HAND WRITTEN NOTES:
OF
CIVIL ENGINEERING

SUBJECT:
RAILWAY ENGINEERING
INTRODUCTION

Important Terms:

Cross-section of a railway track (on curve) was shown in figure:

1. Gauge: Inner distance b/w two rails (distance b/w running faces of two rails).

[Diagram showing cross-section of railway track with labels: Ballast formation, Ballast shoulder, Rail fixture, Soil subgrade, Natural Ground Level, 1 in 20 running.
Different Gauge:

1. Broad Gauge (B.G) = 1.676 m
2. Meter Gauge (M.G) = 1.0 m
3. Narrow Gauge (N.G) = 0.762 m
4. Feather Track Gauge (L.G) = 0.61 m

Coning of wheels:

[Diagram of coning of wheels]
A slope of 1 in 20 is provided on wheel surface and over rails also. This slope provided to wheels is called coning of wheels.

**Purpose:**

1. To keep the train just in central position during movement on a straight track.

2. To reduce the distance travelled on two rails on a curved track.

3. To reduce wear & tear of rails & wheels (A gap below flange and rail shall be maintained).

**Theory:**

On straight tracks, axle moves in central position, so that diameter of wheel on two rails are same. If there is any side movement, dia of wheel over one rail increases and decreases over another, so the axle is diverted back to its central position automatically.
On curved track, the rail is moved outward due to centripetal force, resulting in increase of dia over outer rail. So, distance to be travelled on outer and inner rails are adjusted.

Welded Rails (Long welded rails) (LWR)

To avoid expansion joints, rails are welded. Stress developed due to increase in length of rails due to temp is arrested by fixtures to sleepers.

In case of LWR, rails are not allowed to expand. Thus, stresses are developed. Force developed due to this stress is arrested by fixtures.

\[ \text{If } l = \text{length of rail} \]

Increase in length due to \( T \)° temp:
\[ \text{increase: } \Delta l = l \cdot \alpha T \]

if \( \Delta l \) movement is not allowed

\[ e = \frac{\Delta l}{l} = \frac{l \cdot \alpha T}{l} = \alpha T \]

stress developed \[ \frac{\text{stress}}{\text{strain}} = E \]

\[ \sigma_s = \text{stress} = eE_s = \alpha T \cdot E_s \]

if \( A_s = \sigma_s \) area of steel (one rail)

force developed

\[ F = A_s \cdot \sigma_s = A_s \cdot \alpha T \cdot E_s \]

if one sleeper can provide \( R \) resistance

no. of sleeper required to assist \( F \)

force

\[ n = \frac{F}{R} = \frac{A_s \cdot \alpha T \cdot E_s}{R} \]

if \( s = \) spacing of sleepers

min. length of rail on one side

required so that rail does not move

\[ = (n-1)s \]
Total min. length of LWR so that central portion does not move:

$$l = 2l_1 = 2(n-1)s.$$  

Movement of rails observed, no movement may be 0 to 00.
\[ \text{Min. length} \]
\[ \text{increase in length} = 1 \times 7 \]
\[ \text{strain developed if above increase is not allowed} \]
\[ \frac{\varepsilon}{\varepsilon} = \frac{dT}{E} \]
\[ \text{stress developed} = \alpha T E \]
\[ = 11.5 \times 10^{-6} \times 30 \times 2.1 \times 10^6 \]
\[ = 724.5 \text{ kg/cm}^2 \]

\[ \text{force developed} = A_s \times p_s \]
\[ = 66.15 \times 724.5 \]
\[ = 47925.675 \text{ kg} \]

\[ \text{no. of sleepers} = \frac{47925.675}{300} \]
\[ = 159.15 \]
\[ = 160 \]

\[ \text{min. length of LWR required} = \sigma (n-1) \]
\[ = \sigma (160-1) \times 6 \]
\[ = 190.80 \text{ m} \]
Sleepers:

1. Composite sleeper in India:
   - It is determined to determine suitability of a wooden sleeper for use on a railway track.

\[
\text{CSI} = S + 10H
\]

\[
S = \text{Strength index of timber at 12% moisture content}
\]

\[
H = \text{Hardness index of timber at 12% moisture content}
\]

**CSI values**

- Track sleeper: 783
- Crossing sleeper: 1552
- Bridge sleeper: 1455

2. Sleeper density:
   - No. of sleeper to be used for one rail length. It is denoted by \((n+x)\)
\[ n = \text{length of one rail in meters} \]
\[ x = 3 \text{ to } 6 \]
\[ (n+3) \text{ to } (n+6) \]

**Example:** if sleeper density is \((n+5)\) for 860 track, calculate member of sleepers required per km length of track.

- Length of one rail = 12.8 m \(=\) 13 m
- Sleeper density = \(n+5\)
  \[ = 13 + 5 = 18 \]

- \(12.8 \text{ m} \) — 18 number
- 1000 m — \(\frac{18 \times 1000}{12.8} = 1406\) sleepers

**Minimum depth of ballast cushion:**

![Diagram of rail and ballast cushion]
Minimum depth of ballast

\[ d_b = \frac{S - W}{2} \]

Geometrical Design:

1. Speed of train

Maximum speed that can be allowed on a railway track shall be minimum of following:

1. Safe speed on curve (Martin's formula) - maximum speed.

2. Speed as per S.E. formula (Cont).

3. Speed as per length of transition curve.

4. Maximum speed sanctioned by Railway Board.

1. Safe speed on curve (Martin's formula) -

a) On transitional curve -

(when transition curve has been provided with simple curved)
(1) for BR & rich track -
\[ V_{\text{max}} = 4.35 \sqrt{R-67} \]

for NR -
\[ V_{\text{max}} = 3.65 \sqrt{R-6} \]

b) On non-transitional curve:
(When transition curve has not been provided) -
do if. \( V_{\text{max}} \) for transition curve

1) for BR & ME -
\[ V_{\text{max}} = 0.80 \times 4.35 \sqrt{R-67} \]

li) for NR -
\[ V_{\text{max}} = 0.80 \times 3.65 \sqrt{R-6} \]

c) For high-speed trains -
\[ V_{\text{max}} = 4.58 \sqrt{R} \]

in all above formulas:
\[ R = \text{radius of curve in (m)} \]
\[ V_{\text{max}} = \text{kmph} \]
Angle made at center by one chain length of curve is called degree of curve \( (D^\circ) \).

(i) for 30 m chain length:

\[ \frac{30 \pi R}{180} = \frac{260}{D^\circ} \]

\[ D^\circ = \frac{260 \times 180}{30 \pi R} \]

\[ D^\circ = \frac{1718.9}{R} \]

\[ D^\circ = \frac{1720}{R} \]

<table>
<thead>
<tr>
<th>( D^\circ )</th>
<th>1°</th>
<th>2°</th>
<th>3°</th>
<th>4°</th>
<th>5°</th>
</tr>
</thead>
<tbody>
<tr>
<td>R for 30 m</td>
<td>1720 m</td>
<td>860 m</td>
<td>573 m</td>
<td>430 m</td>
<td>344 m</td>
</tr>
<tr>
<td>R for 50 m</td>
<td>1150 m</td>
<td>575 m</td>
<td>373 m</td>
<td>298 m</td>
<td>230 m</td>
</tr>
</tbody>
</table>
(ii) for a 90 m chain length
\[
\frac{817R}{90} = \frac{860}{90^\circ} \circlearrowleft
\]
\[
\theta = \frac{90 \times 360}{2 \pi R}
\]
\[
\theta^\circ = \frac{1150}{R}
\]

12. Maxm. degree of curve:
\[
BM = 10^\circ
\]
\[
ML = 16^\circ
\]
\[
NL = 40^\circ
\]

3. Versine of curve:

For a chord AB, distance (CD = V) is called versine of curve.

By using property of circle:
\[
AB \times DB = CD \times DE
\]
\[
\frac{c \times l}{a} = V(2R - V)
\]
\[ \frac{1}{4} = \varphi RV \]

\[ V = \frac{\frac{L^2}{2R}}{\varphi RV} \]

**Super-elevation or Cant:**

- **Purpose:**
  1. To counteract the effect of centrifugal force on curve.
  2. To reduce the chance of derailment.
  3. To reduce wear & tear of rails.

