IS code it is taken as \( \frac{s}{\sqrt{2}} \)

Minimum size of weld

Thickness of thicker part

<table>
<thead>
<tr>
<th>Thickness of Thinner Plate Being Joined</th>
<th>Min Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 10</td>
<td>3 mm</td>
</tr>
<tr>
<td>10 - 20</td>
<td>5 mm</td>
</tr>
<tr>
<td>20 - 32</td>
<td>6 mm</td>
</tr>
<tr>
<td>32 - 50</td>
<td>8 mm first min, 10 mm final</td>
</tr>
</tbody>
</table>

For thickness > 50 mm

Special technique like pre-heating is required. Pre-heating is done so that heat does not extract heat from the weld.

If the weld size is small, it will cool faster due to heat extracted by the thicker plate. Hence, rapid cooling leads to brittleness in the weld; hence there should be min size of the weld controlled by the thickness of the thicker plate.

Max size of weld

For square edge, max size = thickness of thinner plate being joined - 1.5 mm
for round edge

\[
\text{Max size of weld} = \frac{3}{4} \text{nominal thickness of round edge}
\]

\[ Y_2 \]

Note: From economy point of view it is better to use smaller size of longer length of weld. And longer.

\[ \text{quantity of steel} = \frac{1}{2} \times w \times s \times x \times L \]

\[ \propto L \times s^2 \]

\[ \text{Force} \propto L \times s \]

**Throat of fillet weld:**

Effective throat thickness is given by:

\[ = k \times \text{size of weld} \]

\[
\begin{array}{cccccc}
\theta & 60-90 & 91-100 & 101-106 & 107-113 & 114-120 \\
\kappa & 0.8 & 0.7 & 0.65 & 0.6 & 0.55 & 0.5
\end{array}
\]

Area resisting shear in fillet weld is \( A \times t \)

where \( L \) is the length of weld \( t \) is the effective throat thickness hence force resisted by fillet weld

\[ A \times t \times 0.4 \times f_y = \text{permmissible shear in shear} \]

where \( f_y \) is throat thickness,
Effective length of fillet weld.

\[ \text{Eff. length} = \text{Leverage} - 2s \]

- In drawings the length shown is the effective length
- Minimum length of weld = 9s (for weld to be 100% effective in load transfer)

Side fillet weld

\[ l \neq b \]

As width increases non-uniformity in stress in the plate increases to make it more uniform plug or slot weld can be used

Intermittent fillet weld

When the length of smallest size weld required to transmit the load is less than the continuous length of joint, intermittent fillet weld is provided.
\[ P = \ell \times t \times 0.4 f_y \]

- \( \ell \) choose min
- \( f_y \)

Find \( \ell \) if \( \ell < (20 + 5) \)

We provide intermittent fillet weld.

The strength of end fillet weld is generally larger than that of side fillet weld; however, for strength calculation we take the strength of the two welds to be same.

**Plug and slot weld.**

![Diagram of plug and slot weld]

Effective connection b/w top & bottom plate

Plug weld is capable of resisting shear however as per codal recommendation it is not considered to be contributing to the strength.

Plug weld is also provided to make stress variation in top plate more uniform.
slot weld -

gap b/2
slots should be
2t

limited overlap

If the overlap is limited and even by providing the largest size of weld all along the available length, if P force can not be resisted than we provide additional length of connection by making slot - The additional length of connection in the above figure will be 4t

width of slot ≥ 3t or 25 mm whichever is greater

R ≥ 1.5t or 12 mm

Note: min overlap in lap joint ≥ 4t or 40 mm whichever is more

fillet weld for truss member

Truss joint should be such that it should be moment free

\[ \frac{P_1}{P_2} = \frac{h_2}{h_1} \]

\[ \frac{h_1 \times E \times 0.4f_y}{h_2 \times E \times 0.4f_y} = \frac{h_1}{h_1} \]

\[ \frac{I_1}{I_2} = \frac{h_2}{h_2} \]
\[(h_1 h_2) \times t \times 0.4 \times f_y = P \]  \hspace{1cm} \text{(1)}

From \ref{eq:1} \& \ref{eq:11} \( h_1 + h_2 \) can be found out.

Case-II

\[ P = (h_1 + h_2 + l_3) \times t \times 0.4 \times f_y \]  \hspace{1cm} \text{(1)}

\( l_3 \) Throat thickness

\[ p_1 h_1 - p_2 h_2 - p_3 \left( \frac{h_1 + h_2}{2} - h_1 \right) = 0 \]

\[ p_1 = l_1 \times t \times 0.4 \times f_y \]
\[ p_2 = l_2 \times t \times 0.4 \times f_y \]
\[ p_3 = l_3 \times t \times 0.4 \times f_y \]

An ISLC 300 with area 4211 mm², \( T = 11.6 \) mm (flange thickness), \( t = 6.7 \) mm (web thickness) is used to transmit a pull of 600 kn. The channel section is connected to a gunit plate 10 mm thick. Design a fillet weld if overlap is limited to 350 mm and welding can be provided only on 3 sides. Use slot weld if required.
Max size of fillet weld on side 1 = Thickness of thinner part -1.5mm (fp edge) = 10 - 1.5 = 8.5mm
Min size = 5mm

Max size of fillet on side 2 = 6.7 - 1.5 = 5.2 mm
Min = 3 mm

If uniform size of weld is to be provided we will adopt 5 mm size.

Max force that can be resisted by providing weld on 3 side and adopting the max possible size:

\[ P_{\text{max}} = 350 \times 2 \times t_1 \times f_y + 300 \times t_2 \times 0.9 \times f_y \]

Possible:
\[ t_1 = 0.75t_1 = 0.7 \times 8.5 \]
\[ t_2 = 0.7 \times 5.2 \]

\[ P_{\text{max}} = 525.7 \text{ KN} \leq 600 \text{ KN} \]

Hence we need to provide slots by adopting a uniform size of weld = 5 mm
Length of weld required = 1

\[ 600 \times 10^3 = \text{length} \times 0.7 \times 5 \times 0.4 \times 250 \]

\[ \text{length} = 17.1428 \text{ mm} \]

Length of slot weld required = 17.1428 - 1000 = 7.1428 mm

Proving two slots

\[ \text{length of slots required} = \frac{7.1428}{2} = 3.5714 \text{ mm} \]

Adopt 180 mm

Width of slot = 3t or 25 mm whichever is more

\[ = 3 \times 6.7 \text{ or } 25 \]

\[ = 20.1 \text{ or } 25 \]

Take b = 25 mm

* In an industrial shed and edge support consisting of two angles 110 x 110 mm is to be connected to 16 mm gusset plate for a tensile load of 650 kN.

Design the moment free welded connection.

\[ A = \frac{650}{200} = 3.25 \text{ mm}^2 \]

\[ \text{Width of slot}\]

\[ = 3 \times 6.7 \text{ or } 25 \]

\[ = 20.1 \text{ or } 25 \]

Take b = 25 mm

**NEA**

Force twisted by angle

\[ (220 - 1.11 \times 150) \geq 3 \times 2 \times 10^3 \]

\[ 220 - t^2 \geq 3166 \]

\[ t > 10.73 \text{ mm} \]

Take t = 12 mm
The best arrangement for transfer of loading to the gusset plate will be when two angles are connected on off side of gusset plate.

Hence each angle will carry 325 kN load.

Hence we have to decide length of weld, size of weld and arrangement of weld.

As the thickness of angle is not given we will proceed on the basis of min size of weld decided on the basis of thickness of gusset plate.

Min. size = 5 mm adopted

Length of weld req. = \( \frac{325000}{5 \times 0.7 \times 10^-3} \) = 928.57 mm

Note:
1. Max. thickness available for angle
2. Max. size available for angle
   a) Equal angle
   b) Unequal angle
3. Max. depth of I section available
(iv) Gauge location for I-section
(v) Min thickness of plate
(vi) Max thickness of plate

\[
\text{Strength of weld} = 0.7 \times 5 \times 100 = 350 \text{ N/mm}^2
\]

\[
l_1 + l_2 = 928.57 - 110 = 818.57 \text{ mm}
\]

\[
P_1 = l_1 \times 7.5 \times 0.4 f_y
\]

\[
P_2 = l_2 \times 0.75 \times 0.4 f_y
\]

\[
F_2 = l_2 \times 0.75 \times 0.4 f_y
= 110 \times 7.5 \times 100 = 385 \text{ KN}
\]

For moment free connection

\[
P_1 h_1 = P_2 h_2 - P_3 \left( \frac{h_1 + h_2}{2} - h_1 \right) + \phi = 0
\]

\[
P_1 + P_2 = 325 - 38.5 = 286.5
\]

\[\phi = 0 \text{ about centroidal axis}
350 l_1 \times 30.9 = 350 l_2 \times 79.1
350 l_1 \times 30.9 = 38.5 \times 10 \times 24.1 = 0
\]

\[
\frac{l_1 + l_2}{2} = \frac{30.9 l_1 - 79.1 l_2}{2} = 26.51
\]

\[
l_1 + l_2 = 818.57
\]

\[
l_1 = 612.72 = 615 \text{ mm}
\]

\[
l_2 = 205.84 = 210 \text{ mm}
\]

Weld Notations

\[\text{Fillet weld}\]

\[\text{Square butt weld}\]

\[\text{Single V}\]

\[\text{Double V}\]
Field butt weld of throat thickness = 6mm and weld is double bevel.

5 mm shop fillet weld of length 125 mm

5 mm weld I = 150 mm

8 mm weld length 200 mm

5 mm weld shop weld.
0. A plate of 150 mm width and 20 mm thickness is welded to another plate by fillet weld as shown in figure. The size of weld is 12 mm throughout. Compute the average shear stress produced in the weld for the full strength of plate if the allowable stress is 150 N/mm² in axial tension.

Note: As in this case the location of slot is not given, the strength of plate cannot be decided hence we will work on full strength of plate at see 1-1.
Full strength of plate $= 150 \times 10 \times 150 = 450 \text{ KN}$

total length of weld $= 2 \times (150 + 70.7) + 50 + 7 \times 60 = 680 \text{ mm}$

Note: Strength of plug weld is not considered in design but we can consider the strength of root weld.

$$S = 0.75 \times f_s = 450 \times 10^3$$

$$f_s = \frac{450 \times 10^3}{680 \times 0.7 \times 12}$$

A circular shaft of diameter 150 mm is welded to a rigid plate by an external all round fillet weld of size 10 mm if a torque of 10 KNm is applied to the shaft find the maximum stress in the weld.

![Diagram of circular shaft and plate with weld](image)

10 mm size fillet weld.
Shear stress = \( \frac{T r}{J} \), where \( T \) is the shear force, \( r \) is the distance from the centroid of the section to the point of application of the shear force, and \( J \) is the polar moment of inertia of the section.

**Exact Calc. of \( J \)**

\[
\int_{0}^{r_0} 2\pi x \, dx = r^2
\]

\( m = 75 \text{ mm} \)

\( r = r_0 + \frac{x}{\sqrt{r^2 - x^2}} \cdot 7 \times 10^{-3} \)

\[
J = \int_{0}^{r_0} 2\pi \left( 80 + \frac{x}{\sqrt{r^2 - x^2}} \right)^3 \, dx = 20.474 \times 10^6 \text{ mm}^4
\]

\[
\frac{1}{J_{max}} = r_0 + 0.7 \times 10^{-3} = 79.949 \text{ mm}
\]

Max. shear stress = \( \frac{10 \times 10^6 \times 79.949}{20.474 \times 10^6} \) = 39.05 N/mm².
**Approximate Calc.**

\[ J = \frac{\pi}{32} \left[ (150 + 2 \times 4.9)^4 - 150^4 \right] \]
\[ = 14 \cdot 3177 \times 10^6 \text{ mm}^4 \]
\[ T = \frac{TY}{J} = \frac{10 \times 10^6 \times 79.95}{14 \cdot 3177 \times 10^6} = 55.83 \text{ kN/mm}^2 \]
\[ J = \frac{\pi}{32} \left[ (150 + 14)^4 - 150^4 \right] = 21 \cdot 318 \times 10^6 \text{ mm}^4 \]

Approximate calc. is quite different from the actual value; a logical calculation will be when the projected width is taken as the actual width of resisting section and it is assumed that the stress is uniform over the cross section of throat.

\[ T_0 = \frac{10 \times 10^6}{2A \left( \frac{75}{\sqrt{2}} \right)^2} = 35.57 \text{ kN/mm}^2 \]
Eccentric connection using weld.

Rivet 2 is subjected to shear and axial tension.

Tension in a rivet will be corresponding to a value in Bending Stress diagram.

In rivet 2 check for safety is done as follows:

\[
\frac{\text{Calculated axial force}}{\text{Permissible axial force}} + \frac{\text{Calculated shear force}}{\text{Permissible shear force}} \leq 1.9
\]
Eccentric connection using welding.

**Case 1**

Weld subjected to direct shear + Torqional shear -

\[ f_r = \sqrt{f_d + f_t + \frac{1}{3} f_t \cos \theta} \leq \sigma_{cr} \]

Shear stress in weld

Size of weld = \( s \)

As welding is continuous area we will work in terms of shear

\[ f_d = \frac{P}{(2a+d) t} \]

Throat thickness

\[ f_d = \frac{P}{(2a+d)(a-7.5)} \]

direct shear on throat of weld

Direction of \( f_d \) is taken along the line of action of force \( P \)

\[ f_t = \frac{T s}{J} \]

Weld will be considered as a line area as shown above. Thickness of the line will be assumed as throat thickness

\[ I_p = I_{xx} + I_{yy} \]
\[ I_{xx} = \frac{td^3}{12} + (axt)(\frac{d}{2})^2 x^2 \]

\[ I_{yy} = (txd) x^2 + \left[ \frac{ka^3}{12} + at \left( \frac{a}{2} - \frac{b}{2} \right)^2 \right]^2 \]

\( r \) is distance of point under consideration from c.g. of weld
\( e \) is \( \perp \) distance of line of action of \( P \) from the c.g. of weld.
Dir. of \( ft \) will be \( \perp \) to the line joining point under consideration to the c.g. of weld and it will be in the same sense as that of the torsional moment.

Resultant shear stress

\[ f_r = \sqrt{f_t^2 + f_t^2 + 2f_t f_t \cos \theta} \]

for safety

**Case II**

Fillet weld subjected to direct load and bending.

Diagram of column and beam with weld details.
Direct shear stress

\[ f_v = \frac{P}{2dt} \]

\[ = \frac{P}{2d(0.75)} \]

\[ \theta \]

At any point along the length of weld the bending stress will be resisted by the fillet weld through shearing action on its throat hence horizontal shear stress in fillet weld \( f_h \)

\[ f_h = \left( \frac{M_y}{I_{yy}} \right) \]

\[ I_{yy} = 2 \left( \frac{td^3}{12} \right) = 2 \left( 0.75d \right) d^3 \]

\[ f_s = \sqrt{f_h^2 + f_v^2} \]

for safety of connection \( f_s < \) permissible shear stress in weld

**Case III**

Butt Weld

Butt weld is just an extension of plate whatever is the stress in the plate, it will be resisted by butt weld

\[ f_s = \text{shear stress in butt weld} = \frac{P}{td} \] (vertical)
$f_b = \text{bending stress in butt weld} = \frac{PE_{yew}}{t_{ey}} = \frac{PE}{t_{ey}^2} - \frac{t_{ey}}{12}$

Checking safety using interaction formula

\[
\left( \frac{f_s}{\text{perm. stress in butt weld}} \right)^2 + \left( \frac{f_b}{\text{perm. bending stress in butt weld}} \right)^2 \leq 1.0
\]

Equivalency Method -

\[
\sqrt{f_b^2 + 3f_s^2} \leq 0.9f_y
\]

based on max distortion energy theorem

Permissible bending stress for flanged section = 165 N/mm² = 0.67f_y

Permissible for solid sections: (□, ○, △) permmissible

Bending stress is 185 N/mm²
A welded bracket connects the a plate to a column flange as shown in the fig. below determine the size of weld if the allowable stress in the weld a 110 N/mm².

Maximum stress will be either at A or at D

At A
\[ f_d = \frac{100 \times 10^3}{550 \times t} = \frac{181.818}{t} \text{ N/mm}^2 \]

\[ f_t = \frac{T_r}{J} = \frac{T_r}{I_w + I_y} \]

\[ I_{xx} = 2 \left( 150 \times t \times 125^2 \right) + \frac{t \times 250^3}{12} = 5.9876 \times 10^6 \text{ t mm}^4 \]

\[ I_{yy} = 250t \times x^2 + \left[ \frac{150t x^2}{12} + (t \times 150)(75 - 40.91)^2 \right] x^2 \]

\[ = 1.3295 \times 10^6 \text{ t mm}^4 \]
\[
T = 100 \times 10^3 \left( 300 - 40.91 \right) = 25.9 \times 10^6 \text{ N-mm}
\]

\[
T_N = \frac{\left( 150 - 40.91 \right)^2 + 125}{2} = 165.908 \text{ mm}
\]

\[
f_t = \frac{T_N}{J} = \frac{587 \cdot 2.96}{t} \text{ N/mm}^2
\]

\[
\cos \theta = 0.657
\]

\[
f_r^2 = f_d^2 + f_t^2 + 2f_df_t \cos \theta
\]

\[
f_y = \frac{720.009}{t} \text{ N/mm}^2
\]

For safety of connection, \( f_r < \) per. shear stress

\[
\frac{720.009}{t} < 110
\]

\[
t > 6.545 \text{ mm}
\]

\[
s > \frac{6.545}{f} > 9.35 \text{ mm}
\]
The weld is subjected to direct shear \( W \) and bending moment \( BM = W L \).

Thus external shears are resisted by fillet weld through shearing action.

Direct shear stress:
\[
\tau_d = \frac{W}{(100 \times 7.56 + 120 \times 7.35) \times 2} \frac{N}{mm^2}
\]
\[
= \frac{W}{134.4} \frac{N}{mm^2}
\]

Bending shear stress:
\[
\tau_b = \frac{M_y}{I} = \frac{W(200) \times 1.60}{I}
\]

\[
I = 2 \times \left(100 \times 7.56 \right) 100^2 + \frac{7 \times 3 \times 120^3 \times 2}{12}
\]
\[
= 9.0048 \times 10^6 \text{ mm}^4
\]

\[
f_s = \frac{W_l}{450.24 \times 10^3} \frac{N}{mm^2}
\]

\[
f_t = \sqrt{f_s^2 + f_d^2} = \sqrt{\left(\frac{W_l}{134.4}\right)^2 + \left(\frac{W_l}{450.24}\right)^2}
\]
\[
= 2.342 \times 10^{-3} W_l \leq 100
\]

Note: accessories exist lateral loads.
**Tension Member**

Comparison between tension and compression member.

<table>
<thead>
<tr>
<th>Tension Member</th>
<th>Compression Member</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) In tension net area is effective</td>
<td>1) Gross area is effective</td>
</tr>
<tr>
<td>2) There is no stability problem</td>
<td>2) There is stability problem</td>
</tr>
<tr>
<td>3) Permissible stress ( \sigma_{at} = 0.6 \text{ fyd} )</td>
<td>3) ( \sigma_{at} &lt; 0.6 \text{ fyd} )</td>
</tr>
<tr>
<td>4) Design is straightforward</td>
<td>4) Design is based on trial and error</td>
</tr>
</tbody>
</table>

**Note:** Theoretically there is no limitation on slenderness ratio of tension member since stability is of little concern.

However, the member may be subjected to compressive load during transportation and erection. Hence, in order to provide adequate rigidity to prevent undesirable lateral buckling and excessive vibration, IS code limits the slenderness ratio values as follows:

\[
\text{Slenderness ratio } \lambda = \frac{l_{eff}}{r_{min}}
\]

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Description</th>
<th>Max. S.F.</th>
<th>Max. S.R.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>A tension member in which reversal occurs due to load other than wind or EA</td>
<td>180</td>
<td></td>
</tr>
<tr>
<td>2)</td>
<td>A member normally acting as a tie in a roof truss or bracing system but subjected to compression due to wind or EA</td>
<td></td>
<td>350</td>
</tr>
</tbody>
</table>
Net Sectional Area

For plates -

A plate may have various modes of failure in tension hence all the modes are required to be examined and the mode which gives the min net area will be the most critical mode.

\[
A_{\text{net}} = Bt - nd't + \left( \frac{b_1^2}{4g_1} + \frac{b_2^2}{4g_2} \right) t = B_{\text{eff}} t
\]

No. of rivets along failure line

for each inclined line \( \left( \frac{p^2}{4g_2} \right) \) is added

\[
A_{\text{net}} = Bt - 2d't + \frac{b^2}{4g_1}
\]
The thickness of plate is 10 mm. Find the minimum net area provided by the plate.

\[ \text{Net area} = 360 \times 10 - 2 \times 22 \times 10 = 3160 \text{ mm}^2 \]

\[ \text{Net area}_{ABCD} = 360 \times 10 - 3 \times 22 \times 10 + \left( \frac{50^2 + 50^2}{4 \times 60 + 4 \times 180} \right) \times 10 
= 3078 \text{ mm}^2 \]

\[ \text{Net area}_{ABFC} = 360 \times 10 - 4 \times 22 \times 10 + \left( \frac{50^2 + 50^2}{4 \times 60 + 4 \times 60} \right) \times 10 
= 2928.33 \text{ mm}^2 \]

\[ \text{Net area}_{ARGH} = 360 \times 10 - 3 \times 22 \times 10 + \frac{50^2}{4 \times 60 + 4 \times 60} \times 10 
= 3044.17 \text{ mm}^2 \]

In this case for calculation of force we have to add the direct value of \( c \).

Hence the most critical section set is \( ABFC \).
Net area for angles

\[ A_1 = (b_1 - d' - \frac{t}{2}) t \]

\[ A_2 = (b_2 - \frac{t}{2}) t \]

Forces are transferred in the member through the gusset plate. The location of transfer is at the location of rivets/welds. This force is then distributed in the whole member through shearing. The connected legs lead over the outstanding leg in the distribution through shear stress. Thus there is a shear lag existing between connected and outstanding leg. The stress in the angle at the location of joint can be shown as follows.

As the stress in outstanding leg is less, the force carried by it will be less to account for this net area is taken as \((A_2 + K \times A_1)\), where \(K\) is a reduction factor for outstanding leg:

\[ A_1 - \text{Area of connected leg} \]
\[ \text{outstanding leg} \]
Area: \((l_1 - \frac{t}{2}) \ t\)

Total area: \((l_1 + l_2 - t) \ t\)

\[
\text{Single angle connected with through one leg only}
\]

\[
A_{\text{net}} = A_1 + kA_2
\]

\[
k = \frac{3A_1}{3A_1 + A_2}
\]

\[
A_1 = (l_1 - d - t/2) \ t
\]

\[
A_2 = (l_2 - t/3) \ t
\]

\[
A_{\text{net}} = A_1 + kA_2
\]

\[
= (l_2 - \frac{t}{2}) \ t
\]

\[
A_{\text{net}} = \min \left\{ \left( \frac{l_1 - t}{2} - d' \right) \ t, \left( l_1 - 2d + \frac{P^2}{4g} \right) \ t \right\}
\]
Case II (a) When two angles are connected on the same side of gusset plate (and are tacked)

\[ A_{net} = A_1 + KA_1 \]

\[ K = \frac{5A_1}{5A_1 + A_2} \]

- \( A_1 \) : Area of connected legs
- \( A_2 \) : Area of outstanding legs

(b) When two angles are connected on the same side of gusset plate and not tacked rivetted.

In this case two angles will behave individually hence net area would be twice the area corresponding to single acute connected to a gusset plate.

Case III (a)

Two angles connected on the gusset plate and tacked along the length.

Tacking rivet
\[ A_{el} = (A_{gou} - \text{Area of hole}) \]
\[ = \frac{1}{2}(A_1 + A_2) - \text{area of hole} \]
\[ = \frac{1}{2}(h_1 + h_2 - t) - 2d't \] (7)

3. When there is no tack riveting along the length. The two angles will behave individually and net area would be twice that corresponding to single angle connected to gusset plate.

Note: Shear lag effect is non-existing in this case (case a)

Q. 09.5

An ISA 75x75x10 is connected to gusset plate by 16 mm diameter rivet through both legs. The pitch on each leg is 80 mm, and the rivets on one leg are staggered by 40 mm w.r.t. those other. Find allowable tensile load on the angle.

\[ 45\text{mm} \]
\[ 45\text{mm} \]

\[ 2 \times 75 \times 75 \times 10 \]

\[ 80 \text{mm} \]

\[ 40 \text{mm} \]

\[ 0 \]

\[ 0 \]

\[ 0 \]

\[ 0 \]

\[ 0 \]
As both the legs are connected legs the angle can be considered as a plate

\[ d^2 = 16 + 1.5 = 17.5 \]

\[ \theta = 100 \]

\[ A_{\text{and} 1-1} = (140 - 12.5) \times 10 = 1225 \text{ mm}^2 \]

\[ A_{\text{and} 2-2} = (140 - 2 \times 17.5 + \frac{40^2}{2 \times 80}) \times 10 = 1100 \text{ mm}^2 \]

\[ A_{\text{total}} = 1100 \text{ mm}^2 \]

\[ P_{\text{req}} = 1100 \times 0.6 \text{ ft} \]

\[ = 165 \text{ KN} \]

**Q.** An IISA 150 x 115 rivetted on one side of a gusset plate by two rows of 22 mm dia rivet through 150 mm leg. It is required to carry a tensile force of 320 KN. Find the thickness of the angle required. The diagram is as shown below.
If: \( 150 - 2 \times 5.5 = 138.5 \) 
\( 150 - 2 \times 3.5 = 140.5 \)
\( \frac{4.6^2 \times t}{4 \times 60} = 109.67 \)

\[ A_{\text{net req.}} = \frac{320}{0.6f_y} = \frac{320 \times 10^5 N}{150 N/mm^2} = 2133.33 \text{ mm}^2 \]

\[ A_{\text{net available}} = A_{\text{net req}} + \frac{k}{2} \]
\[ A_2 = (125 - \frac{t}{2}) \times t \]
\[ A_{\text{net}} = 150 - \frac{t}{2} - 2 \times 13.5 + \frac{40}{4 \times 60} \times t \]
\[ = (148.5 - \frac{t}{2}) \times t \]
\[ = 109.67 - \frac{t}{2} \times t \]

The thickness will be assumed in such a way that the available area should be more than the net area required.

Assume \( t = 8 \text{ mm} \)

\[ A_{\text{net}} = 845.336 \text{ mm}^2 \]
\[ A_2 = 88.8 \]
\[ k = 74 \]

\[ A_{\text{net available}} = 1734.08 \text{ mm}^2 < A_{\text{net req. not safe}} \]

Assume \( t = 10 \text{ mm} \Rightarrow \text{ unsafe} \)

\( t = 12 \text{ mm} \Rightarrow \text{ safe} \)
\[
\begin{align*}
E &= \frac{3A_1}{3A_1 + A_2} = \frac{3\left(109.07 - \frac{t}{2}\right)}{3\left(109.07 - \frac{t}{2}\right) + (115 - \frac{t}{2})} \\
\text{NEA} &= A_1 + E \cdot A_2 \\
2135.3 \cdot 33 &= \left(109.07 - \frac{t}{2}\right)t + \frac{387.21 - 1.5t}{442.21 - 2t} \cdot (115 - \frac{t}{2})t \\
t &= 11.59 \text{ mm} \\
\text{take } t &= 12 \text{ mm}
\end{align*}
\]

Lug Angle:

Lug angle is a short length of angle section used at a joint to connect the outstanding leg of main member thereby reducing the length of joint.
The rivet connecting the outstanding leg of main member with the lug angle should start in advance of all other rivets. This is done to ensure that force in the outstanding leg is effectively transferred to the lug angle when angle members are main members.

Lug angle and their connection with the gusset plate are designed for forces greater than equal to 1.2 times the force in outstanding leg of the main member.

Force in the outstanding leg: \[ F_{\text{out}} = \frac{FA_2}{(A_1 + A_2)} \]

where:
- \( A_2 \) = area of outstanding leg
- \( A_1 \) = area of connected leg

Hence, designed force for lug angle and rivet 2 is:

\[ F_{\text{D}_2} = 1.2 \left( \frac{FA_2}{A_1 + A_2} \right) \]

Connection of outstanding leg of main member with the lug angle should be designed for force \( > 1.4 \) times force in outstanding leg.

rivet 1 will be designed for:

\[ F_{\text{D}_1} = 1.4 \left( \frac{FA_2}{A_1 + A_2} \right) \]
When channel sections are main members:

- Lug angles and its connection with the gusset plate should be designed for the force 1.1 times force in outstanding legs.
- Connection of lug angles with the outstanding legs of channel should be designed for a force greater than equal to 1.2 times the force in the outstanding legs.
- Minimum no. of rivets in lug angles should be 2.
- Rivet 3 should be designed for force in connected leg.

\[
E_3 - 95 \quad 0.2 \quad \text{Page 45}
\]

At

The force is not given, the maximum force that the angle can resist will be found out—under the assumption that, at any section main angle is reduced by a rivet hole.

\[
F_{max} = \left( A_{gross} - A_{hole} \right) \times \sigma_{at}
\]

\[
A_{gross} = (75 + 100 - 10) \times 10 = 1650 \text{ mm}^2
\]

\[
A_{hole} = 0.12 \times 21.5 \times 10 = 215 \text{ mm}^2
\]

\[
F_{max} = (1650 - 215) \times 150 = 215.25 \text{ KN}
\]

\[
A_1 = \left( 100 - \frac{t}{2} \right) \times 150 = 950
\]

\[
A_2 = (75 - 5) \times 10 = 700
\]
Sent standing = \( \frac{FA2}{AI + A_L} = \frac{215.25 \times 790}{1650} = 91.218 \) KN

\( A_I = (100 - 5) \times 10 = 950 \) m²

\( A_L = 700 \) m²

F connected = 123.932 KIN

Rivet value = \( \text{Min} \left( \text{Shearing}, \text{Bearing} \right) \)

\[ r_v = \text{Min} \left( \frac{\pi d^2}{4} \times 100, \frac{d'^2}{4} \times 300 \right) \]

\[ = \left( \frac{\pi 21.5^2}{4} \times 100, 21.5 \times 10 \times 300 \right) \]

\[ = (36.305 \text{ KIN}, 64.5 \text{ KIN}), 51.6 \text{ KIN} \]

\[ r_v = 36.305 \text{ KIN} \]

No. of rivet 3 = \( \frac{123.932}{36.305} = 3.41 \)

Adopt 4 no.

No. of rivet 2

Design of lug angle

Foreign lug angle = 91.318 x 1.2 = 109.582 KIN

\[ \text{Acrt req.} = \frac{109.582 \times 10^3}{150} = 730.55 \text{ mm}^2 \]

Let us adopt JSA 60x60x8

\[ \text{Acrt} = 896 - 21.5 \times 8 = 724 \text{ mm}^2 \leq \text{Acrt req.} \]

Hence adopt a bigger section.
Let us adopt DA 60 x 60 x 10

And provided = 1100 - 21.5 x 10 = 885 mm² > And

No. of rivet 1 = \( \frac{1.4 \times F_{act}}{R_V} \)

= \( \frac{1.4 \times 91.318}{36.305} \) = 3.52

Adopt 4 no

No. of rivet 2 = \( \frac{1.2 \times F_{act}}{R_V} \)

= \( \frac{1.2 \times 91.318}{36.305} \) = 3.018

Adopt = 4 no

O.K.
The thickness of main plate depends on the tearing strength of the main plate that is:

\[ \text{tearing strength} \geq 500 \text{ KN} \]

Tearing strength can be known only if arrangement of rivet is known.

Arrangement of rivet is known only when no. of rivets are known.

No. of rivets are known only when rivet value is known.

Rivet value is known only when we know whether shearing or bearing govern.

The rivets are in double shear, hence shearing strength is known.

Bearing strength will depend on the thickness of main plate by equating shearing and bearing strength of rivet we get the thickness of main plate under the assumption that combined thickness of cover plate is more than that of main plate.

If thickness is not capable of resisting 500 KN load, thickness has to be increased. If thickness is increased, bearing strength will become more than the shearing stress of rivet. Hence the rivet value will be governed by shearing strength of rivet. Hence no. of rivets will be:

\[ \frac{500}{10} \]
No. of rivet will be arranged and tearing strength will be calculated at various sections in terms of thickness t

Taking per shear = 100 N/mm²

Set: Rivet is in double shear

Shearing sl. of rivet = \( \frac{t}{4} \times (2.5)² \times 100 \)

\[ = \frac{t}{4} \times 6.25 \times 100 \]

Assuming the th\(t\) combined thickness of cover plate to be more than main plate

\[ \text{bearing strength} = \frac{d'}{t} \times 6.9 \]

\[ = 23.5 \times 300 \times t = 7.05 \text{ t kN} \]

\[ 86.747 = 7.05 \text{ t kN} \]

\[ t = 12.305 \text{ mm} \]

Max. force that gross area corresponding to this thickness can resist

\[ = 0.8 \times 12.305 \times 250 \times 260 = 480.2 \text{ kN} \leq 500 \text{ kN} \]

Hence thickness of main plate has to be \( t \)

Thus shearing strength govern the rivet value.

\[ \sigma = \frac{500}{86.748} = 5.7 \]

Adopt \( n = 6 \) rivet

The 6 no. of rivet can be arranged in diamond pattern.
Tearing strength of steel 1 : 1

\[
\frac{(250 - 23.5) \times t \times 0.6 \times 260}{t} \geq 500 \times 10^3
\]

\[
t \geq 14.716\text{mm} \quad 14.151\text{m}
\]
\[
af. 2.3
\]
\[
(250 - 2 \times 23.5) \times t \times 0.6 \times 260 + 86.748 \times 10^3 \geq 520 \times 10^3
\]
\[
t \geq 13.572\text{mm}
\]
\[
\geq 13.05\text{mm}
\]

Hence we will adopt thickness 15mm

Let us adopt 16 mm thickness of main plate

& check for strength of cover plate

For cover plate sec. 3-3 is critical

\[
(250 - 3d^2) \times t \times 0.6 \times 260 \geq 500
\]
\[
t \geq 17.85\text{mm}
\]

Adopting thickness of cover plate as 10mm each

For gusset plates as the thickness of main gusset plate as well as combined thickness of two cover plate are more than 12.3 mm hence shearing will govern the rivet value.

hence \[\frac{500}{86.74} = 5.7\]

Adopt 6 rivet
1) Tacking along a, b, c, d
   \[ A_{net} = (A_{gray} - 4A_{hole}) \]

2) Tacking along a + b
   \[ A_{net} = \left[ 2 \times \right] \]
   \[ k = \frac{5A_1}{5A_1 + A_2} \]

3) Tacking along c + d
   \[ A_{net} = \left[ 2 \times \right] \]
   \[ k = \frac{6A_1}{2} \]

4) No tacking
   \[ A_{net} = 4 \times \]
   \[ k = \frac{3A_1}{3A_1 + A_2} \]
Ex. Angle 60 x 60 x 10

\[ A_{total} = (60 + 60 - 10) \times 10 = 1100 \text{ mm}^2 \]

Can 1

\[ 4 \times 1100 - 4 \times 21.5 \times 10 = 3540 \text{ mm}^2 \]

Can 2

\[ k = \frac{5A_1}{5A_1 + A_2} = \frac{5 \times 670}{5 \times 670 + 1100} = 0.7528 \]

\[ A_{hole} = 2 \left[ 670 + 0.7528 \times 1100 \right] = 2976.16 \text{ mm}^2 \]

Can 3

\[ 2 \times \left[ \frac{1}{2} \times 1100 \times 10 - 2 \times 21.5 \times 10 \right] = 3\times 3540 \text{ mm}^2 \]

Can 4

\[ 4 \times \left[ A_1 + kA_2 \right] \]

\[ A_1 = 335 \]

\[ A_2 = (60 - \frac{1}{2}) \times 10 = 550 \]

\[ k = \frac{3A_1}{2A_1 + A_2} = 0.6463 \]

\[ A_{hole} = 2761.86 \text{ mm}^2 \]

Note: Max. deduction due to web hole at any section along the length of member should never be more than max. deduction of hole at any seen at the joint. This is done to ensure that if the joint is not failing member will not fail.
Compressive Member (Purely Axial)

For long columns under purely axial loading the failure is always by buckling and buckling load is given by Euler's load

\[ P = \frac{\pi^2 E I}{L^2} \]

The buckling stress is given by Euler stress

\[ \sigma_{eu} = \frac{\pi^2 E}{L^2} \]

\[ \sigma_{eu} = \frac{\sigma_{ef}}{\sigma_{min}} \]

Where \( \sigma_{min} \) is the minimum radius of gyration of the section and \( L \) is the effective length of compression member which depend on end support condition. However the IS code takes into account the failure by buckling and crushing simultaneously through the use of Merchant Rankine formula

\[ \sigma_{cu} = \text{ksi stress} = \frac{0.6 \sigma_y}{[\sigma_{ef}^{1.4} + (\sigma_y)^{1.4}]^{1.4}} \]

\( \sigma_{eff} \) - Eular stress
\( \sigma_y \) - Yield stress.