![Diagram of forces and cant](image)
outward component of centrifugal force in rail of rail surface = \( \frac{mv^2 \cos \theta}{R} \)

inward component of weight - mg\( \theta \) (there is no force of friction due rail & wheel in lateral dir.)

\[
mg \sin \theta = \frac{mv^2 \cos \theta}{R}
\]

\[
tan \theta = \frac{v^2}{gR}.
\]

Super elevation or Cant
\[
e = c_1 \tan \theta = \frac{c_1 v^2}{gR}
\]

\[
e = \frac{c_1 v^2}{9.81 R}
\]

\[
e = c_1 (0.878 V)^2
\]

\[
e = \frac{c_1 v^2}{12.7 R}
\]

\[c_1 = \text{gauge in meter}\]
\[V = \text{speed in kmph}\]
\[R = \text{radius in meter}\]
Equilibrium Cant:

\[ e = \frac{67V^2}{127R} \]  

Different trains are moving with different speeds. So, we need to design the cant for an avg. speed that is called the equilibrium speed & cant provided for this speed is called equilibrium cant. This is the actual cant that is provided on the track.

**Equilibrium Speed**

\[ V_{eq} = \frac{3}{4} \ max \ speed \]

\[ V_{eq} = \frac{3}{4} \ V_{max} \]

2) if sanctioned speed \(< 50 \ km/\ hr\)

\[ V_{eq} = V_{max} \]
3. Weighted avg. speed

\[ n_1 \text{ trains} \rightarrow V_1 \text{ speed} \]
\[ n_2 \text{ trains} \rightarrow V_2 \text{ speed} \]

Weighted avg. speed

\[ V_{aw} = \frac{n_1 V_1 + n_2 V_2 + \ldots}{n_1 + n_2 + \ldots} \]

\[ V_{aw} = \frac{\sum n v}{\sum n} \]

Max. limit of super-elevation -
(actual cant provided)

<table>
<thead>
<tr>
<th>Type</th>
<th>( e_{max} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)  Bc7</td>
<td></td>
</tr>
<tr>
<td>Speed &lt; 140 kmph</td>
<td>16.5 cm</td>
</tr>
<tr>
<td>Speed &gt; 140 kmph</td>
<td>18.5 cm</td>
</tr>
<tr>
<td>(2)  Mc7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.0 cm</td>
</tr>
<tr>
<td>(3)  Nc7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.6 cm</td>
</tr>
</tbody>
</table>
Cant deficiency = \( (p) \)

Cant required for a high speed train shall be more than actually provided value of cant on track (for equilibrium). There will be a deficiency of cant for this high speed train, this deficiency is called cant deficiency.

Limits of cant deficiency:

1. B6 track
   - \( V < 100 \text{ kmph} \) 7.60 cm
   - \( V > 100 \text{ kmph} \) 10.0 cm

2. H6
   - 5.10 cm

3. H9
   - 3.80 cm
\[ e + w = e_{act} + \Theta \]

**Example:**

For a 3° curve, if actual cant is provided for eqn sp. of 75 kmph on a 3m back, calculate maxm speed that can be allowed on the back.

(2)

For a 3° curve, \( R = \frac{1720}{3} = 573 \text{ m} \)

\[ R = 573 \text{ m} \]

(1) Martin’s Formula

\[ \text{maxm safe sp. on the curve} = 4.35 \sqrt{R - 67} \]

\[ = 4.35 \sqrt{573 - 67} \]

\[ = 97.85 \text{ kmph} \]

(2) From cant formula –

actual cant provided for

\( V_{eq} = 75 \text{ kmph} \)

\[ e_{act} = \frac{0.7 V_{eq}^2}{R} = \frac{0.7 \times 75^2}{187 \times 573} \]

\[ = 0.129 \text{ m} \]

\[ e_{act} = 12.9 \text{ cm} \]

Theoretical cant allowed.
\[ e_{th} = e_{act} + 9 \]
\[ e_{th} = 12.90 + 7.60 \]
\[ = 20.50 \text{ cm} = 0.205 \text{ m} \]

For maximum speed:

\[ e_{th} = 6.7 \cdot V_{\text{max}} \]
\[ 197 R \]
\[ V_{\text{max}} = \sqrt{\frac{197 R \cdot e_{th}}{67}} \]

\[ = \sqrt{\frac{197 \times 0.205 \times 0.205}{1.676}} \]
\[ = 94.34 \text{ kmph} \]

Maximum speed allowed = 94 kmph

Negative Super Elevation:

If outer rail is provided at lower elevation, outer to inner rail
it is called a negative super elevation

in case when a branch curve track is diverging from a main curved track in opposite dirn the
SE provided for main track. will become a negative SE for branch track.

Ques: A branch curved track of 6° curve is diverging from a 3° main curve in opposite direction. Both are 60 track.

1. If max speed allowable on main back is 65 kmph. Calculate actual cant provided and max speed allowed on branch track.

\[
V_{\max} = \frac{65 \text{ kmph}}{3} \text{ main curve}.
\]

\[
V_{\max} = \frac{65 \text{ kmph}}{6} \text{ 6° branch curve}.
\]

for 3° curve \( R_b = \frac{1720}{3} = 573 \text{ m} \)

for 6° curve \( R_b = \frac{1720}{6} = 286 \text{ m} \)

max speed on main curve

\[
(V_{\max})_{\text{main}} = 65 \text{ kmph}.
\]
\[ e_{th} = 0.75 \frac{V_{max}^2}{1.97 R_A} \]

\[ e_{th} = \frac{1.676 \times 65^2}{1.97 \times 573} = 9.73 \text{ cm} \]

\[ e_{act} = e_{th} - D \]

\[ = 9.73 - 7.60 = 2.13 \text{ cm} \]

Actual cant on main track is

+ve. = 2.13 cm

Actual cant on branch track is

-ve. = -2.13 cm

\[ e_{th} \text{ for branch track} = e_{th} + D \]

\[ = -9.13 + 7.60 = -5.47 = 0.0547 \text{ m} \]

\[ V_{max} \text{ for branch track} \]

\[ 0.0547 = \frac{1.676 \times V_{max}^2}{1.27 \times 286} \]

\[ V_{max} = 84.43 \text{ kmph} \]
Transition Curve
A parabolic curve is introduced by the straight portion and curved portion of railway track to serve following purposes

1. To provide S.E. in a gradual manner from 0 to e

2. To reduce the radius of curve (at straight back) to (R at curve back) in a gradual manner.

3. To reduce the effect of sudden jerk when there is a change from straight to curved.

This is called 'Transition Curve'

Requirements of an ideal transition curve:

The curve should be perfectly tangential to its junction points.

At straight junction ➞ Radius of transition curve

\[ R = \infty \]

At curved junction ➞ Radius of curvature

\[ R = R \]
The rate of change of curvature should be same as rate of change of SE (curvature = \( \frac{1}{k} \)).

Types:

- Spiral - for highway
- Cubic parabola - for railway
- Bernoulli's lemniscate

Cubic Parabola - Cubic Parabola is used for railway transition curve.

to a certain value of deflection angle, shape of all these three curves are similar.

cubic parabola is used for railway as it is easy to lay.
Cubic Parabola

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General eqn of cubic parabola:

\[ y = ax^3 + bx^2 + cx + d \]  \( \cdots \)  \( \text{Eq. 1} \)

at \( n = 0 \), \( y = 0 \)

\[ 0 = 0 + 0 + 0 + d \]

\[ d = 0 \]

now differentiate:

\[ \frac{dy}{dn} = 3an^2 + 2bn + c \]

at \( n = 0 \), slope \( \frac{dy}{dn} = 0 \)

then \[ c = 0 \]

again differentiate:

\[ \frac{d^2 y}{dn^2} = 6an + 2b = \frac{1}{R} \]
at \( n = 0 \), \( R = \infty \)

\[
\frac{d^2 y}{d n^2} = \frac{1}{R} = \frac{1}{\infty} = 0
\]

\( 6an + 2b = 0 \) \( \square \)

\( 0 + 2b = 0 \)

\[ b = 0 \]

at \( n = L \)

\[
\frac{d^2 y}{d n^2} = \frac{1}{R}
\]

\[ \frac{1}{R} = 6aL + 2x0 \]

\[
a = \frac{1}{6RL}
\]

\[
y = \frac{1}{6RL} \quad (\text{def \( n \) \ eq \( n \))}
\]

\[
\begin{align*}
dn^0 = \frac{dy}{dn} = \frac{n^2}{2RL} & \quad (\text{slope \ eq \( n \))} \\
& \\
&
\end{align*}
\]

\[ d^2 n^2 = 6an^2 = 6L \times 1 \times n \]

\[
\frac{d^2 y}{d n^2} = \frac{n}{RL} \left( \frac{\text{curvature eq \( n \))}}{R} \right)
\]
maxm slope

$$\theta_{\text{max}} = \frac{L^3}{2RL}$$

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$$\theta_{\text{max}} = \frac{L}{2R}$$

Spiral Angle : - \( (\phi) \)

It is the slope of tangent at any point on the curve.

$$\phi = \theta$$

$$\phi_{\text{max}} = \frac{L}{2R}$$

Deflection Angle : - \( (\alpha) \) -

It is the slope of line joining a point \( (P) \) from origin.

$$\alpha = \tan \phi = \frac{y}{x} = \frac{x^3}{n^3}$$

$$\alpha = \frac{x^2}{6RL} = \frac{L}{3} \times \frac{n^3}{2RL}$$
\[ \alpha = \frac{1}{3} \phi \]

Slight "wedge" = \( \frac{1}{3} \times \text{spiral angle} \)

\[ \alpha_{\text{max}} = \frac{L^2}{6RL} = \frac{L}{6R} = \frac{\phi_{\text{max}}}{3} \]

Providing cubic parabola as transition curve on a railway track.
Two transition curve have been provided on both sides of simple curve.

Transition curve is provided such that 1/2 length is available on both side of initial tangent points ($T_1$ or $T_2$).

\[ \text{eg}:- \]

\[ 0 \quad 700 \quad T_1 \quad 1500 \quad T_2 \quad 2600 \]

If chainage of $V$ is given

1st tangent length = $AV = AT_1 + T_1V = \frac{\ell}{2} + (R+S) \tan \frac{\Delta}{2}$

chainage $A = \text{chainage } V - AV$

chainage of $B = \text{chainage } A + AB$

chainage of $C = \text{chainage } B + BC$

chainage $D = \text{chainage } C + d$. 
Chainage of $O = \text{chainage } C + L$

Angle $\angle POB = \frac{L/R}{2R} = \frac{L}{2R} = \phi$

$\angle BOC = (\theta - 2\phi)$

Length of simple curve

\[ l = \frac{2\pi R}{360} \times (\theta - 2\phi) \]

Shift $s = \frac{L^2}{94R}$

$S = T_1 B = T_1 M - QM$

$= T_3 B - (0Q - 0M)$

$= y - (R - R\cos \phi)$

\[ = \frac{n^3}{6RL} - R(1 - \cos \phi) \]

\[ = \frac{L^3}{6R} - R\left(\frac{28\sin^2 \phi}{2}\right) \]

\[ = \frac{L^3}{6R} - 2R\left(\frac{\phi}{2}\right)^2 \]

\[ = \frac{L^2}{6R} - 2R \times \left(\frac{\theta}{4R}\right)^2 \]

\[ S = \frac{L^2}{124R} \]
Length of transition curve in Railways

There are two approaches:

1st approach:

\[ L = 7.2 \, e \]

\[ e = \text{cants in cm} \]

\[ L = \text{length of T.C. in m} \]

2.

\[ L = 0.073 \, e \cdot V_{\text{max}} \]

\[ e = \text{cants in cm} \]

\[ V_{\text{max}} = \text{Speed in kmph max} \]

\[ L = \text{in m} \]

3.

\[ L = 0.073 \, D \cdot V_{\text{max}} \]

\[ D = \text{cants deficieny} \]

\[ V_{\text{max}} = \text{kmph} \]

\[ L = \text{meter} \]

2nd approach:

\[ L = 4 + 4 \sqrt{R} \rightarrow \text{Railway Board formula} \]

\[ R = \text{radius of curve in m} \]

\[ L = \text{in m} \]
3.

Based on superelevation

\[ L = 3.6e \]

\[ e = \text{ramp in cm} \]

\[ L = \text{length in m} \]

\[ R = \text{radius in m} \]

Length \( L \) as per rate of change of radial acceleration:

\[ L = 3.28 \frac{v^3}{R} - \frac{v^3}{cR} \]

Here \( c = 0.3048 \text{ m/sec}^2 \)

\( v = \text{speed in m/sec} \)

\( R = \text{radius in m} \)

\( L' = \text{length of TC in m} \)

Radial acceleration:

\[ \frac{v^2}{R} \]

\[ \text{at straight junction} = \frac{v^2}{R} - \frac{v^2}{\infty} = 0 \]

\[ \text{at curved junction} = \frac{v^2}{R} \]
change in RA = 0 to \( \frac{u^2}{R} = \frac{v^3}{R} \)

If rate of change of radial acceleration is \( c \).

Total change of RA in T-sec = \( c \cdot T = \frac{u^2}{R} \)

\[ T = \frac{u^2}{cR} \]

\[ L = v^2 \cdot T \]  

\[ T = \frac{L}{v} \]

Equating (1) and (2):

\[ \frac{u^2}{cR} = \frac{L}{v} \]

\[ v^3 = LcR \]

\[ \frac{L = \frac{v^3}{cR}}{cR} \]

Based on rate of change of RA.

\[ c = 0.3048 \]

\[ L = \frac{3.38 v^3}{R} \]
Max\textsuperscript{m} speed based on length of transition curve.

0. For normal speed

\[ V_{\text{max}} = \frac{134L}{e} \]

or

\[ V_{\text{max}} = \frac{134L}{D} \]

Here:
- \( L \) = length of T.C. in (m)
- \( e \) = cant in mm
- \( D \) = cant deficiency in mm

2. For high-speed train

\[ V_{\text{max}} = \frac{198L}{e} \quad \text{or} \quad \frac{198L}{D} \]

3. Equilibrium cant is provided on a 160 track of 4° curve. For an equilibrium speed of 80 km/h, calculate actual cant provided. What max\textsuperscript{m} speed can be allowed on this track. Calculate length of transition curve.
required. If chainage of intersection point is 8052m & defl. angle = 85°, calculate chainage of important points on the curve.

Set out the transition curve at every 10m distance.

Soln:

4° curve \( R = \frac{1720}{4} = 430 \)

\( e_0 = 1.676m \)

\( \phi_{eq} = 80 \text{ kmph} \)

\[ e_{act} = e_0 \left( 1 - \frac{e_0^2}{127R} \right) = \frac{1.676 \times 80^2}{127 \times 430} = 19.64 \text{ cm} \]

\( e_{max} = 16.50 \text{ cm} \)

So actual camt provided = 16.50 cm

Max. speed allowed -

1) Safe speed on curve

\[ V_{max} = 4.35 \sqrt{430 - 67} \]

\[ 82.87 \text{ kmph} \]
as per cant formula -

\[ e_{thv} = e_{acc} + 0 \]
\[ = 16.50 + 7.60 = 24.10 \text{ cm} \]
\[ = 0.241 \text{ m} \]

\[ e_{thv} = \frac{G_{0} V_{\text{max}}^{2}}{127 R} \]
\[ 0.241 = \frac{1.676 \times V_{\text{max}}^{2}}{127 \times 433} \]

\[ V_{\text{max}} = 88.61 \text{ kmph} \]

Max. speed that can be allowed
\[ = 82.88 \text{ kmph} \]

Length of transition curve -
(should be calculated using max speed allowed)

using 1st approach -

1. \[ L = 7.2e \]
\[ L = 7.2 \times 16.50 = 118.8 \text{ m} \approx 119 \text{ m} \]

2. \[ L = 0.073 \times e \times V_{\text{max}} \]
\[ L = 0.073 \times 16.50 \times 82.88 = 99.8 \text{ m} \]
\[ L = 0.073 \times 7.60 \times 82.88 \]

\[ = 45.98 \text{ m} \]

So length of transition curve = 119

using on ia curve

chainage of important points:

\[ V = 805.5 \text{ m} \]

\[ \Delta = 85^\circ \]

\[ \phi = \Delta - 2\phi \]

chainage \( V = 805.5 \text{ m} \)

shift \( S = \frac{L^2}{24R} \frac{119^2}{24 \times 430} = 1.37 \text{ m} \)

1st tangent length
\[ VA = \frac{L}{2} + (R+s) \tan \Delta \]

\[ = \frac{119}{2} + (430 + 1.37) \tan 0.5^\circ \]

\[ = 454.78 \text{ m} \]

Length of simple curve.