For a given grade of steel \( \sigma_{cu} = f(d) \)
As $A_1$ decreases

$\frac{1.2}{l}$ left

Effective length (for column)

S. No.  Description
(1) 85
(2) 
(3) fixed at one end and at other end restrained against rotation but fixed in position 1.2 $l$
(4) 
(5) fixed at one end but at other end partially restrained against rotation 1.5 $l$

Note:

Length adopted by IS code is more than the theoretical value because it is impossible to simulate perfect fixity of support. So we take $1.2l$. If $l < 0.85l$

(6) 

2 $l$

(7) 

2 $l$

The above values are $1.0$ by a battened column.
Effective length of angle strut

Used in trusses and bracing system

<table>
<thead>
<tr>
<th>Description</th>
<th>$l_{eff}$</th>
<th>Allowable compressive stress $(\sigma_{ac})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Continuous single or double angle</td>
<td>$0.7l - 1$</td>
<td>$\sigma_{ac}$</td>
</tr>
<tr>
<td>2. Discontinuous single angle connected with one rivet only</td>
<td>$l$</td>
<td>$0.8 \sigma_{ac}$</td>
</tr>
<tr>
<td>3. Single angle with more than one rivet or weld</td>
<td>$0.85l$</td>
<td>$\sigma_{ac}$</td>
</tr>
<tr>
<td>4. Double angle back to back on opposite side of gusset plate</td>
<td>$0.7 - 0.85l$</td>
<td>$\sigma_{ac}$</td>
</tr>
<tr>
<td>5. Double angle on same side of gusset plate</td>
<td>$l$</td>
<td>$0.8 \sigma_{ac}$</td>
</tr>
</tbody>
</table>

Actual value of effective length depend on the rigidity of joint:

- Continuous
- Discontinuous

\[ \text{As fivity of joint increases, } \begin{cases} \text{left \& vice versa} \end{cases} \]
Maximum Slenderness ratio

S. No. | Description | $A_{max}$
--- | --- | ---
1 | Member carrying compressive load due to dead & live load only | 180
2 | Member subjected to compression due to wind or EQ. | 250
3 | for compression flange of beam | 300
4 | Member normally acting as a tie in a roof truss or bracing system but subjected to reversal of stress due to wind or EQ. | 350

 Rolled Section

\[ \delta_{min} = F_{y}v \]
\[ \delta_{min} = F_{y}v \]
for rolled sections M.O.I. can not be changed. Hence to find out the load carrying capacity \( I_{\text{min}} \) needs to be located. If the section is having at least one axis of symmetry, \( I_{\text{min}} = \sqrt{\frac{I_{\text{min}}}{A}} \) will be about the axis of symmetry or about an axis \( \perp \) to the axis of symmetry. If however the section is having no axis of symmetry then major and minor principal axes need to be located.

In case of built up sections we can change the M.O.I of combined section \( \alpha \)

\[
\begin{align*}
\gamma_{xy} = & 2 \times \gamma_{xy} \text{ of one section} \\
\gamma_{yy_{\text{comb}}} = & 2 \left[ \gamma_{yy_{\text{au}}} + A \left( \gamma_{yy} + \frac{S}{L} \right)^2 \right]
\end{align*}
\]

If spacing b/w seen is zero \( \gamma_{yy_{\text{comb}}} \) in this case will be less than \( \gamma_{xy} \) of combination hence \( I_{\text{min}} \) will be \( I_{yy} \) of combination.

If \( I_{\text{min}} \) is less \( I_{\text{min}} \) will be less so for this \( A \) will be more and hence \( \gamma_{ax} \) will be less.
Thus load carrying capacity will be less.
To increase the load carrying capacity for the given area, $I_{yy}$ of combination needs to be increased. This can be achieved by increasing the spacing. For a given spacing, $S$, $I_{yy}$ of combination will become $S^2$ of comb. For $S > S_0$, $I_{yy}$ of comb. which is fixed. Hence load carrying capacity becomes fixed for all $S > S_0$ and this spacing corresponds to maximum load carrying capacity for a given sectional area. Thus for best utilization of material $I_{yy}$ of comb. = $S^2$ of comb.

Radius of gyration of a sections will not change when put in combination when the axis does not shift.
for a given overall size of column strength of \( f \) will be more than the strength of \( f \) because shifting of axes will be more in (I) than in (II) -

\[ d > d' \]

for maintenance point of view II will be better.

---

55 Code Recommendation

---

\[ \text{Overall} = \frac{\text{left}}{R_{\text{min}} \text{comb}} \]

---

\[ \text{one} = \text{spacing b/w tacking rivets} \]

---

When two components are placed back to back they should be tack rivetted such that stress ratio of individual sec b/w tacking rivet should not be...
more than 40 nor more than \(-0.6 \times\) slenderness ratio
of whole sec.

\[ \text{Diam} \neq 40 \]
\[ \neq 0.6 \times \text{whole sec.} \]

This recommendation safe guards against local buckling of individual sections between tacking rivets.

The spacing b/w tacking rivet however should not be more than 600 mm.

The dia of tacking rivet \(s\) should not be less than the min dia given below.

<table>
<thead>
<tr>
<th>Thickness of member</th>
<th>Dia of tacking rivet up to</th>
</tr>
</thead>
<tbody>
<tr>
<td>up to 10 mm</td>
<td>18 mm</td>
</tr>
<tr>
<td>10 - 16 mm</td>
<td>20 mm</td>
</tr>
<tr>
<td>&gt; 16 mm</td>
<td>22 mm</td>
</tr>
</tbody>
</table>

**Note:**
Tacking rivet recommendation given before 32 t or 500 mm is the max. Spacing b/w tacking rivets in any position. However the limiting criteria in compression will become

\[ \text{Diam} \neq 40 \]
\[ \neq 0.6 \times \text{whole sec.} \]

- Surface of tacking rivet is to hold the two member together and to equalize the stress in the component memb.
If the web of channel is \( > 150 \text{ mm} \) in depth, two rows of tacking rivet can be provided. If angles are \( 125 \text{ mm} \) or more two rows of tacking rivet can be provided.

\[ f_{yw} = 2 \times 150 \times 828 \times 10^{-3} + \left[ \frac{500 \times 10^3}{12} \right] \times 2 \]
\[ = 721.9893 \times 10^6 \text{ mm}^4 \]

\[ f_{yy} = 2 \left[ \frac{5048 \times 10^3 + (100 + 24.2) \times 6.292 \times 10^3}{12} \right] \]
\[ + 2 \times 10 \times 500^3 \]
\[ = 412.5769 \times 10^6 \text{ mm}^4 \]

\[ f_{yy} \text{ is min} \]

\[ E \text{ min} = \sqrt{f_{yy}} = \sqrt{412.5769 \times 10^6} \]
\[ \frac{412.5769 \times 10^6}{2 \times 6.292 \times 10^3 + 5000} \]
\[ = 135.15 \text{ mm} \]

\[ C_{002} = 500 \]
\[ n = \frac{d_{eff}}{h_{min}} = \frac{5.000}{135.155} = 0.036995 \]

\[ \sigma_{ac} = 145 \phi - \left(\frac{145 - 102}{20}\right) \times 6.295 \]

\[ = 140.455 \text{ N/mm}^2 \]

Safe load = \( 140.455 \times 2 \times (6293 + 5000) = 3172.277 \text{ KN} \)

If effective length is 6 m:

\[ n = \frac{6000}{135.155} = 44.393 \]

\[ \sigma_{ac} = 145 - \left(\frac{145 - 132}{20}\right) \times 14.393 \]

\[ = 135.645 \text{ N/mm}^2 \]

Safe load = \( 135.645 \times 2 \times (6293 + 5000) = 3063.667 \text{ KN} \)
\[ I_{xx} = 2 A (r^{2} + y^{2}) \]

\[ I_{yy} = \left[ A r^{2} + A \times 33.1^{2} \right] \times 2 \]

\[ = A \left[ 27.5^{2} + 33.1^{2} \right] \times 2 = 5.107429 \times 10^{6} \]

\[ I_{yy} = \sqrt{\frac{I_{yy}}{A}} = 43.03 \text{ mm} \]

\[ d = \frac{\text{left}}{r} = \frac{2550}{49.94} = 92.727 \]

\[ d_{one} = \frac{30.0}{27.5} = 17.143 \]

\[ f = \frac{1220 - 1320 \times 9.257}{20} = 1227.43 \text{ N/mm}^{2} \]

\[ p = 126.288 \text{ N/mm}^{2} \]
\[ f = 9.900 - \frac{9.00 - 7.20}{2.0} \times 2.727 \]
\[ = 8.7545 \text{ MPa} = 85.882 \text{ N/mm}^2 \]
\[ P = 85.882 \times 2 \times 1377 = 236863 \text{ KN} \]

\[ g = 9.81 \text{ N/m}^2 \]

\[ \text{Note:} \quad \text{If } g = 10 \Rightarrow P = 241325 \text{ KN} \]

As the angle is equal angle \( \theta_{xy} = \theta_{yy} \)

\[ \theta_{xx} = \theta_{yy} \]

When put in combination x axis does not shift but y axis shifts such that

\[ \theta_{yy} \text{ of comb.} > \theta_{xx} \text{ of comb.} \]

hence

\[ I_{\text{min}} \text{ of comb.} = I_{xx} \text{ of comb.} \]

check for local buckling.

\[ d_{\text{ave}} = \frac{30}{8.99} = \frac{30}{1.75} = 17.143 \]

\[ d_{\text{hole}} > d_{\text{ave}} \]

\[ 17.14 \neq 92.7 \times 2700.6 \text{ & } 17.14 \neq 40 \]

safe in local buckling.

local buckling will only be checked load cannot be calculated on the basis of this
A stanchion of eff. length 6 m consists of twin box beam using an ESMB 250 with two plates 2.56 x 10 mm she welded each to the lips of two flanges of ESMB 250 on both sides with 4 mm fillet weld continuous through out.

The height properties of ESMB 250 are:

\[ D = 250 \text{ mm} \]
\[ B = 125 \text{ mm} \]
\[ A = 40.4755 \text{ cm}^2 \]
\[ I_{xx} = 5131.6 \text{ cm}^4 \]
\[ I_{yy} = 334.5 \text{ cm}^4 \]

Calculate the load carrying capacity of the section.

\[ I_{xx} = 5131.6 + 2 \times \frac{1 \times 26}{12} = 8060.933 \text{ cm}^4 \]
\[ I_{yy} = 334.5 \times 10^4 + 2 \left( \frac{260 \times 10^3}{12} + \frac{67.5 \times 260 \times 10}{12} \right) = 27.08 \times 10^6 \text{ mm}^4 \]
\[ J_{min} = 27.08 \times 10^6 \]
\[ \sqrt{\frac{J_{min}}{A}} = \sqrt{\frac{27.08 \times 10^6}{(4.755 + 2.60 \times 10) \times 2}} = 52.1667 \text{ mm} \]
\[ d = \frac{6000}{52.1667} = 115.038 \]
Local buckling phenomenon in cover plates and web plates.

Local buckling phenomenon in webs and cover plate of component member occurs in the form of waves or wrinkles. The critical stress at which local buckling in the form of waves or wrinkle starts is given by:

\[ f_{cr} = \frac{K \pi^2 E}{12(1-\nu^2)(b/t)^2} \]

Larger the value of \( b/t \), smaller is the stress at which local buckling starts in the form of waves or wrinkles.

If \( \frac{b}{t} \) is so adjusted that critical stress for local buckling becomes more than the stress corresponding to overall failure of column, then local buckling will not occur before overall failure of column on this.
basis of code has given the following recommendations:

\[ \left( \frac{b}{t} \right) < 16 \]

Any portion of \( b \) greater than 16 \( t \) should not be taken into account in strength calculation.

For web:

\[ \left( \frac{d'}{t_w} \right) \neq 50 \]

Any portion of \( d' > 50 \) \( t_w \) should not be taken into account in strength calculation.

Note:

The area reduction should be done in such a way that it leads to maximum reduction in strength.
100 fmm

\[ I_{yy} = 2 \times \frac{198^3 \times 6}{12} + \frac{6.8 \times 300}{12} = 7.7677 \times 10^6 \text{mm}^4 \]

\[ I_{xx} = \frac{6 \times 300^3}{12} + 2 \left( \frac{198 \times 6^3}{12} + 198 \times 6 \times 300 \right) \]

\[ = 111.42 \times 10^6 \text{mm}^4 \]

\[ \phi_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{7.7677 \times 10^6}{2 \times 198 \times 6 + 300 \times 6}} = 43.12 \text{mm} \]

\[ N = 81.15 \]

\[ \sigma_a = 977.91 \text{ MPa/cm}^2 \]

\[ P_{safe} = 977.91 \times \left( 2 \times 198 \times 6 + 300 \times 6 \right) \times 10^2 \]

\[ = 41672.7 \text{ N} \]

\[ = 41672.7 \text{ KN} \]
A companion is to be constructed using a F5MC 300 placed back to back.

\[ \delta_{ac} = \frac{f_{c}}{f_{c} - 0.5} \]

\[ \text{Rankine constant} \]

\[ c_{yy} = \frac{3200/2}{1 + \frac{1}{4500} A^{2}} = \frac{800000}{7500 + A^{2}} \]

For max. effecite:

\[ f_{xx} = f_{yy} \]

\[ f_{yy} = \left[ 310.8 \times 10^{4} + 4564 \times \left( \frac{23.6 + 5}{2} \right)^{2} \right] \times 2 \]

\[ S = 183.951 \text{ mm} \]

\[ \gamma_{min} = (\gamma_{xx})_{cool} = (\gamma_{yy})_{cool} = 118.1 \text{ mm} \]

\[ d' = 6.5 \times 6 = 6.5 \times 6 \]

\[ \sigma = 55.0 \text{ kN} \]
Design of compression member required

Design of compression member required finding out area of cross section which depend on $\sigma_{ce}$ (area $A = \frac{F}{\sigma_{ce}}$)

But $\sigma_{ce}$ also depends on the section hence we will not have a direct relation as in the case of tension member.

where permissible stress was constant hence we will go for a trial and error solution.

Step 1: Assume the value of $\sigma_{ce}$

a) For rolled sections choose $\sigma_{ce}$ between 60 — 80 MPa.

b) For built up sections choose $\sigma_{ce}$ 110 MPa

\[ \frac{P}{(\sigma_{ce})_{chosen}} \] = Area required

choose see trial see and find the slenderness ratio and hence $\sigma_{ce}$ for the chosen area
Calculate safe load carrying capacity of the chosen section

\[ p' = \text{area chosen} \times \sigma \text{ chosen} \]
\[ p' = \text{area provided} \times \sigma \text{ provided} \]

- Safe load carrying capacity of trial section \( p' \)
- If \( p' > p \) section is sufficient, otherwise choose another larger section and repeat.

Check for max. limit of slenderness ratio
Check for slenderness ratio can be done at the time when slenderness ratio of chosen section was found out.

8. Design a built-up column 10 m long to carry an axial load of 750 KN. Use 2 channels placed back to back.
   \( l_{eff} = 10 \text{ m} \). The section available are

   ISMC 250
   \[ A = 3867 \text{ mm}^2 \]
   \[ t_{xy} = 96.4 \text{ mm} \]
   \[ t_{yy} = 23.8 \text{ mm} \]
   \[ c_{yy} = 21 \text{ mm} \]

   ISMC 300
   \[ A = 4564 \text{ mm}^2 \]
   \[ t_{xy} = 98.1 \text{ mm} \]
   \[ t_{yy} = 26.1 \text{ mm} \]
   \[ c_{yy} = 23.6 \text{ mm} \]
1. Chose $\sigma_a = 110$ N/mm$^2$

\[\Delta \text{eqq.} = \frac{P}{\sigma_a} = \frac{750 \times 10^3}{110} = 6818 \text{ mm}^2\]

Chose 2 no. of I50 MC 2.50

A provided = $2 \times 3847 = 7734 \text{ mm}^2$

For the chosen area find $\delta_{\text{min}} = \frac{1}{2} - \delta_a$

$\delta_{\text{min}} < \frac{\text{min of } \gamma_{xy} \text{ & } \gamma_{xx}}{2}$

$\gamma_{xy}$ of combination depends on spacing between the seen for most efficient design

$\gamma_{xy \text{ comb}} = \delta_{xy \text{ comb}}$

Thus spacing is so chosen that $\gamma_{xx \text{ of comb}} = \gamma_{xy \text{ of comb}}$

And under the situation

$\delta_{\text{min}} = \frac{1}{2} (\delta_{xx \text{ of comb}})$

$\delta_{\text{min}} = \delta_{xx \text{ of one seen}} = 99.4 \text{ mm}$

$\text{left} = 10 \text{ m}$

$\frac{\text{left}}{\delta_{\text{min}}} = 100.6 < 180 \text{ ex. from the bond}$
\[ \sigma_{u} = 88.478 \text{ N/mm}^2 \]

\[ p^1 = \frac{\sigma_{u} \times \text{Area}_{	ext{pro}}}{600} \]

\[ = 88.478 \times (2 \times 3867) = 645.6 \text{ KN} < 750 \text{ KN safe} \]

Choose 2 ISMC 300

\[ A_{\text{provided}} = 2 \times 4574 = 9148 \text{ mm}^2 \]

\[ \delta_{\text{min}} = \frac{A_{\text{pro}}}{A_{\text{min}}} = 118.4 \text{ mm} \]

\[ d = \frac{L_{\text{eff}}}{\delta_{\text{min}}} = \frac{10,000}{118.4} = 84.67 < 180 \text{ OK} \]

\[ \sigma_{u} = 100.7 - \frac{100.7 - 92.8 \times 24.67}{30} \]

\[ = 94.2 \text{ N/mm}^2 \]

\[ p^1 = \frac{\sigma_{u} \times A_{\text{pro}}}{600} \]

\[ = 94.2 \times 9148 = 859.89 \text{ KN} > 750 \text{ KN OK} \]

Spacing calculation

\[ L_{\text{eff}} = 2 \gamma \gamma \]

\[ 2 L_{\text{eff}} = 2 \left[ \gamma \gamma + A \left( \frac{\gamma \gamma + 5}{2} \right)^2 \right] \]

\[ 2 A \gamma^2 = 2 \left[ A \times \gamma^2 \gamma + A \left( \frac{\gamma \gamma + 5}{2} \right) \right] \]
\[ \varepsilon \delta_{xy}^2 + \gamma_{xy}^2 + \left( \gamma_{yz} + \frac{\varepsilon}{\varepsilon_z} \right)^2 \]

\[ S = 187.1 \text{ mm} \]

Spacing should be chosen 185 mm
under this spacing 5mm will remain \( \gamma_{xy} \)

**Design of lacing and batten**

- **Single lacing**
- **Double lacing**
- **Batten planks**

(I)  
(II)  
(III)  
(IV)
1. Out of (1) + (ii) I is better because accidental total failure of the joint will not decrease the distance 6/5 intermediate connection significantly.

2. If the single lacing is not sufficient to safeguard against local buckling, double lacing will be provided.

Lacing:
- Lacing is in the form of a flat plate or angle. The angle is kept slightly more than 90°.
- Lacing should not be varied throughout the length of the compression member (angle of x seen, etc. should be same).
- Single lacing on opposite side of main component should be mirror image of each other.
- Lacing on adjacent face should be staggered.
- All the top and bottom in the laced column batten plates are provided. They are called end battens.

![Diagram of lacing patterns]

- 1-2 mirror image
- 3-4 mirror image
- 1-3 staggered
Design

1) Angle of inclination of lacing with the longitudinal axis of column should be between 90° to 70° degrees.

2) Slenderness ratio of individual member of column & intermediate connections should not be more than 50 neither more than 0.7 times slenderness ratio of whole section.

\[
\frac{c}{(\frac{c}{T_{\text{min}}})_{\text{one}}} \neq 50
\]

\[
\neq 0.7 \text{ Aspect ratio}
\]

criteria for local buckling

Note: To start with θ angle will be chosen. As spacing has already been chosen for component members, c becomes fixed, this c check for local buckling criteria. If it is not met θ can be increased.