Spiral angle \( \phi = \frac{L}{2R} = \frac{119}{2 \times 430} \text{ rad} \) \( \approx 0.013 \text{ rad} \)

\[ \phi = 7.928 \text{ rad} \]

\[ \phi = 7^\circ 55' 11'' \]

Length of simple curve.

\[ d = \frac{2\pi R}{360} \times (\Delta - 2\phi) \]

\[ = \frac{2\pi \times 430}{360} \times (85^\circ - 2 \times 7^\circ 55' 11'') \]

\[ = 518.92 \text{ m} \]

Chainage \( V = 8052 \text{ m} \)

\[- (VA) = -454.78 \text{ m} \]

Chainage of \( A \) = 7597.23

Chainage of \( B \) = chainage of \( A \) + 119
chainage of C = chainage of B + 1
= 7716.22 + 518
= 8235.14 m

chainage of D = chainage of C + 1
= 8235.14 + 119
= 8354.14 m

Setting of transition curve:

\[ y = \frac{n^3}{GRC} = \frac{n^3}{6 \times 430 \times 119} = \frac{n^3}{307070} \]

<table>
<thead>
<tr>
<th>n</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>0.0032</td>
<td>0.026</td>
<td>0.087</td>
<td>0.208 m</td>
<td>0.407 m</td>
<td>0.703 m</td>
<td>1.17</td>
<td>1.66</td>
<td>2.31 m</td>
</tr>
</tbody>
</table>

\[ \frac{160}{200} \]

<table>
<thead>
<tr>
<th>160</th>
<th>110</th>
<th>119</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.25</td>
<td>4.33</td>
<td>5.48 m</td>
</tr>
</tbody>
</table>
Design the length of a transition curve for a BR track, having a curve with a radius of 12m. The max design speed on the curve is 100 kmph. Also calculate the offsets at every 15m distance and shift of circular curve. Assume want deficiency of 2.60cm.

Q2: Calculate the max permissible speed on a BR track of 2° curve. 8.7 provided is 9cm. Length of transition curve is 325m. Max want deficiency allowed is 10 cm. Max sanctioned speed by Railway Board on the track is 135 kmph.

1. Max speed shall be minimum of following:

   1. Safe speed by Martin's formula

      \[ D^0 = 2^\circ \]
      \[ R = \frac{1720}{2} = 860 \]
      \[ V_{\text{max}} = 4.35 \sqrt{R-6^\circ} = 4.35 \sqrt{860-6.7} = 122.5 \text{ kmph} \]
use high speed formula:

\[ V_{max} = 4.58 \sqrt{R} \]
\[ = 4.58 \sqrt{860} \]
\[ = 134 \text{ kmph} \]

\[ Q \]

as per short formula:

\[ Cen = 9 \text{ cm} \]
\[ cent \text{ deficiency} = 10 \text{ cm} \]
\[ e_{th} = e_{acc} + 20 \]
\[ = 9 + 10 \]
\[ = 19 \text{ cm} = 0.19 \text{ m} \]

\[ V_{max} = \sqrt{\frac{197R \cdot e_{th}}{89}} \]
\[ = \sqrt{\frac{197 \times 860 \times 0.19}{89}} \]
\[ = 111.27 \text{ kmph} \]

\[ R \]

as per length of transition curve:

\[ V_{max} = \frac{198l}{e} \text{ or } \frac{198l}{20} \]
\[ = \frac{198 \times 125}{90} \text{ or } \frac{198 \times 125}{100} \]
\[ = 275 \text{ kmph} \text{ or } = 247.5 \]
1. Maxm. sanctioned speed by railway board = 14.5 kmph.

So maxm. speed allowed = 111 kmph.

2. On a transition curve on 66 track, the speed by railway board formula (Martin's formula) \( V = 4.35 \sqrt{R - 67} \) is 1.35 times speed calculated by cant formula after allowing a cant deficiency of 7.6 m. If actual cant is provided for a speed of 80 kmph. Calculate:

1. Radius of curve.
2. Maxm. speed allowed.
3. Actual value of cant provided.

\[
4.35 \sqrt{R - 67} = 1.35 \sqrt{\frac{127R \cdot \text{eth}}{1.676}}
\]

\[
\text{eth} = \frac{\text{exact} + \Delta}{R}
\]

\[
\text{eth} = \frac{84.46 + 7.6 \times 10^{-3}}{R}
\]

\[
\text{act} = \frac{9V^2}{127R}
\]

\[
\text{act} = \frac{1.676 \times 80^3}{127R}
\]

\[
\text{act} = \frac{84.46}{R}
\]
\[
4.35 \sqrt{R-67} = 1.35 \sqrt{127R + 0.46} + 7.710
\]
\[
4.35 \sqrt{R-67} = 1.35 \sqrt{6400 + 5.758R}
\]
\[(4.35)^2 (R-67) = (1.35)^2 (6400 + 5.758R)\]

\[
18.9325R - 1267.8075 = 11664 + 10.4939R
\]

\[
8.4386R = 13931.8075
\]

\[
R = 1534.5
\]

\[
R = 1535 \text{ m}
\]

3. East = \(\frac{84.46}{1535} = 0.0556\)

4. By marten's formula

\[
V = 4.35 \sqrt{R-67} = 166.66.
\]

By cent formula

\[
V = \sqrt{\frac{127 \times 1535 \times (84.46 + 0.076)}{1.876}}
\]

\[
V = 123 \text{ kmph}
\]

So maximum speed = 123 kmph.
Design of Vertical Alignment

1. Gradients: Max\(m\) slope that can be provided in longitudinal dirn.

2. Ruling Gradient: Max\(m\) gradient that can be provided in most general condition.
   - In plains: 1 in 150 to 1 in 300
   - In hilly area: 1 in 100 to 1 in 150

3. Momentum Gradient:

   ![Diagram of momentum gradient]

   In a particular situation, as shown in fig., when extra momentum gained
   during upward movement can be used for upward movement, A slope slightly
   more than ruling gradient can be provided, that is called momentum
gradient.
c) Pushes Gradient: In very extra ordinary situation, there is no other option, Pusher Gradient can be provided that need extra engine to push the train on steeper gradient.

1 in 75 slope can be provided with one extra locomotive.

d) Gradient in station yards: — Slope in station yards should be min. slope so that train does not move automatically.

Min. slope is required for drainage.
1 in 1000 — for good surface
1 in 200 — for an inferior surface

on station yard
max. slope — 1 in 400
min. slope — 1 in 1000

e) Grade Compensation on curve:

Curve restrict resistance restrict the speed to accommodate the effect of curve. The value of
gradient is slightly reduced at curve location. This reduction of gradient at curve is called grade compensation.

$B_0 = 0.04 \%$ per degree of curve $= 0.0004 B^0$
$M_0 = 0.03 \%$ per degree of curve $= 0.0003 B^0$
$N_0 = 0.02 \%$ per degree of curve $= 0.0002 B^0$

e.

Ruling gradient $= 1 \text{ in } 150$

curve of $3^\circ$ on a $B_0$ Track.

Grade compensation $= 0.04\%$ per degree

$= 0.04 \times \frac{1}{3} = 0.0012$

$= \frac{0.0012}{100}$

Compensated gradient $= \frac{1}{150} - 0.0012$

$= 5.467 \times 10^{-3}$

$= \frac{1}{182.9}$

Diagram:

- Grade compensation: 0.0012
- $1m$
- $0.04\%$
- $150$
- $182.9$
Vertical Curve

1. Summit Curve
2. Valley Curve

1. Summit Curve

2. Valley Curve
Summit Curve:

1st gradient = $+\theta_1^{\circ}$

2nd gradient = $-\theta_2^{\circ}$

If $\delta$ = rate of change of gradient per chain length

Length of summit curve

\[ L = \frac{(\theta_1 - \theta_2)}{\delta} \]

General formula

\[ L = 2n \text{ chains} \]
If RL of C is known

\[ \text{RL of A} = \text{RL of C} - \frac{I_1 \times \text{nd}}{100} \]  

\[ \text{RL of B} = \text{RL of C} - \frac{I_2 \times \text{rd}}{100} \]  

\[ \text{RL of E} = \frac{\text{RL of A} + \text{RL of B}}{2} \]  

\[ \text{RL of D} = \frac{\text{RL of C} + \text{RL of E}}{2} \]  

Here, l = length of one chain (30 m or 300 m)

if P is a point on the curve having co-ordinates (x, y)

A simple curve parabola is used for summit curve.

General eqn of curve -

\[ y = an^2 + bn + c \]  

This eqn -

\[ y = an^2 + bn + c \]  

at \( n = 0 \), \( y = 0 + 0 + c \)  

\[ y = c \]
The slope eqn \( y' = 2an + b \)

at \( n = 0 \)

\( \frac{dy}{dn} = g_1 = 0 + b \)

\( b = g_1 \)

\[ y = an^2 + gn + c \]

To find out RL of \( P \)

RL of \( R \) = RL of \( A \) + \( g_1 n \)

\( = c + g_1 n \)

RL of \( P \) = RL of \( R \) - \( h \)

\( h = RL \) of \( R \) - RL of \( P \)

\( h = c + g_1 n - y \)

\( h = c + g_1 n - (an^2 + gn + c) \)

\( h = -an^2 \)

\(-a\) is a constant. Put \( k = -a \)

\[ h = kn^2 \]
$$e_1 = \text{rise per chain length (for +g)}$$
$$= \frac{g_1 \times \ell}{100}$$

$$e_2 = \text{fall per chain length (for -g)}$$
$$= -\frac{g_2 \times \ell}{100}$$

For last point on curve
$$F_B = h = n(e_1 - e_2) = n(e_1 - e_2)$$

For point B, \( n = 0 \)
$$h = k n^2$$
$$h = k (\Delta n)^2$$
$$h = k \cdot \Delta n^2$$
$$h = n(e_1 - e_2)$$

$$k \cdot \Delta n^2 = n(e_1 - e_2)$$
$$k = \frac{e_1 - e_2}{\Delta n}$$

Final value
$$h = k n^2 = \left(\frac{e_1 - e_2}{\Delta n}\right) n^2$$

For different point on the curve
$$n = 0, 1, 2, 3, \ldots \text{no of chains}$$
A summit curve has two grade +0.65% and -0.85%. Rate of change of gradient per chain length is 0.15%. Chainage & RL of point of intersection is 2500 m. and 350.50 m respectively.

Calculate length of summit curve and RL. and chainage of different points on the curve. Find out radius of the curve also. (Use 30 m chains).

\[ g_1 = +0.65\% \]
\[ g_2 = -0.85\% \]
\[ n = 0.15\% \]

length of curve = \( \frac{g_1 - g_2}{n} = \frac{+0.65 - (-0.85)}{0.15} \)

\[ = 10 \text{ chain (9n)} \]

\[ n = 5 \text{ chains} \]

R.L of A = R.L of B - \( \frac{0.65 \times 150}{100} \)

\[ = 350.50 - \frac{0.65 \times 150}{100} \]

\[ = 349.525 \text{ m} \]
\[ RL \text{ of } C = RL \text{ of } B - \frac{0.85 \times 150}{100} \]
\[ = 350.50 - \frac{0.85 \times 150}{100} \]
\[ = 349.925 \]

\[ RL \text{ of } E = RL \text{ of } A + RL \text{ of } C \]
\[ = 349.525 + 349.925 \]
\[ = 349.375 \text{ m}^2 \]
\[ RL \text{ of } A = RL \text{ of } B + RL \text{ of } E = \frac{350.50 + 349.375}{2} = 349.9375 \text{ m} \]

\[ k = \frac{k_1 a^2}{h_m} \]

\[ k = \frac{e_1 - e_2}{\eta n} \]

\[ e_1 = \frac{g_1 d}{100} = \frac{0.65 \times 30}{100} = 0.195 \]

\[ e_2 = \frac{g_2 d}{100} = \frac{-0.85 \times 30}{100} = -0.255 \]

\[ k = \frac{0.195 + 0.255}{4 \times 5} = \frac{0.45}{20} = 0.0225 \]

\[ \eta = \frac{k n^2}{0.0225 x^2} \]

\[ \eta = 0.1, 0.2, 0.3, \ldots \]
<table>
<thead>
<tr>
<th>Points</th>
<th>Chainage</th>
<th>RL on point on 1st tangent (0.65/100 x 30 = 0.195)</th>
<th>( h = k x^2 )</th>
<th>RL on point on curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2350</td>
<td>349.525</td>
<td>0</td>
<td>349.525</td>
</tr>
<tr>
<td>1</td>
<td>2380</td>
<td>349.720</td>
<td>0.0995</td>
<td>349.925</td>
</tr>
<tr>
<td>2</td>
<td>2410</td>
<td>349.915</td>
<td>0.0998</td>
<td>349.925</td>
</tr>
<tr>
<td>3</td>
<td>2440</td>
<td>350.110</td>
<td>0.2025</td>
<td>349.9175</td>
</tr>
<tr>
<td>4</td>
<td>2470</td>
<td>350.305</td>
<td>0.368</td>
<td>349.975</td>
</tr>
<tr>
<td>5</td>
<td>2500</td>
<td>350.500</td>
<td>0.5625</td>
<td>349.9375</td>
</tr>
<tr>
<td>6</td>
<td>2530</td>
<td>350.695</td>
<td>0.818</td>
<td>349.885</td>
</tr>
<tr>
<td>7</td>
<td>2560</td>
<td>350.890</td>
<td>1.1985</td>
<td>349.7875</td>
</tr>
<tr>
<td>8</td>
<td>2590</td>
<td>351.085</td>
<td>1.44</td>
<td>349.645</td>
</tr>
<tr>
<td>9</td>
<td>2620</td>
<td>351.280</td>
<td>1.8225</td>
<td>349.457</td>
</tr>
<tr>
<td>10</td>
<td>2650</td>
<td>351.475</td>
<td>2.25</td>
<td>349.225</td>
</tr>
</tbody>
</table>

Valley curve:

\[ RL_{of}P = RL_{of}Q + h \]
A curve has a down grade of 0.8\% followed by an up grade of 1.2\%. Allowable rate of change of grade is 0.2\%.

- RL & chaining of point of intersection are 1300 m & 130.50 m. Calculate RL and chaining of different points on valley curve (use 20 m chain).

\[ Q = \frac{17000}{40} = 430 \text{ m} \]

- Cant = 19.2 cm

- Max. design speed = 100 kmph

- Cant deficiency = 19.6 cm

- Max. speed allowed

1. As per Martin's formula,

\[ V_{\text{max}} = 4.35 \sqrt{R - 67} \]

\[ = 4.35 \sqrt{430 - 67} \]

\[ = 82.88 \text{ kmph} \]

2. Speed as per cant formula:

- Theoretical cant

\[ e_{\text{th}} = \text{Cant} + D \]

\[ = 19.2 + 7.60 \]

\[ = 19.6 \text{ cm} = 0.196 \text{ m} \]
\[ V_{\text{max}} = \frac{127 \text{ R.e.m}}{2g} \]
\[ = \frac{127 \times 430 \times 0.196}{1.676} \]
\[ \approx 79.9 \text{ kmph} \]

So max\textsuperscript{m} speed allowed on back = 100 kmph.

So max\textsuperscript{m} speed = min\textsuperscript{m} of above three
\[ = 79.9 \text{ kmph} \]

using 2\textsuperscript{nd} approach -

(1) \[ L = 4.4 \sqrt{R} \]
\[ = 4.4 \sqrt{430} \]
\[ \approx 91.24 \text{ m} \]

(2) \[ L = 3.6e \]
\[ = 3.6 \times 12 = 43.2 \text{ m} \]

(3) \[ L = 3.28 \frac{V^2}{R} \]
\[ R = \frac{3.28 \times (0.78 \times 80)^3}{430} \]
\[ \approx 83.9 \text{ m} \]

\[ L = 91.24 \text{ m} \approx 92 \text{ m} \]
\[ \text{Shift} = \frac{L^2}{Q^2} = \frac{92^2}{24 \times 430} = 0.89 \text{m} \]

Eq. of T.C

\[ y = \frac{x^3}{GRL} = \frac{u^3}{GRL} = \frac{n^3}{GRL} \]

\[ 62 \]

\[ \text{Trail dist} \]

<table>
<thead>
<tr>
<th>n</th>
<th>0</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>75</th>
<th>90</th>
<th>92</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>0.014</td>
<td>0.113</td>
<td>0.884</td>
<td>0.91</td>
<td>1.78</td>
<td>3.07</td>
<td>3.28</td>
</tr>
</tbody>
</table>
Turnout - Turnout is an arrangement to divert the train from one track to another, to get better flexibility of movement on different tracks.