Effective length of lacing bar

\[
L_{eff} = l \text{ for single lacing (riveted)}
\]

\[
= 0.7l \text{ for double lacing (when there is connection at the overlap)}
\]

\[
= 0.7l \text{ for welded lacing}
\]

\[l \rightarrow \text{ Distance b/w inner ends of rivet or weld}\]

Slenderness ratio of lacing bar should not be more than 145

\[
\frac{L_{eff}}{\delta_{\text{min}}} \neq 145
\]
\[ b \quad \delta_{\min} = \gamma_{YY} = \sqrt{\frac{f_{yy}}{A}} = \sqrt{\frac{bt^3}{6t^2}} = \frac{t}{\sqrt{12}} \]

\[ \frac{I_{eff}}{t/\sqrt{12}} \neq 17.5 \]

**Min Width of Lacing Bar**

Nominal dia of rivet (mm):
- 22
- 20
- 18
- 16

Min thickness of lacing bar

Min thickness \( \neq \frac{t}{40} \) for single lacing

\( \frac{t}{60} \) for double lacing

**Force for which lacing should be designed.**

The lacing should be designed for shear force in column arising due to:
- a) B.M.
- b) Lateral loading
- c) Incidental Eccentricity

\[ \frac{t}{30} \quad \text{unavoidable} \]
For incidental eccentricity transverse shear = 3.5% of axial load in the member is taken into account. This shear is divided equally between all transverse lacing system in parallel planes.

\[ \frac{V}{L} \]

Lacing bar will be designed for tension and compression the force will be taken equal to \( \left( \frac{V}{2 \sin \theta} \right) \).

Check for tension

\[ (b-d') t \sigma_{at} \geq \frac{V}{2 \sin \theta} \]

Check for compression

\[ \frac{V}{2 b x t} \leq \sigma_{cu} \] (corresponding to effective min. = \( \frac{1}{f_{m}} \))
Design for connection of lacing bar

\[ \frac{V}{2 \sin \theta} \leq R_u \]

Single shear

\[ \frac{V}{2 \sin \theta} \]

Note

Shear at sec. 1 will correspond to \( \frac{V}{2 \sin \theta} \)

Shear at sec. 2 \( \geq \frac{V \cot \theta}{2} \)

Normally check is done for \( V \cot \theta \) However if \( \frac{V}{2 \sin \theta} \) becomes more than \( V \cot \theta \) means

\[ \frac{V}{2 \sin \theta} > \frac{V \cot \theta}{2} \Rightarrow \theta > 60^\circ \]

sec. 1 will become critical hence
check should be done for \( \frac{V}{2 \sin \theta} \) force but normally 0 is around 45... so \((v \cot \theta)\) governs.

\[
P = 250 \text{ kN}
\]

\[
A = 4 \times 1.903 = 7.612 \text{ mm}^2
\]

\[
\sigma_{tu} = \frac{750 \times 10^3}{7.612} = 98.13 \text{ N/mm}^2
\]

\[
\delta_{ep} = 82.247 \text{ mm}
\]

\[
\delta_{st} = 82.247 \text{ mm}
\]

\[
\frac{\delta_{eff}}{\delta_{min}} \geq 114.29 \text{ mm}
\]

\[
s_{min} = s_{min} = 114.29 \text{ mm}
\]

\[
s = A \frac{r^2}{2
\]

\[
s = (4 \times 1.903) 114.29^2 = 99.93 \times 10^6 \text{ mm}^2
\]

\[
S_{min} = 9 \times \left[ 1.77 \times 10^4 + \left( 100 - 28.4 + \frac{5}{2} \right) \times 190.3 \right] = 99.3 \times 10^6
\]

\[
s = 77.09 \text{ mm}
\]

Adapt \( s = 80 \text{ mm} \)
taking \( j = 40 \text{ mm} \) \( > 0.5 \frac{c}{c_{yy}} \)

\[ \text{tan} \theta = \frac{200}{c/2} \]
\[ c = \frac{400}{\text{tan} \theta} \]

assuming \( \theta = 45^\circ \)
\[ c = 400 \text{ mm} \]

Check for local buckling

\[ \frac{c}{c_{yy}} \mid \text{ind} \neq 50 \]
\[ \frac{c}{c_{yy}} \mid \text{dubek} \neq 0.7 \]
\[ \frac{900}{c_{yy}} \neq 50 \]
\[ \frac{900}{19.4} = 46.2 \neq 50 \quad \text{OK} \]
Note: for slenderness ratio 83.247 spacing is 77.09
If spacing is increased \( \delta \) min will increase hence \( \lambda \) will fall thus for spacing equal to \( \lambda \) it would be less than 82.247 if local buckling criteria is satisfied for longer slenderness ratio.

Design of Lacing

Local buckling criteria is satisfied for \( \lambda < 45 \) in single lacing hence single lacing is sufficient.

\[
\lambda = \frac{l}{n} = 282.843 \text{ mm}
\]

\[
\text{Dia of wire} = 6.08 \sqrt{\frac{E}{\sigma}} = 6.08 \sqrt{\frac{210}{1913}} = 1.91 \text{ mm}
\]

Min width of lacing = 61 mm

Min thickness = \( \frac{l}{40} = \frac{282.843}{40} = 7.07 \text{ mm} \)

Check for slenderness ratio of lacing

\[
\frac{\lambda}{\delta_{\text{min}}} \neq 145
\]

\[
\frac{\delta_{\text{eff}}}{t/\sqrt{n}} \neq 145
\]

\[
\frac{282.843 \sqrt{n}}{8} \neq 145
\]

122.47 \neq 145

Ok.
check for tension

\[ V = 3.5\% \text{ of } P \]

\[ \frac{2.5 \times 750}{100} = 18.75 \text{ KN} \]

Tensile force in hacing \( f = \frac{V}{a \sin \theta} = 13.25 \text{ KN} \)

Tearing strength \( > 13.25 \)

\[ (60 - 21.5) \times 8 \times 0.6 f_y \geq 13.25 \]

\[ 46.2 \text{ KN} > 13.25 \text{ KN} \]

Check for compression

\[ n = 122.5 \]

\[ \frac{n}{120} = 67.1 \]

\[ 130 = 59.7 \]

\[ \sigma_{ac} = 65.27 \text{ KN/mm}^2 \]

\[ \rho' = \frac{\sigma_{ac}}{f_y} \]

\[ 65.27 \times 60 \times 8 = 313.3 \text{ KN} > 13.25 \text{ KN} \]

Safe in compression

Design for connection.

\[ V_{weld} = 18.75 \text{ KN} \]
Rivet value = 3 Min (shearing, bearing)

\[ \text{Shearing strength of rivet} = 36.3 \text{ KN} \]

No. of rivet = \( \frac{18.75}{36.3} \)

Adopt 1 rivet

Check for bearing

64.5\(\text{KN} \) > 18.75 \(\text{OK} \), safe in bearing

51.6 \(\text{KN} \) > 13.25 \(\text{OK} \)

Design of End batten

\[ b = 100 \text{ mm} \]

\[ d + 2(180-28.4) = 223.2 \text{ mm} \]

\[ d = \text{Eff. depth of end batten} \]

\[ d \neq a \neq 223.2 \]

\[ d \neq 2b \neq 250 \]

Adopt \( d = 225 \text{ mm} \)

\[ d \pm d + 2 \text{ edge distance} = 225 + (2 \times 20) \times 2 = 305 \text{ mm} \]
Thick = \frac{1}{50} = \frac{200}{50} = 4

\text{depth} \ 8 \text{mm}
Design of End batten.

The effective length of batten column is taken 10% more than that of faced column.

Y-Y plane is the plane i to the plane of batten.

Batten on opposite faces should be mirror image. No of batten should be such that it divides the column longitudinally in not less than 3 parts.

Effective length of battened column is taken 10% more.

Design Specification:

\[
\frac{c}{(\sqrt{\text{min}})}_{\text{brc}} \times 50
\]

\[
\neq 0.7d_{\text{weld}}
\]
Effective depth of intermediate batten should not be less than \( \frac{3}{4} \) times the distance b/w centroid of component members, the eff. depth however should not be less than twice the width of 1 component member in the plane of batten.

\[
d = s + 2cy_y
\]

\[
d' = \frac{3}{4} d' \neq 2b
\]

For End batten:

Effective depth of end batten \( d \neq d' \neq 2b \)

The thickness of batten should not be less than \( \frac{l}{50} \)

where \( l \) is the distance b/w innermost connecting line of rivets or weld

\[
l = (s + 2y)
\]
Check for safety

Batten should be designed to carry B.M. and shear arising due to transverse shear. Transverse shear for incidental eccentricity

\[ v = \frac{2.5}{100} \text{ Pascal} \]

Hence batten plate will be designed for

Shear = \[ \frac{vc}{2l} \]

Moment = \[ \frac{vc}{t} \]

Check for shear

Permissible average shear stress = \[ 0.4 \times fy \]

\[ \left( \frac{vc}{2l} \right) \leq 0.4 \times fy \]

Check for bending

Permissible average bending stress = \[ 185 \text{ N/mm}^2 \]

\[ \left( \frac{vc}{t} \right) \leq 185 \text{ N/mm}^2 \]

or

\[ \left( \frac{t \cdot D^3}{12} \right) \leq 165 \text{ N/mm}^2 \]
Design of connection will be done as eccentric connection in which rivet group is subjected to transverse shear of \( (\frac{Vc}{2t}) \) and torsional moment \( (\frac{Vc}{t}) \).

From the torsional moment (or torsional shear) and transverse shear, shear force in extreme rivet will be found out and this should be less than rivet value for safety of connection.

**Welded Connection**

\[ \text{Overlap} \leq 4t \]

\( t \rightarrow \text{thickness of plate} \)

Total length of weld at the edge of batten \( \leq \frac{D}{2} \)

Practically we provide continuous connection hence their recommendation will not be required to be checked.
Column splices

Top storey column & bottom story column are of same size.

Top & bottom storey columns are of different size but top storey column flange resting completely on bottom storey flange.
In case of complete bearing of top story column flange on bottom story column flange the design is as follows:

If the column ends are milled all of the direct force will be assumed to be transferred through direct bearing of top story to bottom story, i.e., SS code 122.

If moments are not acting, the purpose of splice plate will only be to hold the two columns together.

However, to find out the size of splice plate, it will be assumed that 50% of the direct load is transferred through splice plates.

Hence, force on one splice plate = \( \frac{1}{4} \)

Area of splice plate: \( \frac{\pi}{4} \left( \frac{\sigma_{sw}}{f_y} \right) \)

As there is no buckling, i.e., \( \sigma_{sw} = 0.6f_y \)

\[ b \times t = \frac{(9/4)}{0.6f_y} \]

\( b \rightarrow \) width of column flange

- If the column ends are not milled, design force = \( \frac{1}{2} \) for one splice
- If the column is subjected to direct load as well as moment

**Case 1:** When column ends are milled
All direct force will be assumed to be transferred through direct bearing. Splice plate however will be designed for forces generated due to moment. Hence

\[ \text{design force} = \left( \frac{M}{l} \right) \]

\[ l = \text{spacing b/w splice plate} \]

(Case \(b\))

When column ends are not milled

Design force = \( \left( \frac{p}{2} + \frac{M}{l} \right) \) in compression.

= \( \left( \frac{p}{2} - \frac{M}{l} \right) \) in tension. (When \( v \) becomes \( \omega \) ve)

Web splice are provided to resist shear in the column.

\( \frac{v}{2(t)} \leq 0.4 \) fy

Web splice are available for providing splice plate \( d = 2h_2 \)

\( a \) should be chosen smaller than this.

Rivets here will be designed to be in double shear.
When top story column flange is not resting on the bottom story column flange,

in this case the total load is assumed to have been transferred through the column flanges to the base plate and then through the base plate to the bottom story column flanges. The connection will be designed by usual methods. However, the base plate needs to be designed.

\[ b_1 = \text{Depth of top story column} \]
\[ T_b = \text{Thickness of base plate} \]

\[ (D_b - T_b) = b_1 \]

\[ D_b - T_b = a \]

\[ M_{\text{max}} = \frac{P(a - b)}{4} \]

For bending stress,

\[ \sigma = \frac{P(a - b)}{I} \]

165 for flanged

185 for solid column
$$\sigma_{pl} \cdot \frac{bt^2}{6} = \frac{P(\alpha - h)}{t}$$

from here calc. t

Note:

i) If packing is required, thickness of packing will be equal to the gap to be filled.

ii) In this case if thickness of packing is greater than 6 mm, additional rivets will be provided, on packing extension and no. of such rivets is equal to 2.5% of the no. of rivets obtained from normal calculation per 2 mm thickness of packing.

Mean of partial bearing all the direct load will be assumed to have been transferred from top story column to bottom story column through splice plate.

Question:
2 ISMB 300 sections are spliced using compression splices. The applied load is 300 KN, S.F. is 75 KN and applied moment is 20 KNm. Assume column ends to be not milled. Design column splice.

As the column ends are not milled, all of the direct force will be transferred through the splice plate. Hence force in splice plate: \( F = \frac{P}{2} + \frac{M}{l} \)

Adopt: \( l = 300 \) mm to be on safer side.

The forces in splice are:

\[ \frac{P}{2} + \frac{M}{l} \]
\[ \frac{300}{2} + \frac{20}{3} \]

216.07 KN comp.

\[ \frac{P}{2} = \frac{M}{l} \]
\[ \frac{300}{2} - \frac{20}{3} \]

83.33 KN comp.
Design force for column splice = 216.67 kN
Permissible stress in compression in splice = 0.6fyf

splice will not buckle individually

\[
\text{area required} = \frac{216.67 \times 10^3}{0.6 \times 250} = 1744.47 \text{ mm}^2
\]

Width of splice plate is taken as the width of column flange

\[
b \times t = 1744.47
\]

\[
b = 250
\]

\[
t = \frac{1744.47}{250} = 7.0
\]

Adopt \( t = 8 \text{ mm} \)

Length of column splice will depend on no. of rivets

Design of Connection

Rivets will be designed for force of 216.67 kN
Adopting the dia. of rivet as 18 mm and choosing power driven field driven

For stress in rivet in shear = 90 MPa

\[
\text{bearing} = \frac{19.8 \times 1290}{19.8 \times 8 \times 170}
\]

\[
\text{min} \left\{ \frac{A}{L} \right\} = 26.87 \text{ kN} < 42.12 \text{ kN}
\]

Rivet value min \( \left\{ \text{Shear, Bearing} \right\} \)
\[ \frac{N}{F_n} = 26.87 \]

- No. of level = \[ \frac{218.67}{26.87} = 8.06 \]

- Adopt to No. \((\text{even no. due to symmetry})\)

\[ 12.8 \]

\[ f = 8f + 4c \]
\[ = 8 \times 2.5d + 4 \times 2d' \]
\[ = 28d - 18x18 \]
\[ = 504 \text{mm} \]
\[ = 20xd + 8d' \]
\[ = 520 \text{mm} \]

splice plate on the other side will also be of same section & connection detailing will also be same.

Design for shear splice:

Area required to resist shear
\[ = \frac{75}{0.4f_y} \]
\[ = \frac{75 \times 10^3}{100} = 750 \text{mm}^2 \]

\[ 2at > 750 \]

- Adopt width of splice \(125 \text{mm} \)

\[ \frac{750}{125} = 6 \]

\[ \geq 3 \text{mm} \]

- Adopt sl thickness of plate = 6 mm
Rivets are in double shear hence

\[
P_v = \min \left\{ \frac{2 \times \pi \times 12^2}{4} \times 90, \quad \frac{6 \times 90}{x + 6} \right\}
\]

\[
P_v = \min \left\{ 53.75, \quad 90.014 \text{ KN} \right\}
\]

\[
P_v = 90.014 \text{ KN}
\]

No. of rivets = \[
\frac{75}{90.014} \approx 0.8
\]

Adopt 2 rivets.

**Note:**

ISSC → Is a § I section

1. An ISHB 300 in lower story and ISHB 200 in the upper story has been used.

- ISHB 300
  - D = 300
  - B = 250
  - T = 10.6
  - \( t_o = 7.6 \)

Column load is 650 KN design the base plate placed between top & bottom story column.

for the top story column flange is not resting at all on the bottom story column flange hence we require the base plate.

Size of the base plate will be 300 x 250 x t
\[
\frac{P(a-b)}{4} \leq M.O.R
\]

\[
10^{-3} \times 650 \left(289.4 - 191\right) \leq \frac{\sigma_{\text{per}} \cdot Z}{185} \cdot \frac{6}{6}
\]

\[
t \geq 45.54 \text{ mm}
\]

Adopting \( t = 50 \text{ mm} \)

# Column Bases
If steel column is directly placed on a concrete pedestal, the concrete will get crushed. To safeguard against crushing of concrete area of load transferred should be increased so that the resulting stress on concrete is less than the permissible stress in direct compression in concrete.

The various types of column bases are:

a) Slab Base
b) Grussetted base
c) Grillage Foundation

![Diagram of column bases]

Cleat Angle
ISA 60x60x8

(Slab base)

(Grusseted base)
Grillage Foundation

Note: Grouted base is better if the column loads are heavy.

It provides greater rigidity at the joint.

When the load is heavy and soil is weak or when the foundation is to be laid at shallow depth we provide grillage foundation. Grillage foundation provides larger area at the base hence the pressure on the soil reduces.

Grillage foundation is also provided in case of temporary construction or to temporarily support the structure.

When the column load is purely axial

Assuming that column transfers a load of \( P \) at the base plate

Size of base plate will be decided as

\[
\frac{P}{oc} = \text{Area of base plate}
\]
Note: If applied load is given we should take into account the dead wt. also hence the design force for base plate will be 1.1 x P where P = applied load on column and 1.1 takes into account the dead wt. which is taken as 10% of applied load.

For purely axially loaded column

\[
\tau = \sqrt{\frac{3w (a^2 - b^2)}{a_b} \left( \frac{a^2 - b^2}{t} \right)}
\]

- \( t \) : larger overhang
- \( t \) : smaller overhang

\( l \) : Thickness of base plate

\( a_{bs} \) : Res. bending stress in base plate (185 N/mm^2)

If \( a \) & \( b \) are smaller and larger overhangs

\( w \): load on base plate

Area of base plate

\[
M_{11} = \frac{wa^2}{2}
\]

\[
M_{22} = \frac{wb^2}{2}
\]

\[
\sigma_{11} = \frac{M_{11} x t}{2}
\]

\[
\sigma_{22} = \frac{M_{22} x t}{2} + \frac{1}{2} x \frac{t^3}{12}
\]
\[
\sigma_a = \sigma_{bs} \Rightarrow t \geq \sqrt{\frac{3 \omega (a^2 - b^2)}{\delta_{bs}}} 
\]

Assuming \( \mu = 0.35 \) for steel,

\[
t_{\text{min}} = \sqrt{\frac{3 \omega}{\delta_{bs}}} \left( a^2 - \frac{b^2}{4} \right)
\]

Note: The plate is critical in bending at section 1-1 hence thickness is decided on the basis of bending however shear for shear can be done as

\[
\frac{\omega \times 1 \times a}{1 \times t} \leq 0.4 f_y
\]

If the column is subjected to load as well as moment, net base per is to be calculated and bending stress at critical section is evaluated. This bending stress is shown to be less than the permissible bending stress this analysis helps us in finding out thickness of base plate.
Slab base subjected to moment and axial loading

\[ \text{Net stress} = \frac{P}{BD} \left( 1 - \frac{6e}{D} \right) \]

If \( c < \frac{D}{6} \)
\[ \frac{6e}{D} < 1 \]

For design \( \frac{P}{BD} \left( 1 + \frac{6e}{D} \right) \leq \sigma_c \)

To avoid stress in compression.
Using this formula we can choose $l$ & find out other
(choose either of $B_1D$ & cal. other)

Thus size of base plate will be known -

Thickness of base plate

Section 1-1 is the critical section for bending

\[
\sigma_{A_1} = \sigma_{11} - \mu \sigma_{22} \\
\quad = \frac{m_{11} \frac{t}{2}}{t^3/P_2} - \frac{\mu m_{22} \frac{t/2}{1+3/P_2}}{t^3/P_2} \\
\sigma_A = \frac{6}{t^2} \left( m_{11} - \mu m_{22} \right) \leq \sigma_{ob}
\]
\[ t \geq \frac{6 (e_{\text{in}} - e_{\text{out}})}{\sigma_{\text{bs}}} \]

If value of \( e \) is not given we can neglect it by doing so we will be on safer side.

\[ \text{max. stress} = \frac{P}{BD} \left(1 + \frac{6e}{d}\right) \]
\[ \text{min. stress} = \frac{P}{BD} \left(1 - \frac{6e}{d}\right) \]

If \( e > \frac{d}{6} \),
\[ \text{min. stress} = \frac{P}{BD} \left(1 - \frac{6e}{d}\right) \text{ becomes } \frac{P}{BD} \text{ Terminate} \]

Due to tensile stress there will be loss of contact between plate and concrete. If the final length of contact situation minimum stress will be zero. For this

\[ \frac{2P}{BD} \]

\[ 3 \left(\frac{d}{2} - e\right) = D'' \]
Hence final length of contact \( = \frac{3}{2} \left( \frac{b}{2} - c \right) \)

\[ e = \frac{M_v}{P} \]

and Max. comp. stress \( = \frac{2P}{3B \left( \frac{b}{2} - c \right)} = \frac{2P}{B \cdot 3 \left( \frac{b}{2} - e \right)} \)

\[ e > \frac{b}{6} \]

In the starting \( e \) will be known \( \left( \frac{M_v}{P} \right) \). Hence \( D \) will be chosen such that \( e \leq \frac{b}{6} \). In that situation stress will be compressive through out and design will be done as discussed in previous case.

However if \( D \) is given, then if \( e \geq \frac{b}{6} \), there would be loss of contact. Hence final length of contact and maxm compresive stress will be calculated.

From this stress distribution maxm bending stress at critical rech 1-1 will be calculated and should be \( \leq \) per. Bending stress.

\[ \sigma_c = 3.75 \text{ MPa} \]

\[ f_y = 250 \text{ MPa} \]

\[ T = 15 \text{ mm} \]

\[ s_{bt} = 1 \text{ mm weld} / \text{ mm} = 76 \text{ N} \]

\[ f_{bt} = 0.7 f_y = 175 \text{ N} \text{ per mm} \text{ strav in bending tension} \]
Shearing force that can be resisted by a weld = 572 x 76

Design of base plate

\[ C = \frac{M}{P} = \frac{5.5}{700} = 7.857 \text{ mm} \]

Choose \( D \geq 6E \)
\[ \geq 471.43 \text{ mm} \]

Choose \( D = 500 \text{ mm} \)

Max. stress

\[
\frac{P}{BD} \left( 1 + \frac{6E}{D} \right) \leq \sigma_c
\]

\[
\frac{700}{B \times 500} \left( 1 + \frac{6 \times 78.57}{500} \right) \leq 3.75
\]

\[ B \geq 725.37 \text{ mm} \]

Adopt \( B = 730 \text{ mm} \)

Note: By increasing the value of \( D \), overhang portion will increase, but stress will decrease. However, by choosing smaller \( D \), overhang will reduce but stress will increase. Final selection is generally done based on economy i.e. \( V \) of steel \( B \times D \times t \) should be less. Ne
Streng D.11

\[ \frac{P}{8D} \left(\frac{1 - 6e}{D}\right) = 109 \text{ N/mm} \]

Note: if connection has been made using angles the length of plate should be chosen such that the connecting angle could be accommodated in the chosen length.

\[ \sigma_A = \frac{M_{n1} \cdot l/2}{I/12} - \frac{M_{n2} \cdot l/2}{I/12} \leq \sigma_{k3} \]

\[ x = 79.26 \text{ mm} \]

Force on 1 mm width:

\[ \frac{1}{2} \left(2.641 + 8.726\right) \times 150 \times 1 \]

\[ F = 477.45 \text{ N} \]

\[ \sigma_{n1} \leq 477.45 \times 79.26 = 37848.275 \text{ N/mm} \]

\[ U = \frac{2.641 \times 265^2}{2} = 92728.6 \text{ N/mm} \]
\[
\frac{6}{t^2} \left( M_u - u M_n \right) \leq 0.7 \times 250
\]
\[
t \geq 34.135 \text{ mm}
\]

Similarly
\[
\frac{6}{t^2} \left( M_{u1} - u M_n \right) \leq 0.7 \times 250
\]
\[
t \geq 53.43 \text{ mm}
\]

Let us adopt max. thickness
\[
t = 55 \text{ mm}
\]
\[
B = 730 \text{ mm}
\]
\[
a = 500 \text{ mm}
\]
\[
t = 55 \text{ mm}
\]

Design of weld.

As the column ends are milled all the direct load will be assumed to have been transferred through direct bearing on the base plate hence weld should be designed only to resist the moment.

\[
\frac{M}{d} = \frac{55 \times 10^6}{200} = 275 \text{ MN}
\]

\[
x \times 5 \times 76 \geq 275 \times 10^3
\]

\[
\text{min. size} = 10 \text{ mm}
\]
\[
\text{max. size} = 15 - 1.5 = 13.5 \text{ mm max.}
\]
\( F_{xtc} = 27.5 \times 10^3 \)

- \( F \geq 361.84 \text{ mm} \)
  - Adopt \( F = 370 \text{ mm} \)

10 mm fillet weld

---

Note:

If there is tension at the base, the bolt will come under tension and we need to design the bolt as well. The bolt can be designed if the tension in the bolt can be estimated. A detailed determination of \( T \) will be based on the following fig.

However a conservative estimate of tension in the bolt is as follows.
Once tension is known the dia of bolt and length of embedment can be calculated.

\[
\pi \phi \times \text{Lemb} \times \sigma_a \geq T \quad (i)
\]

Per adhesive stress b/w bolt & cone depends on grade of steel

\[
\frac{\pi \phi^2 \times \sigma_t}{t} \geq T \quad (ii)
\]

\( \phi \) = A Dia at the root of thread

From (i) & (ii) dia of bolt & length of embedment can be calculated.

If a solid round steel column is supported over a square base plate then for purely axial load thickness \( t \) is given by

\[
\frac{9 \omega L}{16 \sigma_{ts}} \frac{t}{(8 - 3t)}
\]
\( s + 1.5 (d_0 + 75) \text{ mm} \)

\( W \rightarrow \text{KN} \)

\( t \rightarrow \text{mm} \)

\( \sigma_{b3} \rightarrow \text{MPa} \) [144]
Design of Beam

For a beam to be safe

a) It should be safe in bending (primary criteria)
   safe in shear
   deflection

b) safe in local buckling (secondary criteria)
   of flange plate

c) safe in web crippling

d) safe in web buckling

Laterally restrained and unrestrained Beams

Beam can buckle laterally if the compression flange is weak hence to safeguard against lateral buckling

a) Beam can be laterally restrained by using cross beams

b) By inserting the compression flange inside the flange-gusset slab
(c) by making compression flange heavy.

\[ \bar{\sigma}_{bc} = \sigma_{bt} \]

per. stress in bending

\[ \sigma_{bt} = 165 \text{ MPa} \]

per. stress in bending comp.

For laterally restrained beam

\[ \sigma_{bc} < \sigma_{bl} \]

Design of beam (laterally restrained beam)

1) Beam is designed for bending and checked for other criteria.

\[ \frac{M_{\text{max}}}{\bar{\sigma}_{bc} \text{ or } \sigma_{bl}} = 2 \text{ rep.} \]

Choose section that provides \( Z > 2 \text{ rep.} \).

When \( Z \text{ rep.} \) can not be provided by using single rolled section, we will adopt built up section.

Note: Most suitable arrangement for deflection criteria.

One of the most common built up section is I seen with flange plates.
In I section and flange plate combination the largest I section available is used and deficiency in \( z \) is met by using plates. 

\[
\begin{align*}
\text{I}_{\text{eq}} &= \text{I}_{\text{rolled}} + \frac{5}{2} \text{I}_{\text{plate}} \quad \text{I}_{\text{eff}} = \text{I}_{\text{rolled}} + \frac{5}{2} \text{I}_{\text{plate}} \\
2_{\text{ref.}} &= 2_{\text{rolled}} + \frac{4p \times (D/2)^2 \times 2}{D/2} \\
\frac{2_{\text{ref.}} - 2_{\text{rolled}}}{D} &= Ap
\end{align*}
\]

The above approach gives the approximate value of \( Ap \) the area of plate chosen is more than \( Ap \).

The increase in \( Ap \) should cater for the area of hole on the tension flange because in tension net area is effectively built up.

The beam should be checked for safety in bending as follows:

\[
\frac{M \text{Y}_{\text{comb}}}{I_{\text{geom}}} \leq \sigma_{yl}
\]

\[
\frac{M \text{Y}_{\text{ten}}}{I_{\text{geom}}} \times \left( \frac{\text{Gross area of tension flange}}{\text{Net area of tension flange}} \right) \leq \sigma_{lt}
\]

\( I_{\text{geom}} \) corresponds to \( na \) of section neglecting holes.
Check for shear

\[ V_{\text{max}} \leq 0.4 f_y \]

\[ \frac{V_{\text{max}}}{b \times t_0} \rightarrow \text{Max. s.f. in Beam} \]

\[ \frac{V_{\text{max}}}{b \times t_0} = \text{Av. Shear Stress} \leq 0.4 f_y \text{ per av. shear stress} \]

Note:

\[ \text{Max. shear stress} \leq 0.95 f_y \]

\[ \left( \frac{V_{\text{max}}}{b \times t_0} \right) \]

\[ \text{Max. shear stress} \leq 0.95 f_y \]
Check for Deflection:

Max. Permissible deflection \( \Delta_{\text{max}} \) = \( \frac{\text{span}}{325} \)

\[ \Delta_{\text{max}} = \frac{5 \cdot w_a t^2}{384 \cdot E I} \leq \frac{1}{325}. \]

Check for secondary criteria:

Local buckling of flange plate -

Note: Secondary criteria need not to be checked for rolled section because sections are designed in such a way that overall failure of the section takes place before the local failure has had the chance to take place.

\[ \frac{a'_{\text{t}}}{t} \neq 50 \]
\[ \frac{b'_{\text{t}}}{t} \neq 50 \]

Plates are simply supported placed one over the other if however two plates are welded so that they behave as 1 unit

\[ \frac{b}{2t} \neq 16 \]

\[ \frac{b'}{3t} \neq 16 \]
This criteria should be checked at the time of selection of plate only.

Check for web crippling.

Web crippling takes place at the location of heavy point load. The heavy point load can come in the span and at the support. The point load are dispersed as shown below.

For no web crippling:

\[ \frac{P}{\alpha t w} \leq \sigma_{bw} \]  

(per bearing stress)

Web crippling takes place at location where section provides least area of resistance. Such location is first encountered at the root of fillet. Hence web crippling takes place at the root of fillet.

For safety against web crippling the bearing stress at the root of fillet should be less than permissible bearing stress (0.75 fy)
\[
\frac{P}{8(1 + 0.05(h_2 + t_2))} \leq \sigma_{bt}
\]

At support:

\[
\frac{P}{8(1 + 0.05(h_2 + t_2))} \leq \sigma_{br}
\]

**Note:** If the beam is carrying UDL, web crippling needs to be checked at the support.

The support width is decided based on the bearing failure of supporting structure as well as web crippling of steel section.

**Web Buckling:** Web buckling occurs due to heavy concentrated, diagonal compression due to shear. To check for safety in web buckling.

Diagram showing:
- Cross-section of a beam with annotations.
- Diagram illustrating force application and dimensions.

Area at N.A.: \( Y_{tw} = A \)
To check for web buckling, the web is treated as a column.

\[ \text{Stiffness ratio} = \frac{dw\sqrt{L}}{tw} \quad \text{where} \]

\[ dw = b - 2h \]

\[ tw \quad \text{thickness of web} \]

The area resisting compression is calculated as follows (Fig II):

For safety against web crippling,

\[ \text{Carrying capacity} = \sigma_{ax} \times A > p \]

\[ \sigma_{ax} = \text{Res. stress in axial compression obtained from} \]

\[ \text{Stiffness ratio} = \frac{dw\sqrt{L}}{tw} \]

\[ A \rightarrow \text{Applied load} \]

Note: The web can be treated as a column which is fixed at the two flange ends hence

\[ \delta_{eff} = \frac{dw}{L} \]

\[ \delta_{min} = \sqrt{\frac{I_{min}}{A}} \]

\[ a \]

\[ \delta_{eff} = \frac{(dw/1)}{tw/\sqrt{J_{1/2}}} = \frac{dw\sqrt{L}}{tw} \]

\[ \delta_{min} = \sqrt{\frac{a^{3}}{12 \times a \times tw}} = \frac{tw}{\sqrt{J_{1/2}}} \]
Generally if beam is safe in web wrinkling it will be safer in web buckling.

### Design of rivets

No of rivet per pitch length = 2

\[
\frac{V A Y \times p}{T_w} = \text{Shear force per pitch length} \leq n R_v
\]

\( n \) = no of rivet/pitch length

\( L \) = for safety

Rivet diameter will be assumed and pitch will be calculated for max. shear.

Same pitch can be adopted throughout the span.
In reality, as shear force decreases towards the midspan, the pitch should increase towards the midspan.

**Curtailment of Plate**

Theoretical cutoff point is the location at which B.M. is equal to MOR of continuing section.

\[
\begin{align*}
\text{B.M. at } x_3 & \text{ location} \\
& = \frac{wL}{2} \left( \frac{L}{2} - x_3 \right) - \frac{w}{2} \left( \frac{L}{2} - x_3 \right)^2 \\
& = \frac{wL}{2} \left[ \frac{x_3^2}{2} - x_3 \frac{L}{3} - \frac{L^2}{4} - x_3^2 + \frac{L x_3}{2} \right] \\
& = \frac{wL}{2} \left( \frac{L^2}{4} - x_3^2 \right) \\
\frac{wL}{2} \left( \frac{L^2}{4} - x_3^2 \right) &= 0.12 \times 72
\end{align*}
\]

find \( x_3 \)

At least one plate must continue through out the span.
BM = 300
$I_m = 89.9 \times 10^6 \text{mm}^4$
$Z = 59.7 \times 10^4$
$b = 190 \text{mm}$
$t = 7.7 \text{mm}$

$M_{\text{max}} = \frac{Wt^2}{8} = \frac{40 \times 6.75^2}{8} \text{kNm} = 227.813 \text{kNm}$

$Z_{\text{req.}} = \frac{M_{\text{max}}}{0.641} = \frac{227.813 \times 10^6}{165} = 1.38 \times 10^4 \text{mm}^2$

No rolled section can provide $Z = Z_{\text{req.}}$.

Hence plates will be used to increase the $Z$ value.

Hence let us use ISMB 400 and cover plate as shown.

Approx area of plate req. $Z_{\text{req.}} - Z_{\text{rolled}} = \frac{D}{b} = \frac{(1.38 \times 10^4 - 102 \times 10^4)}{b}$

$= \frac{36 \times 10^4}{400} = 900 \text{mm}^2$

Let us choose $A_p = 1200 \text{mm}^2$

Adopting $t = 8 \text{mm}$

Width req. $= \frac{1200}{8} = 150 \text{mm}$

Check for bending:

Area of tension flange = Area of flange plate + Area of flange of rolled sec.
As the thickness of flange of rolled section is not given the gross and net area of flange can be calculated from the area of flange plate only.

\[
\frac{\text{Gross area}}{\text{Net area}} = \frac{150 \times 8}{150 \times 8 - 21.5 \times 8 \times t}
\]

\[
= 1.9
\]

\[
I_{\text{gross}} = 2.05 \times 10^6 \times \frac{1}{12} \left[ \frac{150 \times 8^3}{12} + 150 \times 8 \times 204^2 \right]
\]

\[
= 304.89 \times 10^6 \text{ mm}^4
\]

\[
G_{\text{ax}} = 208 \text{ mm}
\]

\[
\frac{G_{\text{ax}}}{N_{\text{ax}}} = \frac{227.813 \times 10^6 \times 208 \times 1.4}{304.89 \times 10^6}
\]

\[
= 217.58 > 165
\]

Let us adopt area of plate 1800 mm\(^2\) and staggering of rivets.

hence

\[
B \times t = 1800 \text{ mm}^2
\]

adopt \( t = 10 \text{ mm} \)

\[
B = 180 \text{ mm}
\]

\[
\frac{G_{\text{ax}}}{N_{\text{ax}}} = \frac{1800}{1800 - 21.5 \times 10}
\]

\[
= 1.136
\]

\[
I_{\text{gross}} = 2.05 \times 10^6 \times \frac{1}{12} \left[ \frac{180 \times 10^3}{12} + 180 \times 205^2 \right]
\]

\[
= 356.32 \times 10^6 \text{ mm}^4
\]
check for bending: \[ \frac{m}{I} \times \frac{6A}{NA} \]

\[= \frac{227.813 \times 10^6 \times 210 \times 1136}{356.32 \times 10^6}\]

\[= 152.92 \text{ N/mm}^2 \leq 165 \text{ N/mm}^2\]

check for shear:

Area: \[420 \times 8.9 = 3738 \text{ mm}^2\]

Av. shear stress: \[\frac{V_{\text{max}}}{3738}\]

\[= \frac{135 \times 10^6}{3728} = 36.12 \text{ N} < 1000 \text{ N/mm}^2\]

Check for Deflection:

\[\Delta_{\text{max}} = \frac{5L^4}{384EI}\]

\[= \frac{5 \times 40 \times 6750^4}{384 \times 2 \times 10^5 \times 356.32 \times 10^6}\]

\[= 15.17 \text{ mm}\]

\[\frac{\text{Span}}{3.25} = \frac{6750}{325} = 20.77\]

\[15.77 < 20.77\]

\[\text{OK}\]

Note: \[\text{seen from the} \]

point of view.
Check for Secondary Criteria

Assuming \( g = 80 \text{mm} \)

\[ h_2 = 32.8 \text{mm} \]

---

Check for local buckling of flange plate

\[ b = \frac{180 - 80}{2} = 50 \text{mm} \]

\[ \frac{b}{t} = \frac{50}{10} = 5 < 16 \]

---

Check for web crippling:

The beam is carrying only horizontal point loading at an end support, hence web crippling will be checked at support.