These are one of the most weak locations of railway track, so very strong material is required. High manganese steel is used for points & crossing.

Diff 'n component are shown in fig.
Component of turnout:

a) Point (Switch):

- Heel of switch
- Toe of switch
- Heel divergence
- Flange way clearance
- Tongue Rail
- Flange way depth

Section at heel of switch

1. Heel Divergence: Distance b/w running faces of stock rail and tongue rail at heel of switch.
In India

<table>
<thead>
<tr>
<th>BR</th>
<th>13.7 to 13.3 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>NY</td>
<td>12.1 to 11.7 cm</td>
</tr>
<tr>
<td>NO</td>
<td>9.8 cm</td>
</tr>
</tbody>
</table>

(2) Flange way clearance:
- Distance btw the adjacent faces of stock rail and tongue rail at head of switch.

- Value in India:
  - For 1 in 12 crossing = 6.3 cm
  - For 1 in 8 1/2 crossing = 6.6 cm

(3) Flange way depth:
- Vertical distance btw top of rails to heel block.

(4) Throw of Switch:
- The distance by which toe of tongue rail moves side ways.

- Value in India:
  - BR = 9.5 cm
  - HO/NG = 8.9 cm
Switch angle:

Heel: \( \theta \)

Toe

\( (h-t) \)

\( f \)

\( t \)

\( s_1 \) = actual length of tongue rail

\( s_2 \) = Theoretical length of tongue rail

\[ \sin \beta = \frac{h-t}{s_1} = \frac{h}{s_2}. \]

\[ \beta = \sin^{-1} \left( \frac{h-t}{s_1} \right) = \sin^{-1} \left( \frac{h}{s_2} \right). \]
Crossing:

→ Crossing should be rigid to sustain severe impact loads, vibration etc.
→ Medium or high mag. Magnes. steel is used.

Important Points:

1. ANC - Actual Nose of crossing due to blunt face.
2. TNC - Theoretical nose of crossing intersection points of two rails.
3. Angle of crossing or Number of crossing:

\[ \theta \]

No. of crossing = \frac{spread \ of \ two \ legs \ of \ crossing}{length \ of \ rail \ from \ TRC}

(68)

b) Cole's Method: (Right Angle Method)

\[ \tan \alpha = \frac{1}{N} \]

\[ N = \cot \alpha \]

\[ \alpha = \cot^{-1} N = \tan^{-1} \left\{ \frac{1}{N} \right\} \]

\[ \alpha = \text{angle of crossing} \]

\[ \ln N = \text{number of crossing} \]

This method is used on Indian railway.
eg. - If No. of crossing is 1 in 12

\[ \cot \alpha = \frac{1}{12} \]
\[ \tan \alpha = \frac{1}{11.25} \]
\[ \alpha = \tan^{-1} \frac{1}{11.25} = 4.96 \]
\[ \alpha = 4^\circ 45' 49.11'' \text{ Arc} \]

<table>
<thead>
<tr>
<th>No. of crossing</th>
<th>( \alpha )</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 in 6</td>
<td>9° 37' 44''</td>
<td>Used in symmetrical split</td>
</tr>
<tr>
<td>1 in 8(\frac{1}{2})</td>
<td>6° 42' 35''</td>
<td>Used in station yard where space is restricted and speed is low</td>
</tr>
<tr>
<td>1 in 12</td>
<td>4° 45' 49''</td>
<td>Used on station yard of main line</td>
</tr>
<tr>
<td>1 in 16</td>
<td>3° 34' 35''</td>
<td>Used on high speed tran. on main line of Bölna tracks</td>
</tr>
</tbody>
</table>

Centre Line Method:

\[ \tan \frac{\alpha}{2} = \frac{1}{N} = \frac{1}{2N} \]
\[ \cot \frac{\alpha}{2} = 2N \]

\[ N = \frac{1}{2} \cot \frac{\alpha}{2} \] - No. of crossing
\[ \frac{a^2}{b^2} = \cot^{-1} (2N) \]

\[ \alpha = 2 \cot^{-1} (2N) \] \text{ angle of crossing}

This method is used in US & UK.

5. Gossesles Triangle Method:

\[ \sin \frac{\alpha}{2} = \frac{b}{N} \]

\[ \sin \frac{\alpha}{2} = \frac{1}{\sqrt{N}} \]

\[ \cotie \frac{\alpha}{2} = 2N \]

\[ N = \frac{1}{2} \cotie \frac{\alpha}{2} \] \text{ no of crossing}

\[ \alpha = 2 \cotie^{-1} (2N) \] \text{ angle of crossing}

This method is used in tramways.
Design Calculation of Turnout

Components:

1. Curve lead (CL): The distance from the toe of switch to TNC measured along straight lead rail.

2. Switch lead (SL): The distance from the toe of switch to heel of switch measured along straight rail.

3. Lead (L) or Crossing lead: Distance from heel of switch to TNC measured along straight rail.
Relation: \( CL = SL + L \)

4. Radius of curve:

\[ R = \text{radius of curve} \]
\[ R_0 = \text{outer radius} = R + \frac{c}{2} \]

5. Heel Divergence (\( \theta \))

6. Switch Angle (\( \phi \))

7. Angle of crossing (\( \alpha \))

\[ N = \cot \alpha \]

There are three methods:

1. Method 1:

   In this method, the curve starts from the toe of the switch and ends at TNC.
   Given values: \( c, \phi, \alpha \) or \( N \)

   Curve lead (\( CL \)):

   \[ CL = BD = AH \]

   \[ \tan \frac{\alpha}{2} = \frac{c}{eL} \]

   \[ CL = \frac{c}{\tan \frac{\alpha}{2}} = c \cot \frac{\alpha}{2} \]
(b) \( CL = BO = BF + FD \)
\[ BF + AF \]
\[ CL = C \cot d + m \cos ed \]
\[ = C_1 N + C_2 \sqrt{1 + N^2} \]
\[ = C_1 N + C_2 N \]
\[ = 20N. \]

CL = 20N.

most widely used

(c) using property of circle.
\[ AH \times HA' = 0H \times H D' \]
\[ (CL)^2 = 0x(2R_0 - R) \]
\[ (CL)^2 = 0y(2R_0) \]
\[ \frac{1}{2} (2R_0 - R) \]

\[ CL = \sqrt{2R_0 \cdot R} \]

(2) \text{ Radii} (R \& R_0) \quad -

\[ R_0 = 0H \quad = \quad 0H + H D' \]
\[ \tan \alpha = \frac{CL}{OH} \quad \rightarrow \quad OH = CL \cot \alpha \]
\[ R_0 = CL \cot \alpha + 0H \]
\[ = 0H + 2\sin \alpha N \cdot N \]
\[ R_0 = 0H + 2\tan \alpha N \]

\[ As \quad per \quad Indian \quad Railway \]
\[ R_0 = 1.5 \alpha + 2\alpha N \]

\[ R = R_0 - \frac{5}{2} \]

\[ \text{Lead} \]
\[ \checkmark \quad Switched \quad Unit \quad (SL) \quad - \quad 0 \]
\[ \text{Property of circle} \quad - \quad E \]
\[ (\frac{1}{2} \alpha - R) \quad (\text{SL})^2 \]
\[ (\frac{1}{2} \alpha - R) \quad (\alpha - R) \]
\[ R \times 2R_0 = \text{SL}^2 \]
\[ s_l = \sqrt{280 \cdot h} \]

4. Lead Crossing lead - \[ L = C_L - S_L \]

**Method-2**

In this method, the curve starting from head of switch \( L \) ends at TNC.

Only two values are to be calculated:
1. Lead or crossing lead \( L \)
2. Radius \( R \) or \( \infty \).