---

\[ 135 \times 10^3 \]

\[ 8.9 \times 2.10 + \sqrt{31.8 \times 10} \]

\[ 53.39 \leq 187.5 \]

Safe in web crippling
Check for web buckling

\[ T = \frac{d_0 \sqrt{3}}{t_w} = \frac{(D - 2h_2) \sqrt{3}}{t_w} \]

\[ = \frac{(400 - 2 \times 32.8) \sqrt{3}}{8.9} = 65.07 \]

\[ \sigma_{a_1} = 110.207 \text{ MN/m}^2 \]

Area of resisting section

\[ = (210 + 210) \times 8.9 \]

\[ = 3738 \text{ mm}^2 \]

Carrying capacity

\[ = 3738 \times 110.207 = 411.95 \text{ KN} \]

\[ > 135 \text{ KN} \quad \text{Safe} \]

Rivet Design

\[ \frac{V_{gy}}{I} \leq f_v \]

because no of rivets per pitch length = 1

\[ 180 \times 10 = 1800 \text{ mm}^2 \]

\[ \bar{y} = 201 \text{ mm} \]

\[ f_{Ax} = 356.72 \times 10^6 \]

\[ V = 175 \times 10^5 \text{ KN} \]

\[ \text{Rivet dia} = 20 \text{ mm} \]

\[ k_v = 3.63 \text{ KN} \]
\[ p \leq 259.68 \text{ mm} \]

**Note:**

16t x 12t 200 mm is a recommendation for dead loading. In this case, limit would be 32t or 300 mm. Hence, max. pitch will be 320 mm.

\[ \frac{1}{160} \]

Min. pitch = 300 mm

Adopt \( p = 250 \text{ mm} \).

The pitch can be increased towards the mid span after even we take it constant throughout.

**Permissible stresses**

\[ \sigma_{tk} = \sigma_{bt} = 0.65 \sigma_y \]

\[ \sigma_{shear} = 0.4 \sigma_y \]

\[ \sigma_{bt} = 0.75 \sigma_y \]

All these permissible values are increased by \( 33\frac{1}{3} \% \) if effect of wind or \( \varepsilon \theta \sigma \) is taken into account.
For the restrained beam like this max. b.m. will occur at the support.

\[ \sigma = \frac{14.235 \times 10^6 \gamma}{ \frac{E}{I_{yx}} + \frac{10^6 \gamma x}{I_{yy}}} \]

at N.A. Bending stress in ksi.
\[ \left( \frac{M \times 10^6}{I_{yy}} \right)y + \left( \frac{W \times 10^6}{I_{yy}} \right)x = 0 \]

\[ ax + by = 0 \]

\[ y = -\frac{a}{b} x \]

Location of N.A. shows B. stress will be maxm either at A or B.

**Note:** Instead of finding the location of N.A. by above approach, we could have located it under the concept that N.A. lies b/a minor axis and the direction of resultant moment for safety \( \sigma_a \leq \sigma_{perm} \):

\[ \frac{19.235 \times 10^6}{1558.1 \times 10^2} + \frac{W \times 10^6}{170.7 \times 10^2} \leq 16.5 \text{ MPa} \]

\[ \sigma_a \leq \sigma_{perm} \]

\[ W \leq 26.6 \text{ KN} \]
Grillage Foundation

Permissible Shear:
If the grillage foundation is encased in concrete, the permissible shear is increased by $\frac{33}{3} \%$.
If effect of wind and $E$ is also taken into account, permissible shears are increased by 50%.

Note:
For bending stress, normally is 0.67 $\text{fy}$. 
For all the shear types in grillage foundation, $1.33 \times 0.67$ $\text{fy}$ is used. 
For bending stress in grillage foundation, when effect of wind or $E$ is also taken into account, $1.5 \times 0.67$ $\text{fy}$
Separation must keep the beam properly spaced such that the distance between edges of adjacent beam is not less than 75 mm.

\[ \frac{f}{L.B} \leq \text{Bearing capacity of soil} \]

Choose L of B on this criteria.

**Top tier Beams**

\[ M_{max} = \frac{1}{2} \frac{f}{L} \frac{(1-a)}{L} \times \frac{L}{2} \]

\[ M_{max, x} = \frac{f}{8} (1-a) \]

The above \( M_{max} \) and \( M_{max,x} \) are for all top tier beam taken together. Hence to design top tier beam no. of
beam needs to be selected.

To start with area of base plate can be calculated as follows

\[ A = \frac{P}{f_u} = axb \]

Once the size of base plate is known no of top tier beam can be selected, no of top tier beam should be such that it could be accommodated within width b

Hence max. b.m. in one top tier beam = \( \frac{P(l-a)}{8n} \)

Where n = no of top tier beam.

Max. s.f. in one top tier beam = \( \frac{P(l-a)}{2L \times n} \)

\( P \) is the load in which effect of dead wt already been taken into account.

\[ Z_{eq} = \left\{ \frac{P(l-a)}{8 \times n} \right\} \frac{165 \times 1.33}{165 \times 1.33} \]

Choose seen and check for shear.

Bottom Tier beam.

\[ \text{Where } \frac{p(b)}{f_u} \text{.} \]

\[ \text{Mom. for all bottom tier beam taken together } = \frac{P(b-a)}{8} \]
\[ V_m = \frac{P(l-a)}{2L} \]

Note: If there is a chance of web crippling it will be taken in top tier beam only hence top tier beam can be checked for web crippling also.

8. Load on a column is 2500 kN the safe bearing capacity of soil is 250 kN/m² and permissible bearing stress in conc. is 4000 kN/m². Design grillage foundation.

Set: Load on base plate = 1.1 x 2500 = 2750 kN
Cheesing square base plate

\[ a = \frac{1.1P}{25} = \frac{2750}{25} = 110 \text{ mm} \]

\[ a = 0.25a = 0.25 \times 110 = 27.5 \text{ mm} \]

\[ L = \frac{1.1 \times 2500}{25} = 3.316 \text{ m} \]

\[ V_{max} = \frac{1.1P(l-a)}{2L} = 1031.25 \text{ KN} \]

\[ M_{max} = \frac{1.1P(l-a)}{8} = 876.56 \text{ KNm} \]

\[ Z_{eq} \text{ for all beams taken together} = \frac{876.56 \times 10^6}{165 \times 1.33} = 4000 \text{ cm}^3 \]
Let us choose 3 no. of top tier beams.

2 eq. for $I = \frac{4000}{3} = 1333.33$ 

3. Let us choose ISMB 450

$A_{\text{max}} = 1350.7 \text{ cm}^2$

$b = 150$

So, 3 no. of ISMB 450 can be accommodated within the base plate.

\[ V_{\text{max,one}} = \frac{1.1 P(1-a)}{2Lx3} = 343.75 \text{ KN} \]

\[ A_{\text{v, shear stress}} = \frac{343.75 \times 1.4}{450 \times 1.4} = 81.26 \text{ kN/m} \]

So, for

\[ h_j = 35.4 \text{ mm} \]

\[ \frac{1.1 \times 2500}{3} = 916.7 \text{ kN} \]

\[ \frac{916.7 \times 10^3}{(0.4 + 25.3h_j)h_j} = 1.33 \times 0.75 f_y \\
100.26 \leq 247.175 \text{ MPa} \]
Max spacing of bottom tie beam is generally 200 to 250 mm.

No. of bottom tie beam = \(\frac{P(B-h)}{8}\)

\[ Z_{req} = \frac{876.56 \times 1.33}{133 \times 16.5} = 4000 \text{ cm}^3 \]

Space available = 3.9 m

No. of bottom tie beams are generally 10 - 20

Let us choose 15 no.

\[ Z_{req, \text{one}} = \frac{4000}{15} = 266.7 \text{ cm}^3 \]

Choose ISMB 225

\[ Z_{provided} = 305.9 \text{ cm}^3 \]

\[ 14\times75 + 15\times110 < 3400 \]

\[ 2700 < 3400 \text{ hence can be accommodated} \]

Check for shear

Shear in one bottom shear beam = \(\frac{1031.25}{15}\)

= 68.75 kN

Av. shear stress = \(\frac{68.75}{6.5 \times (225) \times 100}\)

\[ < 1.33 \times 100 \]

\[ \Box \]
Note: If the size of column is given to us, thickness can be calculated.
In conventional design method structure is designed for strength of steel till it yields at a point in the section.

However if yielding occurs at one point in section it does not mean collapse of the member. Due to plastic deformation and strain hardening of material, the particles which were less stressed will be brought into action so that structure is actually able to resist greater load.

In the modern design, strength of steel beyond the point of first yield is utilized and this method of design is called plastic method of design.

Simply Stress-strain curve.

\[
\text{Shear} \quad \text{by}
\]

- Strain hardening range has been omitted in this case which in fact will add to the margin of safety.
- Plastic design can be applied only to redundant structure.
- In the case of simple members load causing first yield is most critical because at that load only large unacceptably deformation occurs.
strain deformation even at the time of collapse is not much hence plastic analysis can be applied only to redundant str.

Assumptions in plastic analysis

1. Material should possess ductility so that it can be deformed in the plastic stage.
2. Strain distribution is linear.
3. Relation between tensile stress and tensile strain is comp. strain should be some
4. Joints should be sufficiently strong to transfer the moment. All joints should be rigid joint.

Plastic Bending of Beams

\[
C = \frac{1}{2} \frac{f_t}{b} (D-x) b
\]

\[
T = \frac{1}{2} f_t x b
\]

\[
\frac{f_c}{(D-x)} = \frac{f_t}{x}
\]

\[
f_c (D-x) b = f_t x b
\]

\[
(D-x)^2 = x^2
\]

\[
b(D-x) \cdot \frac{(D-x)}{2} = \frac{b x \cdot x}{2}
\]

Moment of complete about N.A = Moment of tension area about N.A.

This is the location of C.M.
Within elastic limit, Centroidal axis is the N.A.

N.A. will remain at centroidal axis if section is symmetrical about the axis of bending. (In fig. 1 & 2)

In fig. (v)

\[ C = T \]

\[ f_y A_c = f_y A_T \]

\[ A_c = \frac{A_T}{C} \]

N.A. is equal area axis

\[ C_B = T \cdot B = M_p \]

\( M_p \) is full plastic moment capacity of section

In fig. (iii)

N.A. is centroidal axis

\[ T \alpha = C \alpha = M_y \]

\[ M_y \rightarrow M.O.R. \text{ at the time of first yield} \]

\[ = \text{Yield moment} \]

\( M_p \) is plastic moment capacity which depends on section only.

A fully plastic section can not resist any further moment and if loading is applied beyond \( M_p \), unrestrained rotation will take place at the fully yielded section thus we assume as if a plastic hinge has formed. Plastic hinge can be understood as ruptured hinge in which rotation does not take place up to certain load and beyond that load a large deformation in the form of rotation will take place.
Formation of plastic hinge at one location does not mean the collapse of the structure. For collapse sufficient no. of plastic hinge must develop to make the structure unstable.

\[ \frac{W_s}{m_r} \]

Due to plastic hinge formation fixed end strand behaving as hinge.

For complete collapse of structure the margin available \( W_s - W_t \) beyond the point of first yield \( W_y - W_t \).

For complete yielding of the section margin available beyond the point of first yield \( W_s - W_t \).

Definition of Plastic hinge - A plastic hinge can be defined as a yielded zone due to flexure in a structure in which infinite rotation can take place at a constant resisting moment \( (m_p) \) of the section.

Length of plastic hinge:

Yielded zone

\[ P \]
Some important point about plastic hinge:

1. A section is said to develop a plastic hinge when due to flexural stress at every point of the section is equal to yield stress.

2. Plastic hinge develops first at section subjected to greatest curvature.

3. Due to formation of plastic hinge one after the other, the distribution of moments take place. Sufficient no. of plastic hinges have to be developed to render the structure to unstable or collapse state.

4. No. of plastic hinge required for complete collapse of the structure is \((s+1)\).

   Where \(s\) is the degree of redundancy.

\[ D_3 = 2 \]

For vertical loading:

\[ D_3 = 2 \]

No. of plastic hinge required for complete collapse of \(s+1 = 3\)

\[ D_1 = 2 \]

For partial collapse of \(s+1 = 2\)

No. of plastic hinge = 2

For complete collapse no. of plastic hinge = 2 + 1 = 3

Length of plastic hinge depends on loading & shape of section.
6) for purpose of analysis plastic hinge will be assumed as a point about which plastic rotation takes place.

7) M.O.E. of normal hinge is zero & M.O.E. of P.H. is M_p

8) Plastic hinge is expected to form at
   a) Fixed ends
   b) At the location of point load.
   c) At the point of sudden change in geometry.
   d) At points of zero shear in a span subjected to U.D.L. or U.V.L.

9) When two sections join at a point plastic hinge forms in a section of smaller M_p

If section is loaded up to a moment of M_y and then unloaded complete recovery will take place. However if moment applied on the section is > M_y, then unloading takes place recovery will not be complete.
Shape factor,

\[ \text{Shape factor} = \frac{M_p}{M_y} \]

It shows the reserve of strength of a rebar beyond the point of first yield.

In elastic range

\[ M_y = f_y \cdot z \]

\[ z \rightarrow \text{Section modulus} \]

\[ z = \frac{I_{NA}}{y_{max}} \]

\[ M_y = \frac{I_{NA}}{y_{max}} \cdot 0.85 \text{ about } C_{G1} \]

In plastic range

\[ M_p = \frac{C(\bar{y}_1 + \bar{y}_2)}{1} + T(\bar{y}_1 + \bar{y}_2) \]

\[ = f_y \cdot \frac{A}{2} (\bar{y}_1 + \bar{y}_2) \]

\[ M_p = f_y \cdot s \] when \( s \rightarrow \) Plastic Modulus

\[ = \frac{A}{2} (\bar{y}_1 + \bar{y}_2) \]
shape factor = \frac{A}{I_y} = \frac{\bar{y} - \bar{y}_c}{I_y} = \frac{5}{2}

\frac{A}{2} \left( \frac{\bar{y}_1 + \bar{y}_u}{2} \right) - Z

\text{shape factor of rectangular section -}

\begin{align*}
S.F. &= \frac{A}{2} \left( \frac{\bar{y}_1 + \bar{y}_u}{2} \right) - \frac{bd}{2} \left( \frac{d}{2} \right) \\
&= \frac{6}{4} = \frac{3}{2} = 1.5
\end{align*}

\text{circular section -}

\begin{align*}
S.F. &= \frac{A}{2} \left( \frac{\bar{y}_1 + \bar{y}_u}{2} \right) - \frac{\pi d^4}{8} \left( \frac{4(\pi d)}{3\pi} \right) \\
&= \frac{\pi d^4}{8} \left( \frac{1}{3\pi} \right) - \frac{7}{8} \left( \frac{1}{\pi} \right) \\
&= \frac{1}{6\pi} - \frac{1}{2\pi} = 1.618
\end{align*}

Plastic modulus for circular section = \frac{d^3}{6}
\[ S_f = \frac{A}{2} \frac{y_1 + y_2}{z} \]

\[ = \frac{A}{2} \left( \frac{a}{\sqrt{2}} + \frac{1}{3} x z \right) \]

\[ = \frac{a^4}{12} \frac{\sqrt{2}}{a} \]

\[ = \frac{a^4}{3 \sqrt{2}} \frac{\sqrt{2}}{12} \]

\[ = \frac{a^2}{3 \sqrt{2}} \frac{1}{\sqrt{2}} \]

\[ = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} = \frac{2}{1} = 1 \]

\[ \frac{t}{d} = 1 \]

\[ S = \text{plastic modulus} \]

\[ = \frac{d^3}{6} - \frac{(d-2t)^3}{6} \]

\[ = \frac{d^3}{6} \left( 1 - \left( \frac{d-2t}{d} \right)^2 \right) \]

\[ = \frac{d^3}{6} \left( 1 - \left\{ 1 - \frac{8t^3}{d^3} + \frac{3 \ell^2}{d^3} + \frac{3t^2 \ell}{d^3} \right\} \right) \]

\[ = \frac{d^3}{6} \left[ 6 \frac{1}{d^3} \right] = d^2 \ell \]

\[ \frac{1}{2} \]

\[ J_{\text{max}} = \frac{\pi d^4}{64} - \frac{T \ell (d-2t)^4}{64} \]

\[ = \frac{d}{2} \]
\[ \frac{\eta d^2}{32} \left[ 1 - \left( 1 - \frac{2t}{d} \right)^{d} \right] \]

\[ \frac{\eta d^3}{32} \left[ 1 - \left( 1 - \frac{8t}{d} \right)^{d} \right] \]

\[ \frac{\eta d^2}{32} \times \frac{8t}{d} = \frac{\eta d^2 t}{4} \]

S.F. = \frac{d^2 t}{\pi d^2 t}

\[ S = \frac{5}{2} \]

\[ Z = \frac{1}{\gamma} = \frac{a \cdot (\frac{a}{2})^2}{\gamma} \]

\[ = \frac{a \cdot \left( \frac{a}{2} \right)^2}{2 \frac{a^3}{2} \times \frac{a^3}{24}} = \frac{a^3}{3} \times \frac{a^3}{2} \]

\[ = \frac{a (\frac{a}{2})^2}{2 \frac{a^3}{2} \times \frac{a^3}{24}} \]

\[ \text{Area} = \frac{1}{2} \left\{ \frac{a}{2} \cdot \frac{a}{2} \times \frac{1}{2} \right\} = \frac{a^2 \sqrt{3}}{8} \]

\[ \text{Area of equilateral triangle} = \frac{a^2 \sqrt{3}}{4} \]

\[ \gamma = \frac{1}{3} \left( \frac{2a + x}{a + x} \right) \times \left( \frac{\sqrt{3}a}{2} - \frac{x \sqrt{3}}{2} \right) \]

\[ a = \frac{\sqrt{3} \times x}{2} \]

\[ e = \frac{1}{3} \left( \frac{2 \sqrt{3} + 1}{\sqrt{3} + 1} \right) \frac{x}{2} (\sqrt{3} - 1) = 0.19x \approx 0.268a \]
Length of plastic hinge

\[ \frac{m_p}{m_y} = \frac{\frac{l}{2}}{\left(1 - \frac{L_p}{l}\right)^2} = 1.5 \]

\[ L_p = \frac{l}{3} \]
\[
\frac{M_p}{M_y} = \frac{\omega_1^2}{8} \frac{w_0^2}{8} - \frac{\omega_2^2}{8} - \frac{\omega_2 w_0}{8} \times y
\]

\[
\phi = \sqrt{1 - \frac{M_y}{M_p}}
\]

\[
l_p^2 = \frac{1.5 l_2^2 - 1.5 l_p^2}{12}
\]

\[
l_p^2 = \frac{l_2^2}{12}
\]

\[
l_p = \frac{l}{\sqrt{3}}
\]

Load factor and factor of safety

Load factor = \frac{\text{collapse load}}{\text{working load}}

Note:

Plastic Design

\[
\left(5 \times \frac{t}{t_f} \times \text{load factor} \right)
\]

\[
\left(5 \times 1.5 \right) \times \omega_2 \leq M_p
\]

Elastic Design

\[
\frac{w_0^2}{8} \leq \frac{f_l}{s}
\]

\[
\frac{f_y}{s} \leq \text{F. of S.}
\]
load factor = \frac{\text{catastrophic load}}{\text{working load}} = \frac{W_c}{W_w}

= \frac{M_p}{a_{\text{per.2}}} \cdot \frac{f_{y.s}}{\left(\frac{f_y}{f_{o.s}}\right)^2}

\text{load factor} = (\text{factor of safety}) \times \text{shape factor}

\text{Moment curvature Relationship (Rectangular Section)}

\text{So long as moment is less than } M_y

\text{Elastic condition prevails}

\frac{M}{I} = \frac{f_y}{Y} = \frac{F}{K}

M = \frac{E_0}{E} \cdot \frac{E_0}{K} \cdot \text{curvature}

\frac{M}{M_y} = \frac{IC}{K_y}

\text{As the stress start yielding moment-curvature relationship becomes non-linear}

\text{Even in the yielded zone the strain variation is assumed}
to be linear.

\[ \text{strain } \varepsilon = \frac{y}{y} = yk \]

\[ k = \frac{e}{y} \]

Strain \( \alpha d_A = \frac{f y}{E} = k e \)

Curvature \( (\kappa) = \left\{ \frac{f y}{E e} \right\} \)

\[ \frac{k}{k y} = \frac{h}{2e} \]

\[ M = c_1 \alpha_1 + c_2 \alpha_2 : T_1 \alpha_1 + T_2 \alpha_2 \]

\[ c_1 = \frac{f y}{E} \left( \frac{h}{2} - e \right) b \]

\[ d_1 = h - \left( \frac{h}{2} - e \right) = \left( \frac{h}{2} + e \right) \]

\[ c_2 = \frac{1}{2} \frac{f y}{E} e b \]

\[ \alpha_2 = \frac{4e}{3} \]
\[ M = G x_1 + C_2 x_3 \]
\[ = f_y b \left( \frac{h^2}{4} - c^3 \right) + \frac{2 f_y e^2 b}{3} \]
\[ = f_y \frac{b h^2}{c} \left[ \frac{6}{4} - \frac{6 e^2}{h^2} + \frac{2}{3} \frac{e^2}{h^2} \times 6 \right] \]
\[ = f_y \frac{b h^2}{c} \left[ 1.5 - \frac{2 e^2}{h^2} \right] \]
\[ \frac{M}{M_y} \left[ 1.5 - \frac{2 e^2}{h^2} \right] = \frac{M}{M_y} \]
\[ \frac{c}{h} = \frac{1}{2 e} \]
\[ \frac{e}{h} = \frac{1}{2 e} \]

\[ \frac{M}{M_y} = \left( 1.5 - \frac{F_y e^2}{2 c^2} \right) \]

Non-linear curve

**Question:** Design a simply supported beam of span 6m to support a point load of 100 KN acting at its mid-span.

\[ f_y = 250 \text{ MPa} \quad \text{and load factor} = 1.75 \]
collapse load = 100 x 1.75 = 175 kN

Beam will collapse when plastic hinge forms at midspan, below the point load.

B.M. at A = M_p = \frac{175 x 8}{4} = 262.5 kNm

M_p = f_y \cdot S

S = \frac{262.5 \times 10^6}{250} = 10.5 \times 10^5 \text{ mm}^2

\frac{S}{Z} = \text{shape factor}

Z = \frac{S}{SF} = \frac{10.5 \times 10^5}{1.15}

Note: shape factor for I section = 1.15

Z_{eq} = \frac{10.5 \times 10^5}{1.15} = 91309 \text{ cm}^2

Choose: I5MB - 400

Z = 1022.9 \text{ cm}^2
Important theorems in plastic analysis
In the plastic analysis following conditions must be satisfied.

1. Equilibrium condition

\[ \Sigma F = 0 \]
\[ \Sigma M = 0 \]

In all types of analysis equilibrium eq. is always satisfied.

2. Mechanism condition

At collapse sufficient no of plastic hinge must be develop so as to transform a part or whole of the structure into a mechanism leading to collapse.

For complete collapse of the structure no of plastic hinge required to be formed = \((n+1)\)

\[ n \rightarrow \text{Degree of redundancy of structure.} \]

3. Yield condition

At collapse B.M. at any section must not exceed the fully plastic moment capacity of the section.

If all the above 3 cond. are satisfied, a unique lowest value of collapse load will be achieved.

Based on the above 3 cond. we get the following theorems based on which plastic analysis is performed/done.

a) Upper Bound theorem, or kinematic theorem
b) Lower Bound theorem or static theorem.
Upper Bound Theorem: This theorem satisfies equilibrium and mechanism condition.
Load determined by assuming a mechanism will always be greater than equal to the collapse load \( (P \geq P_c) \).

\( P_c \) - Collapse load.

This theorem can also be stated as:
Of the various possible mechanism, the correct mechanism is the one for which the collapse load is min.

Lower Bound Theorem:
Load determined on the basis of any collapsed B.M.O., in which B.M. at any \( x \) is less than plastic moment, will be less than or equal to actual collapse load i.e. \( P \leq P_c \).

This theorem satisfies equilibrium and yield cond.

Methods of Analysis:

a) Static Method.
b) Kinematic Method.

Static Method:

\( G \) steps:
1) Select the redundant force
   Moment will be taken as redundant.

\( D_3 = 1 \)
3) Draw few BMD and redundant BMD.

3) A combing BMD is drawn in such a way that a mechanism is formed.

4) Collapse load is found out by working out the equilibrium equation.

5) It is checked that B.M at every such is less than $M_r$.
Kinematic Method.

Locate the possible places of plastic hinges and ascertain the various possible mechanisms.

The collapse load is found out by applying the principle of virtual work.

A bird of collapse mechanism is drawn and it is checked that $\sigma_1$ at any section is not more than $M_p$.

\[ \sigma_1 \]

No. of plastic hinge required = 2

\[
\begin{align*}
- M_p \theta & - M_p \theta - M_p \theta \quad (L_c \theta = 0) \\
& = 3M_p \theta - \frac{w_c \theta x}{2} \\
& \quad W_c = \frac{6M_p}{x}
\end{align*}
\]
Kinematic Method.

Locate the possible places of plastic hinges and ascertain the various possible mechanisms.

Collapse load is found out by applying the principle of virtual work.

BMD of collapse mechanism is drawn and it is checked that BM at anyseen is not more than $M_p$.

$\text{No. of plastic hinge required} = 2$

\[ - M_p \theta - M_p \phi - M_p \phi \int \frac{W_c}{M_p} \, d\theta = 0 \]

\[ = 3M_p \phi \cdot \frac{W_c \cdot \ell}{M_p} \]

\[ W_c = \frac{6M_p}{\ell} \]
1) Find \( M_p \) if \( c \) is the collapse load.

2) Find the position of load for collapse load to be min.

\[ q = 1 \quad \text{(for vertical loading)} \]

No of hinge required for complete collapse \( = k + 1 = 2 \)

\[ \text{Cinematic method} \]

\[ -M_p \theta - M_p \phi - M_p \phi + 1 c \cdot b \phi = 0 \]

\[ a \theta = b \phi \]

\[ -\frac{2M_p \theta}{b} - M_p \phi \frac{a \theta}{b} + c a \theta = 0 \]

\[ M_p = \frac{c a b}{a + 2b} \]
\[ (ii) \quad \frac{C_{ub}}{a+2b} = \frac{M_p}{c} \]
\[ c = \frac{M_p}{ab} (a+2b) \]
\[ = \frac{M_p (a+2b)}{(a+2b) b} \]
\[ \text{for } c \text{ to be min} \]
\[ \frac{dc}{db} = 0 \]
\[ \left( \frac{M_p (a+2b)}{(a+2b) b} \right) - \left( \frac{M_p (a+2b)}{(a+2b) b} \right) (l-2b) = 0 \]
\[ (l-b) b - (l+b) (l-2b) = 0 \]
\[ l b - b^2 - (l^2 - 2bl + b^2) = 0 \]
\[ l b - b^2 - l^2 + b^2 + 2b^2 = 0 \]
\[ b^2 + 2bl - l^2 = 0 \]
\[ b = \frac{-2l \pm \sqrt{4l^2 - 4l^2}}{2} \]
\[ b = -l \pm \sqrt{l^2} \]
\[ b = l (\frac{l}{2} - 1) \]
\[ b = 0.414 l \]

That in case of hinged cantilever with point load collapse load will be min when load is at 0.414l from the hinged end.
Find $M_p$

In this case, location of plastic hinge needs to be decided first.

**Method 1**

The plastic hinge will form at support and second plastic hinge will form somewhere in span, the location of plastic hinge will be at the point of zero shear because the beam is loaded with U.D.L.

\[ SF = 0 \text{ at } x \text{ distance from hinged end} \]

\[ V - Cx = 0 \]
\[ VX - \frac{Cx^2}{L} = M_p \]
\[ Cx^3 - \frac{Cx^2}{L} = M_p \]
\[ \frac{C}{L}x^2 = M_p \]

At support A
\[ V - Cx = -M_p \]
\[ Cx - \frac{Cx^2}{L} = -Cx^2 \]

\[ C \frac{M_p}{L} = \frac{C}{L}x^2 \]
\[ 2M_p + C \left( \frac{x^2}{L} \right) = 0 \]
\[ C \left( \frac{x^2}{L} \right) - C \left( \frac{L-x}{L} \right)^2 = 0 \]
\[ 2x^2 - L^2 - x^2 + 2Lx = 0 \]
\[ x^2 + 2Lx - L^2 = 0 \]
\[ x = 0, 1.414L \]
\[ x^3 + 2.8x - l^2 = 0. \]
\[ x = 0.414l \]

By Kinematic Method

\[-M_p \theta - M_p \phi - M_p \phi + \int_{0}^{\theta} \left( \text{area under deflected curve} \right) \, dx = 0 \]

\[-2M_p \theta - M_p \phi + \int_{0}^{\theta} \left( \frac{l}{2} \right) \, dx = 0 \]

\[ \phi = \frac{l}{x} \theta \]

\[ \theta = \frac{l}{x} \theta \]

\[-2M_p \theta - M_p \left( \frac{l-x}{x} \right) \theta + \int_{0}^{\theta} \left( \frac{l}{2} \right) \, dx = 0 \]

\[-M_p \left( 2 + \frac{l-x}{x} \right) + \int_{0}^{\theta} \left( \frac{l}{2} \right) \, dx = 0 \]

\[-M_p \left( \frac{l+x}{x} \right) + \int_{0}^{\theta} \left( \frac{l-x}{x} \right) \, dx = 0 \]

\[ M_p \frac{l}{2} = \frac{\int (l-x) \, dx}{2x} \]

For \( M_p \) to be max, \( \frac{dM_p}{dx} = 0 \)

\[ \frac{cl}{2} \left\{ \frac{1}{x} \right\} - \int_{0}^{\theta} \left( \frac{l-x}{x} \right) \, dx = 0 \]

\[ \frac{2(\theta + x)}{2} - \int_{0}^{\theta} \left( \frac{l-x}{x} \right) \, dx = 0 \]

\[ (l+x)(l-x) - (l-x) \frac{l}{x} = 0 \]

\[ l^2 - 2lx + x^2 - l^2 + l^2 = 0 \]

\[ -x^2 - 2lx + l^2 = 0 \]

\[ x = 0.414l \]
\[ M_p = \frac{c \left( \frac{a + 1}{a} \right)}{2} = \frac{c \left( \frac{3.1415}{a} \right)^2}{2} \cdot (0.085^2) \cdot \frac{c}{a} = \frac{(3-2.5)}{2} \cdot \frac{c}{a}^2 = 0.6863 \left( \frac{c}{a}^2 \right) \frac{8}{8} \]

Find value of \( a \) for simultaneous collapse of \( AB \) & \( AD \)

**Collapse BMD**

For simultaneous collapse of \( AB \) & \( AD \), plastic hinge should form at \( B \) at \( A \) & in b/w \( A \) & \( B \)

\[ M_p \]

\[ \frac{c^2}{8} \]

\[ \frac{c}{8} \]

\[ \frac{1}{8} \]

\[ A \]

\[ B \]

\[ C \]

\[ D \]

**Note:**

Plastic hinge forms at \( 0.414 L \) when moment at the hinged end = 0. But if there is moment at the hinged end, plastic hinge will not form at 0.414 \( L \).
\[
\frac{ca^2}{2} = M_t \\
\frac{cg^2}{8} = 2M_t \\
\frac{cg^2}{8} = 2ca^2 \Rightarrow \frac{g^2}{8} = \frac{2ca^2}{2} \\
a^2 = \frac{g^2}{8} \\
a = \frac{g}{2\sqrt{2}} = 0.353 \text{ l}
\]

Note:

If \( a < 0.353 \text{ l} \),

\( \text{AB will collapse} \)

If \( a = 0.353 \text{ l} \),

\( \text{AB and BD will collapse} \)

If \( a > 0.353 \text{ l} \),

\( \text{BD will collapse} \).
Plastic moment = \( M_p \)

Find collapse load = \( w \)

\( R = 1 \)

No of p.h. for complete collapse = \( 1 + 1 = 2 \)

One will fail at support and other at \( x \) is from A

\[
2M_p = \frac{w (l-x)^2}{2} = 0
\]

\[
\sum M_A = 0
\]

\[
M_p = \frac{wx^2}{2} + \frac{w(l/3)^2}{2} = 0
\]

From (1) and (II)

\[
\frac{w(x-x)^2}{2} = \frac{wx^2}{2} - \frac{wx^2}{2}
\]

\[
(l-x)^2 = 2x^2 - 2l^2
\]

\[
g(l-x)^2 = 18x^2 - 2l^2
\]

\[
g(\frac{x^2 + x^2 - 2lx}{3}) = 18x^2 - 2l^2
\]

\[
11l^2 - 9x^2 + 18lx = 0
\]

\[
9x^2 + 18lx = 11l^2 = 0
\]

\[
x = 0.4907l
\]

By putting \( M_p = \frac{w(x-x)^2}{4} \)

Collapse load \( w = \frac{4M_p}{(l-x)^2} = 15.421 \frac{M_p}{l^2} \)
\[(g-x)\phi = \phi - \frac{(l-x)\phi}{x}\]

\[-mp\theta - mp\theta - mr\phi + \frac{1}{2} mlx\phi - \frac{1}{2} \frac{l}{3} x \frac{\phi}{3} = 0\]

\[-2 \frac{mp}{x} \theta - \frac{mr}{2} \phi + \frac{ml}{2} (l-x)\phi - \frac{m-1}{18} \frac{(l-x)\phi}{x} = 0\]

\[-mp\theta \left[ \frac{l+x}{x} \right] + \frac{ml}{2} (l-x)\phi - \frac{m-1}{18} \frac{(l-x)\phi}{x} = 0\]

\[m\phi = \frac{w (l-x)}{2} \left[ \frac{1 - \frac{\phi}{l+x}}{l+x} \right]\]

\[= \omega \frac{(l-x)(9x-l)}{18 (l+x)}\]

\[\phi x \left( \frac{(l-x)(9x-l)}{l+x} \right) = \frac{k}{(l+x)}\]

\[
\frac{d\phi}{dx} = 0
\]

\[(l+x)(-18x^2 + 10lx) - (-9x^2 + 10lx - l^2) = 0\]

\[-18lx^2 + 10l^2 - 18x^2 + 10lx + qx^2 - 1y^2 = 0\]

\[-9x^2 - 18lx + 11l^2 = 0\]
Beam of uniform \( x \) sec.
\( M_P \) is constant

\( R = 2 \)
No. of P.M. = 3

find collapse load \( w \)

3 hinges will form 2 at the support 4

I will form b/w A B b/c w x s.f. will be zero only in A B

\[ m_1 \left( \begin{array}{c} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{array} \right) m_2 \]

3 \( m_n = 0 \) of left FBD

5 \( M_D = 0 \)

\[ 2M_P = \frac{w x^2}{l} \]

\[ M_P = \frac{w x^2}{2} \]

\[ \frac{4}{l} \left( \frac{1}{2} - x \right)^2 = \frac{w}{l} \left[ \frac{3l^2}{4} - 2lx - 6lx + 9x^2 \right] \]

\[ 4x^2 = 4x^2 - 8lx + 3l^2 \]

\[ 8lx = 3l^2 \]

\[ x = \left( \frac{3}{8} \right) \]

\[ \text{collapse load } w = \frac{4M_p}{x^2} = \frac{4M_f \times 64}{9l^2} = \frac{256M_f}{9l^2} = 28.44M_p \]
By Kinematic Method

\[
\begin{align*}
-m_p \dot{\theta} - m_p \dot{\theta} &= -m_p \phi - m_p \dot{\phi} + \frac{1}{2} \omega \left( \frac{g}{2} - x \right) \left( \frac{g}{2} + x \right) \phi + \left( \frac{1}{2} - x \right) \dot{\phi} + \frac{1}{2} \phi \right] \frac{1}{2} \frac{x}{x} = \omega \\
-2 m_p \left( \frac{g}{2} - x \right) \phi &= -2 m_p \left( \frac{g}{2} - x \right) \phi + \frac{1}{2} \omega \phi \left( \frac{g^2}{4} - x^2 \right) + \frac{\omega \phi}{2} \left( \frac{g}{2} + x \right) = 0 \\
-2 m_p \left( \frac{g + x}{2} \right) - 2 m_p + \frac{\omega}{2} \left[ \frac{g^2}{4} - x^2 + \frac{g}{2} + x^2 \right] &= 0 \\
\frac{g}{2} - x &= \omega + \frac{g}{2} - x \\
4 m_p \left( \frac{g}{2} - x \right) &= \frac{\omega}{4} \left( \frac{g^2}{4} + 4 x \right) \\
\omega &= \frac{32 m_p}{3} \frac{1}{(g - 2x)(1^2 + 4 x)} \\
\frac{\omega}{dn} - \frac{k_1}{l^3 + 4 x^2 - 2 x x^2 - 8 x^2} &= 0 \\
\frac{1}{2} l^2 - 16 x^2 &= 0 \\
\frac{1}{2} l^2 &= 16 x^2 \\
\frac{\phi}{x} &= \frac{1}{8} \\
\left( \frac{g}{2} - x \right) &= \frac{4}{2} - \frac{1}{8} = \frac{a - t}{8} \\
A_{22} &= \left( \frac{3}{4} \right) \\
A_{21} &= \left( \frac{3}{4} \right)
\end{align*}
\]
In case of fixed beam with constant cross-section, location of plastic hinge in the span will be at the location of max. moment in free BM.
Find collapse load \( w \)

Degree of redundancy = 2

No. of p.h. connections = 3

Case 1 mode of failure

When two members join at a point, plastic hinge forms in the member of smaller \( m_p \).

\[
-5 \cdot m_p \cdot 0 + \frac{1}{2} \cdot \frac{3}{4} \cdot 8 \cdot \frac{3}{2} \cdot w = 0
\]

\[
w = \frac{40m_p}{9}
\]

Case 2

\[
2 \cdot m_p \cdot \left( \frac{w}{x} \right)
\]

\[
4 \cdot m_p = \frac{w \cdot x^2}{2}
\]

\[
m_p = \frac{w \cdot x^2}{8}
\]

\[
3 \cdot m_p = w \left( \frac{1}{2} - x \right) \left[ \frac{3}{4} + \frac{1}{4} \cdot x \right]
\]

\[
\frac{3 \cdot 6x^2}{8} = w \left( \frac{3}{4} - x \right) \left( \frac{3}{4} - \frac{x}{2} \right)
\]

\[
\frac{3x^2}{8} = (1-2x)(3x-2x)
\]

\[
3x^2 = \frac{8}{8} \cdot \frac{3x^2}{3} = 3x^2 - 8x^2 + 4x^2
\]
Out of various possible mechanism correct mechanism is the one for which loading is min hence mechanism 1 is the correct mechanism & correct collapse load will be

\[
\left( \frac{40 \text{ MP}^2}{g^2} \right)
\]

Beam is of uniform x seen find \( w \) collapse load

Note

Two mechanism needs to be checked

Mechanism 1 - L.P.M. at A + B

Mechanism 2 - L.P.M. at A + C
Continuous Beam.