![Diagram of a curve with various points labeled and a graph showing the relationship between different variables.](image-url)
Deflection angle $= \alpha - \beta$

$\angle F A H = \angle F E H = \frac{\alpha - \beta}{2}$

$\angle H A I = \alpha - \frac{\alpha - \beta}{2} = \frac{\alpha + \beta}{2}$

$\angle A O E = \alpha - \beta = \text{deflection angle}$

$\angle A O H = \frac{\alpha - \beta}{2}$

1. Lead or crossing lead -

$$\tan \left( \frac{\alpha + \beta}{2} \right) = \frac{\omega - h}{L}$$

$$L = \frac{(\omega - h)}{\tan \left( \frac{\alpha + \beta}{2} \right)}$$

Lead or crossing lead

$$L = (\omega - h) \cot \left( \frac{\alpha + \beta}{2} \right)$$

2. Radius -

On $\triangle A H D$

$$\sin \left( \frac{\alpha - \beta}{2} \right) = \frac{A H}{R_0}$$
\[ R_0 = \frac{AH}{\sin\left(\frac{a-\beta}{2}\right)} \quad \text{(1)} \]

In \( \triangle AEH \):

\[ \sin\left(\frac{\alpha + \beta}{2}\right) = \frac{AE}{2AH} \]

\[ AE = (c-h) \]

\[ \sin\left(\alpha + \beta\right) = \frac{AE}{\sin\left(\frac{\alpha + \beta}{2}\right)} \]

\[ AH = \frac{1}{2} AE \]

\[ AH = \frac{(c-h)}{2\sin\left(\frac{\alpha + \beta}{2}\right)} \]

Put \( AH \) in eqn (1):

\[ R_0 = \frac{(c-h)}{2\sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\alpha + \beta\right)} \]

\[ R_0 = \frac{c_0-h}{(\cos\beta - \cos\delta)} \]

\[ R = R_0 - \frac{\alpha}{2} \]
Method - 3
Curve is starting from heel of switch and ends at starting point of straight portion provided before TNC.

\[ L = r \cos \alpha + (c - h - r \sin \alpha) \cot \frac{\alpha + \beta}{2} \]

\[ R = \frac{(c - h - r \sin \alpha)}{\cos^3 \alpha - \cos \alpha} \]

Derivation is same as method 2.
Q. What will be angle of switch and hull divergence if

theoretical length of switch = 5.10 m

thickness of tongue rail at toe = 0.65 cm

Actual length of tongue rail = 4.80 m

\[
\sin \beta = \frac{h}{s_2} = \frac{h - b}{s_1}
\]

\[
\frac{h}{s_2} = \frac{h - 0.65}{s_1}
\]

\[
\frac{h}{510} = \frac{4.80}{s_2}
\]

\[
h = \frac{0.65 \times 510}{510 - 4.80} = 11.05 \text{ cm}
\]

Switch angle \( \sin \beta = \frac{h}{s_2} = \frac{11.05}{510} \)

\[
\beta = 1^\circ 14'.9.42'' \text{ Ans.}
\]
Calculate necessary elements to set out a turnout taking from a straight line back.

No. of crossing = 1 in 12.

Redivergence = 12 cm.

The curve is starting from toe of switch and ends at TNC.


**Solution:**

\[
\begin{align*}
\text{CL} &= 26N \\
&= 2 \times 1.676 \times 12 \\
&= 40.224 \text{ m} \\
\text{R₀} &= 1.5 \pi \cdot 16.76 + 2 \times 1.676 \times 12^2 \\
&= 485.202 \text{ m}
\end{align*}
\]
\[ SL = \sqrt{g R_0 h} \]
\[ = \sqrt{0.4 \times 4.85 \times 202 \times 0.12} \]
\[ \therefore SL = 10.819 \text{ m} \]
\[ L = CL - SL \]
\[ \therefore L = 99.404 \text{ m} \]

Q. Calculate necessary elements to set out a turnout using following data:

- Code track
- No. of crossings: 11 in 16
- Switch angle: 1°42' 30"
- Head divergence: 13.5 cm
- The curve is starting from bulb of switch & ends at TNC

Solution:

\[ K - L - x - b \]
\[ \alpha = \cot^{-1}(N) \]
\[ \alpha = \tan^{-1}\left\{ \frac{1}{N} \right\} \]
\[ \alpha = \tan^{-1}\left\{ \frac{1}{16} \right\} \]
\[ \alpha = 3^\circ 34' 35'' \]

Switch Angle \[ \beta = 1^\circ 42' 30'' \]

\[ L = (h - h) \cot \left( \frac{\alpha + \beta}{2} \right) \]
\[ = 1.676 - 0.135 \]
\[ \frac{\tan \left( 3^\circ 34' 35'' + 1^\circ 42' 30'' \right)}{2} \]

\[ L = 83.39 \text{ m} \]

\[ R_0 = \frac{h - h}{\cos \beta - \cos \alpha} \]
\[ = 1.676 - 0.135 \]
\[ \frac{\cos 1^\circ 42' 30'' - \cos 3^\circ 34' 35''}{2} \]

\[ R_0 = 1095.27 \text{ m} \]
Calculate necessary elements to set out a 1 in 8\(\frac{1}{2}\) turnout taking off from a straight 860 track, with its curve starting from the switch and ending at a distance of 864 mm from TNC.

Track divergence is 136 mm.

Switch angle = 1°34'27"

Make a free hand sketch and show the calculated values:

\[
\alpha = \cot^{-1} \left( \frac{8\frac{1}{2}}{8} \right)
\]

\[
\alpha = 6°42'35.41"
\]

\[
\beta = 1°34'27"
\]

\[
L = \alpha \cos \alpha + \left( c_0 - h - x \sin \alpha \right) \cot \alpha + \beta
\]

\[
L = 0.864 \cos 6°42'35.41" + \left( 1.676 - 0.136 - 0.864 \sin 6°42'35.41" \right) \cot \left( \frac{6°42'35.41" - 1°34'27"}{2} \right)
\]

\[
L = 1.389 m
\]

\[
R = \frac{d_0 (c_0 - h - x \sin \alpha)}{\cos \beta - \cos \alpha}
\]
\[ = 1.676 - 0.136 - 0.864 \times \sin 64.35^\circ \]

\[ \cos 1^\circ 34' 27'' - \cos 6^\circ 42' 35.41'' \]

\[ \theta = 29.35 \text{ m} \]
CROSSOVER

Type-1: Crossover by two parallel track with a straight intermediate portion.

\[ \text{CD} = \text{G} = \text{gauge} \]

In this case, total length of crossover:

\[ = \text{DG} + \text{LH} + \text{HJ} \]

\[ = 2\text{GN} + S + 2\text{GN} \]

\[ \text{S} = \text{strength portion of crossover b/w two turnout} \]
\[ \cos \alpha = \frac{E_D}{E_D} = \frac{e_7}{e_7} \]

\[ E_D = \frac{e_7}{\cos \alpha} = e_7 \sec \alpha \]

\[ EL = LD - E_D = (\theta - \omega) - e_7 \sec \alpha \]

\[ \tan \delta = \frac{EL}{LH} \]

\[ LH = EL \cot \delta \]

\[ S = EL \cot \delta \cdot \frac{1}{2} \]

\[ S = (\theta - \omega) \cdot \frac{e_7}{\sqrt{1 + \frac{1}{N^2}}} \cdot N \]

\[ S = (\theta - \omega) \cdot \sqrt{1 + N^2} \cdot N \]

Overall length of crossover:

\[ L = 4GN + S \]

\[ L = 4GN + (\theta - \omega)N - e_7 \sqrt{1 + N^2} \]
A crossover occurs b/w two parallel box track, of same crossing number 1 in \( \frac{3}{4} \) with straight intermediate portion b/w the curves. Distance b/w centre of tracks is 5m.

Find the overall length of crossover.