Seam is of uniform x-sect find the collapse load.

In case of continuous beam failure of individual span is considered and the correct mechanism will be the mechanism corresponding to which loading will be minimum.

In this case failure of AB & BC is considered as two mechanisms.

Mechanism 1. Failure of AB

Beam segment AB will fail if plastic hinges are developed one at B and other somewhere b/w A & B

\[ \delta = \frac{w_1 \cdot 4.442}{2} - \frac{w \cdot (4.442)^2}{2} \]

\[ \delta = 1213 \cdot w \cdot l^2 \]

If plastic moment at B and in the span are same, plastic hinge will form at \( 4.442 \cdot l \) from the hinged end A.
The continuous beam shown above has uniform cross-section; find the value of \( M_p \). The given loads are collapse load.

For the continuous beam each of the span will be considered separately for its failure and \( M_p \) required for each of the span will be calculated, if a uniform section is to be provided, we will adopt \( M_p \) corresponding to largest \( M_p \).

For failure of AB two hinges are required. It will form at B and other at \( 0.414 \times l \) from hinged end (end A).

\[
M_p = \frac{6864 \text{ w} \cdot l^2}{2} \times \frac{1}{3} = 128.7 \text{ kNm}
\]

If instead of collapse load working load is given and \( M_p \) required is to be found out, we will have to load the structure with collapse load.

\[
\text{Collapse load} = \text{load factor} \times \text{working load}
\]
Span 8c - 8c will fail with x = 311. Two will form at B & C and one at location of zero shear.

180 + 25 = 205

Location of zero shear:

\[ x = \frac{17.5}{30} = 0.5833 \text{ m} \]

\[ \text{Free B.M.D ordinates: } 175 \times 5.833 - 30 \times \frac{5.833^2}{2} = 510.47 \text{ kNm} \]

\[ 2M_p = 510.47 \]

\[ M_p = 255.237 \text{ kNm} \]

\[ (\ldots...) \]

\[ 2M_p = \frac{30 \times 5.833^2}{2} \]

\[ M_p = 255.17 \text{ kNm} \]

For failure of CD hinge will form at C & D below either of point loading.

CD Mechanism 1

\[ \text{Free B.M.D ordinates: } \]

\[ \text{Moment Diagram: } \]
\[ \frac{m_p}{2} \times \frac{m_p}{2} = 200 \]
\[ \frac{2}{2} \times m_p = 200 \times 100 \]
\[ m_p = 200 \]

Mech. (2)

\[ \frac{m_p}{4} + \frac{m_p}{4} = 250 \]
\[ \frac{5m_p}{4} = 250 \]
\[ m_p = 200 \]

hence largest \( m_p \) will be selected thus \( m_p \) required is 255.178 KNm

8. A continuous beam as shown below is subjected to a collapse load system where each span has a uniform span. If under the action of collapse load system all the spans should collapse, determine the plastic moment required for each span. Assume middle span to be lightest

\[ M_p \text{ BE } \leq M_p \text{ AB } \]

[Diagram of the beam with loads and spans]
When two columns meet at a point, a plastic hinge forms corresponding to smaller $M_p$.

**Span BC**

\[ 2M_{pe} = \frac{wt^2}{8} \]

\[ M_{pe} = \frac{wt^2}{16} = \frac{160 \times 25}{16} = 250 \text{ KNm} \]

**Span AB**

\[ M_{pAB} = \frac{M_{pa} + 250}{2} = 1000 \]

\[ M_{pa} = 587.33 \text{ KNm} \]

**Span CD**

\[ M_{pcd} = \frac{160 \times 2 \times 18.75^2}{2} = 382.8125 \text{ KNm} \]
Given loads are collapse loading.

Mech 1

\[ \frac{M_p}{L} = \frac{2 \times 20 \times 3}{4} \]
\[ M_p = 100 \text{ KNm} \]

Mech 2

\[ \frac{3M_p}{L} = \frac{100 \times 4}{4} \]
\[ M_p = \frac{200}{1} = 66.67 \text{ KNm} \]

Mech 3 combined.

\[ 2M_{p0} + 2M_{p0} = 200 \times 1.5 \theta + 100 \times 2 \theta \]
\[ 4M_{p0} = 500 \theta \]
\[ M_{p0} = \frac{500}{4} \theta \text{ KNm} \]

Section will be provided corresponding to \( M_p = 125 \text{ KNm} \).
The collapse of beam will occur only corresponding to Max. 

To draw collapse end moment at B needs to be calculated.
Plastic Analysis for Portal Frames.

In the portal frame various mechanism considered are:

1) Beam Mechanism
2) Sway Mechanism
3) Gable Mechanism
4) Combined Mechanism
5) Joint Mechanism.

Beam mechanism in AC

Sway Mechanism.

Beam mechanism in CE

Mech. 1

Mech. II

Mech. III
When beam & sway mechanism are considered combinely as in this case if we consider mech 2 & 3, no of hinges will be five but max. no. of hinge that can be formed is 4 only hence one of the hinge needs to be eliminated. The common hinge b/w beam & sway mechanism are 2 & 3.

At 2, the moment from beam and sway mechanism are opposite in nature but they are of same nature at 3 hence in combined mechanism hinge at 2 will be eliminated.
Combined Mechanism

\[(1+3)\]

\[\text{214}\]

\[\theta = L_2 \phi\]

Find collapse load \(w\) and draw collapse BM for the portal frame shown above.

\[k = 3\]

max. no. of plastic hinge = 4

beam mechanism BC
At a joint due to smaller \( x \)'

\[ 20 = 4 \phi \]

\[ -2M_p \theta - 2M_p \phi - 2M_p \phi - 1.5M_p \phi + W - 30 = 0 \]

\[ + 4M_p \theta + 3.5M_p \theta + \frac{30}{4} = 3W_0 \]

\[ \theta = \frac{53}{24} \quad M_p = 2.108 \]

**Sway mechanism**

\[ 0 = \frac{W}{2} \]

\[ 60 = 4 \phi \]

\[ -4M_p \theta - 3M_p \phi = -\frac{W}{2} \]

\[ + 4M_p + 4.5BM_p = 3W \]

\[ W = \frac{25}{3} \quad M_p = \frac{2833}{2} \]

\[ 60 = 4 \phi \]

\[ \theta = 4 \theta \]

\[ \phi = 1 \phi \]

\[ -2M_p \theta - 2M_p \phi - 2M_p \phi - 1.5M_p \phi \]

\[ -3M_p \phi = \omega \frac{\omega}{4} - \omega \phi \]

\[ -4M_p \theta - 3.5 \times 3M_p \theta - 3x + 5M_p \phi \]

\[ \omega = 1.85 \ quad M_p \]
Out of various possible mechanisms correct mech. is one for which collapse loading is min hence collapse load is 1.854 MP.

**Collapsed B.M.O.**

\[
\begin{align*}
\sum M &= 0 \quad \text{(i)}
\end{align*}
\]

\[
H_A + H_B + \frac{W}{2} = 0
\]

\[
\sum M = 0 \quad \text{for CN position}
\]

\[
1.5mp + 1.5mp + H_B \times 4 = 0
\]

\[
H_B = -\frac{3}{4} \text{ MP} = -0.75 \text{ MP}
\]

\[
H_A = -H_B - \frac{W}{2} = -1.77 \text{ MP}
\]
PLATE GIRDERS

Plate girders (beams made using plates) are adopted when span and loading becomes large. For span up to 10 m, web plates may be used but for larger span (15 to 30 m), plate girders are used.

As the plate girders are deeper, chances of web buckling increase; hence, web has to be supported by using stiffeners. Stiffeners actually increase the moment of inertia which bear the load and hold it below the point of failure.

\[ a, b \text{ are unsupported length} \]

Horizontal stiffeners

Vertical stiffeners/transverse stiffeners
Design of Web Plate

Web primarily resist shear hence web is designed for shear.

For unstiffened web permissible web av shear stress = 0.4fy

\[ \sigma = f \left( \frac{d}{t_w}, c \right) \]

Per av shear stress decreases if \( \frac{d}{t_w} \) increases.

Economical depth is given by

\[ d = \frac{1.1}{\sigma_{bt} \times t} \]

Thickness will be assumed and \( d \) will be calculated from economical depth formula and it will be ensured that

with this dimension \( \frac{V_{max}}{d \times t} \) \leq \text{per av shear stress}.

Spacing of vertical stiffeners will be chosen such that the calculated average shear stress is less than equal to per av shear stress.

Design of Flange Plate

\[ I = 2 \left( A_f \left( \frac{d}{L} \right)^2 + \frac{b u d^3}{12} \right) \left( A_f + \frac{A_w}{6} \right) \frac{d^2}{2} \]
Flange plate is designed to resist bending moment. 

\[ M_{O.R} = \sigma_{bt} \times Z \]

\[ M_{o.m.r} = \sigma_{bt} \times \frac{S}{d^{1/2}} \]

\[ = \sigma_{bt} \left[ \left( A_f + \frac{A_w}{6} \right) \frac{d}{2} \right] \]

Effective flange area on compression side

\[ I = 2 \left( A_f + \frac{A_w}{6} \right) \left( \frac{d}{2} \right)^2 \]

On tension side reduction in flange area will occur due to formation of holes hence effective area of flange plate has to be increased. The effective flange area for tension side is taken as 

\[ \left( A_f + \frac{A_w}{8} \right) \]

\[ M_{max} = \sigma_{bt} \left( A_f + \frac{A_w}{8} \right) d \]

Area of flange plate calculated using this formula will be more.

For design purpose flange area is calculated than on tension side and same is adopted for compression side.
Check for bending

\[
\frac{M_{max} \cdot Y_{max}}{I_{max}} \leq \sigma_{bc}
\]

\[
\frac{M_{max} \cdot Y_{max} \cdot (G.A. \text{ of tension flange})}{I_{max} \cdot (\text{Net area of tension flange})} \leq \sigma_{bc}
\]

Tension flange will comprise of flange plate, flange angles and area of web plate included by flange angle.

**Design of Stiffner**

1. \( \frac{d}{t_w} < 1.5 \) No stiffner required (no horizontal stiffner)

2. \( 1.5 \leq \frac{d}{t_w} \leq 2.0 \) Only vertical stiffner is required

3. \( 2.0 \leq \frac{d}{t_w} \leq 2.5 \) Apart from vertical stiffner one horizontal stiffener is provided at a distance of \( \frac{2}{5} \) of the distance of compression flange from N.A.

\[
\text{If } \frac{d}{t_w} \in [2.50 - 4.00] \text{ One more hor. stiffener is provided at N.A.}
\]

\[
250 < \frac{d}{t_w} \leq 400
\]
for stiffened web min thickness of web = \( \frac{d}{400} \) for unstiffened web \( t_{min} = \frac{d}{85} \)

- Design of vertical stiffeners

Spacing is selected such that permissible shear stress is greater than \( \frac{V}{dbw} \).

The max. & min. spacing of a vertical stiffener are 1.5 \( d \), or 0.33 \( d \), respectively.

![Fig. (I)](image1)

![Fig. (II)](image2)

To decide about the type of stiffener to be provided we work on actual length of web however to design the vertical stiffener i.e. to decide on the cross-section of stiffener required we work on \( d_3 \) (actual unsupported length).

\( d_3 = \text{twice the clear distance from N.A. of the beam to the compression flange} \)

\[ d_3 = 2x \]

Larger & smaller clear dimension should be maintained below 20 to 60 & 150 to 25.
Vertical stiffener should be provided on one angle two sides of web. If it is provided on one side it should be staggered.

\[ \text{Eros at point } \frac{1.5d}{2t_a} \]

The moment of inertia of angle should not be less than

\[ I \geq \frac{1.5d^3}{t_w^3} \quad \frac{t_w^3}{c^2} \]

Connection Design

Connection should be designed for s.f

\[ S_f = \frac{12.5 \cdot t_w^2}{h} \quad \text{N/mm} \]

Where \( h \) is the outward of stiffener from the web.

\[ \frac{f_w}{12.5 \cdot b_o^2} = \text{Pitch} \]

\( b_o \geq 16 \cdot t \) for angle where

\( t \) is the thickness of member. 
In case of flat plate cantilever:

\[ t = \text{thickness of plate} \]

Vertical stiffener is primary supposed to resist shear buckling.

**Horizontal stiffener**

1. \( 200 < \frac{d_2}{t_c} \leq 250 \)
2. \( 250 < \frac{d_2}{t_c} \leq 400 \)

\[ F_{tx} \neq \frac{d_3 t_c^3}{3} \]

Connection of hor. stiffener will be done in some way as vert stiffener.

Horizontal stiffener primarily safeguards against buckling due to bending compression.

**Design of load bearing stiffener**

\[ A_t = \text{crossing area} \]

\[ E_t = \text{thickness of hang angle} \]

\[ t_{web} = \text{web thickness} \]

\[ A_i = \text{load bearing stiffener} \]
1. Outstanding leg of the stiffener is assumed to provide the necessary bearing area; hence bearing area required is found out.

\[ \frac{1}{b \cdot t} = \text{bearing area req.} \]

\[ \sigma_{xx} = \text{perm. bearing stress} = 0.75 \text{ fy} \]

2. Unequal angle is chosen in such a way that the overhang portion provides the necessary bearing area. Smaller leg will be connected to the web larger leg will be overhang.

3. Once the angle is chosen overall buckling is to be checked.

4. To check for overall buckling, the stiffener is assumed to be a column. The permissible load capacity of that column should be greater than applied load.

To find out \( \lambda \) is required and

\[ \lambda = \frac{\text{leff}}{\delta_{\text{min}}} \]

Where leff is taken as 0.71

\( l \) = overall depth/length of stiffner

The section resisting compression is assumed to be consisting of a pair of stiffeners together with a length of web on either side of centre line of stiffener, where available, equal to 20 times the web thickness.
A = shaded area
= 40 \times b + 2a

Load carrying capacity:
\[ \sigma_{\text{cr}} \times A > P \]

Web splices

In Fig. (1):
- Moment splice
- Shear splice
- Better way
In Fig. (i) when splice plate will be assumed to carry %\% shear at the splice location and moment equal to \( M_w \) where \[
M_w = \frac{M_{ld}}{I}.
\]

where \( M_{ld} \) B.M. at splicing section
\( I_u \) = m.o.i. of web plate
\( I \) = geo.m.o.i of original section about bending axis.

\[
\frac{V}{2d^2t} \leq 0.4 f_y
\]

\[
M_{OR} = \frac{0.6t \times 2}{6} \geq M_w
\]

Connection will be designed as eccentric connection

For drift s.f. \( \beta \) = \( h \)

\( f \) Torsional moment = \( M_w \)

In Fig. (ii) We provide two sets of plate

1. Shear splice \( \rightarrow \) assumed to resist only shear (\( M_{ld} \)).
2. Moment splice \( \rightarrow \) shear at splicing seen

\( \rightarrow \) assumed to resist moment \( M_w \).
Moment splice

\[ F_d \alpha = M_d \]

\[ F = \frac{M_d}{\alpha} \]

\[ 2(b-d') \times t_f \times f_y \geq F \]

\[ n = \frac{F}{R_u} \]

This design is for tension and some are provided for compression.

Shear splice

\[ \frac{V}{2d_w \times t} \leq 0.4 f_y \]

\[ N = \frac{V}{P_u} \]
Industrial Building.

- King Post
  - Span < 6 m

- Queen post truss
  - 9 m

- Fink truss

- Howe truss
  - 12-18 m

- Howe truss (flat)
  - Up to 24 m

- Compound Fink
  - 12-18 m

- Saw teeth

- Gable

- Gable for sun light

- $H$: Height of truss
  - Slope of truss = $\frac{H}{l} = 2\left(\frac{H}{L}\right)$

- Pitch of truss = $\frac{H}{L}$

- Slope = $2 \times$ Pitch
Deeper trusses have smaller deflection as compared to shallow trusses.

Member forces are larger in shallow truss as compared to deep truss.

For smaller span height of truss is normally taken as \( \frac{1}{6} \)th of the span.

\[
\frac{h}{L} = \frac{1}{6}
\]

Slope = \( \frac{1}{6} \)

For larger span depth may be taken as \( \frac{1}{10} \)th of the span.

Galvanised iron

If GI sheet is used as roof covering pitch of truss may be taken as 1 in 6

For AC (Asbestos cement) sheet roof covering pitch may be taken as 1 in 12

Lower pitch is advantageous for wind pressure

Spacing of truss
Normally, spacing is taken as \( \frac{4}{5} \) of span. Span up to 15m.

For economy in roof:

\[
\frac{\text{cost of truss}}{\text{area}} = \frac{\text{cost of purlin}}{\text{area}} \times 2 + \text{cost of roof covering}
\]

Area = Area of floor + or plan area of building

- **Purlins**

  Purpose of purlin is to support the roof sheet.

- **Sag rod - Tension member**

  Purlins are channel, angle or I section.
Max spacing b/w purlin is 1.4 m

Purlins are subjected to biaxial loading

Hence design of it should be done as biaxial bending.

However if slope of roof truss is < 30°

We may avoid design purlin by biaxial bending.

In this case (β < 30°) angle purlins can be used and it will be designed as uniaxial bending condition.

In this case Dead wt. & w.b. are assumed to be acting 1 to the roof coverage

\[
M_{max} = \frac{W \cdot L}{10}
\]
- \( l \) = span of purlin
- \( s \) = spacing between
- \( w \) = Total load resisted by purlin

\[
Z_{req} = \frac{wl/10}{\sigma_{lc}}
\]

\[
Z_{xx} \geq \frac{wl/10}{\sigma_{fc}}
\]  
- Choose angle such that

\[
x \geq \frac{L}{60}.
\]

- Angle chosen should be such that
- Width of angle \( I \) to roof covering \( \geq \frac{L}{45} \)
- Width of angle \( I \) to roof covering \( \geq \frac{L}{60} \)
- \( I \) to -

\[
\text{Sag and resist tangential component of D.L. due to}
\]
\[
\text{sheet (roof covering) of purlin itself.}
\]

Try to make overlap connection at the location of purlin.
for slope up to 1:3

For flatter slope, overlap length can be 1/3.

End lap

Side lap

Side lap for A.C. sheet: \( \frac{1}{2} \) corrugation length lap
For G.I. sheet: it is full corrugation length lap.

Note: When two sheets overlap at location, two sheets are cut at corners. This process is called mitring.
Most of the time in a yr. sun moves from east to west through south hence direct sunlight will come in the building from south which will cause shadow formation. To avoid it windows should be provided on north side so that diffused light come inside the building which does not cause any shadow formation.