Refer to previous trig. 
\[
\alpha = \tan^{-1} \left( \frac{1}{8.5} \right) = 6^\circ42'35.4''
\]

To get length of straight portion \( s \)

\[
\begin{align*}
\overline{CD} & = \theta = 1.676 m \\
\overline{ED} & = 6 \cdot \sec \alpha = 1.676 \sec 6^\circ42'35.4'' \\
& = 1.680 m \\
\overline{EL} & = (\theta - \theta) - \overline{ED} \\
& = (15 - 1.676) - 1.680 \\
& = 1.636 m \\
s & = EL \cot \alpha = 1.636 \\
& \tan 6^\circ42'35.4'' \\
& = 13.91 m
\end{align*}
\]

Overall length \( L = 40N + s \)
\[
= 4 \times 1.676 \times 8.5 + 13.91 \\
= 70.89 m
\]
Type 2 - Crossover with curved portion

between two turnout:

\[ C_2 \]

\[ R_2 \]

\[ R_1 \]

\[ C \]

\[ R_1 \]

if 1st turnout is 1 in N_1

2nd turnout is 1 in N_2

1. Radius: \( R_1 \)

\[ R_{10} = 1.50 + 20N_2^2 \]

\[ R_1 = R_{10} - \frac{\alpha}{d} \]

Radius: \( R_2 \)

\[ R_{20} = 1.50 + 20N_2^2 \]

\[ R_2 = R_{20} - \frac{\alpha}{d} \]
\[ A D = D_1 C = D_1 B - CB = R_1 - d \]

\[ d = \text{center to center distance b/w tracks} \]

In triangle \( D_1 O_2 A \):

\[ \begin{align*}
D_1 O_2 &= R_1 + R_2 \\
O_2 A &= R_2 + AD = R_2 + (R_1 - d) = R_1 + R_2 - d
\end{align*} \]

Overall length of crossover:

\[ L = \sqrt{O_1 O_2^2 - O_2 A^2} \]

\[ L = \sqrt{(R_1 + R_2)^2 - (R_1 + R_2 - d)^2} = \sqrt{(d R_2 + d R_2 - d)^2} \]
A crossover is found b/w two parallel tracks using 1 in 12 turnout on one side & 1 in 16 turnout on other. Partition b/w two turnout is also curved. Find out overall length of crossover b/w these two b/w back centre to centre.

\[ \theta = 5.20 \text{m} \]

**Turnout**

\[ \text{Turnout} \, \theta = 1 \text{ in } 12 \]

\[ N_1 = 12 \]

Radius \[ R_{10} = \frac{1.50 + 2.90N_1^2}{2} \]

\[ = 1.5 \times 1.676 + 2 \times 1.676 \times 12^2 \]

\[ = 485.202 \]

\[ R_1 = \frac{R_{10} - \theta}{2} = 485.202 - \frac{1.676}{2} \]

\[ R_1 = 484.364 \text{ m} \]
Turnout \( \beta \) 1 in 16 \( \therefore N_2 = 16 \)

\[
R_{20} = 1.56 + 20N_2^2
\]
\[
= 1.5 \times 1.676 + 2 \times 1.676 \times 16^2
\]
\[
= 860.626 \text{ m}
\]

\[
R_9 = R_{20} - \frac{c_9}{2}
\]
\[
= 860.626 - \frac{1.674}{2}
\]
\[
= 859.788 \text{ m}
\]

\[
O_{102} = R_1 + R_2
\]
\[
= 484.364 + 859.788
\]
\[
= 1344.152 \text{ m}
\]

\[
O_{2A} = R_1 + R_2 - \beta
\]
\[
= 1344.152 - 5.20
\]
\[
= 1338.952 \text{ m}
\]

Overall length of cross over

\[
L = \sqrt{O_{102}^2 - O_{2A}^2}
\]
\[
L = \sqrt{(1344.152)^2 - (1338.952)^2}
\]
\[
L = 118.12 \text{ m}
\]
Diamond Crossing

\[ AB = BC = CD = DA \]

In \( \triangle CDF \):
\[ CD = C \cos \alpha \]

\[ FB = DF \]
\[ \tan \theta = \frac{C \sin \alpha}{DF} \]
\[ DF = C \cot \alpha \]

Diagonal \( AC \):
\[ AC = C \cos \alpha \]
\[ \theta \beta = \gamma H \theta \]
\[ \tan \frac{\alpha}{2} = \frac{HD}{NH} \]
\[ HD = AH \tan \frac{\alpha}{2} \]
\[ HD = \frac{AC \tan \alpha}{2} \]
\[ HD = \cot \sec \frac{\alpha}{2} \times \tan \frac{\alpha}{2} \]
\[ HD = \frac{1}{\cot \sec \frac{\alpha}{2}} \]

\[ BA = \varphi H \theta \]
\[ BA = \cot \sec \frac{\alpha}{2} \]

**Q.** Design a diamond crossing between two bar tracks crossing each at an angle of 1 in 10.
\[ \ln N = \ln 10 \]
\[ N = 10 \]
\[ \cot \alpha = 10 \]
\[ \alpha = \cot^{-1}(10) \]
\[ \alpha = \tan^{-1}\left(\frac{1}{10}\right) \]
\[ \alpha = 5^\circ 42' 38.14'' \]

Length \( AB = BC = CD = DA \)
\[ = 69.60 \text{sec} \alpha \]
\[ = 1.676 \]
\[ \frac{2 \sin 5^\circ 42' 38.14''}{2} \]
\[ = 16.84 \text{ m} \]

Length \( EB = DF \)
\[ = 0.1 \cot \alpha = 1.676 \times 10 = 16.76 \text{ m} \]

Diagonal \( AC = 6 \cos \sec \frac{\alpha}{2} = \frac{1.676}{\sin(5^\circ 42' 38.14'')} \]
\[ = 33.645 \text{ m} \]

Diagonal \( BD = 6 \sin \frac{\alpha}{2} = \frac{1.676}{\cos 5^\circ 42' 38.14'')} \]
\[ = 1.678 \text{ m} \]
TRACTION & TRACTIVE EFFORT:

To understand this topic three important points are:

1. Tractive effort: This is the sole force applied by engine on driving wheels for movement of train.

2. Hauling Capacity: This is the frictional force available by the driving wheels and rails. It depends upon the weight on driving wheels and coefficient of friction.

\[
F_f = UR = H.C.
\]

\[
H.C = F_f = UR = fr.W
\]

\[
HC = \text{in} \ \text{lb} \ \text{trawl}
\]
Total Resistance - Total Resistance offered by train due to various reason during movement.

For movement of train:

\[ T_e \geq H.C > \text{Total Resistance} \]

Tractive Effort \((T_e)\) :

\[ T_e = \frac{P}{A} \]

But if we take an example of steam engine:

If \( A = \text{area of piston} \)
\( \alpha = \text{volume of piston} \)
\( P = \text{pressure diff' on two side} \)
\[ l = \text{length of stroke} \]
\[ T_e = \text{Tractive effort generated on wheel} \]
\[ d = \text{Dia of wheel} \]
\[ n = \text{number of cylinders} \]

Power generated = Work done

\[ \frac{\pi}{4} (d^2) x p x a x n = T_e x \pi x d \]

\[ T_e = \frac{n p d^2}{a D} \]

Tractive effort \( T_e \propto \frac{1}{d} \)

Velocity of train \( V \propto d \)

Diameter of driving wheel should be selected such that sufficient tractive effort can be generated without exceeding the speed much.

\[ ^2 \text{Hauling Capacity} : (H.C.) \]

It is the frictional force available between driving wheels of locomotive and rails.
\( w_a \) = wt on each pair of driving wheels
\( n \) = no. of pairs of driving wheels
\( \mu \) = coeff. of friction
\( H.C. = \mu W = \mu \cdot n \cdot w_a \)

Value of \( \mu \)

At low speed \( \mu = 0.30 \)
At high speed \( \mu = 0.10 \)

During movement

Value of \( \mu \) considered \( \mu = 0.20 \)

or \( \mu = \frac{1}{6} = 0.166 \)

Designation of a locomotive

A locomotive is designated as \( n_1 - n_2 - n_3 \)
\[ n_1 = \text{Total no. of front wheel} \]
\[ n_2 = \text{Total no. of driving wheel} \]
\[ n_3 = \text{Total no. of rear wheels} \]

For hauling capacity

\[ n = \text{nos of pairs} = \frac{n_2}{2} \]

\[ \text{4-8-2 locomotive} \]

\[ \frac{2}{2} \frac{2}{2} \frac{2}{2} \frac{2}{2} \]

\[ 4 \]

\[ 8 \]

\[ 2 \]

Max. axle load \( \rho \):

\[ B67 = 28.56 \, \text{t} \]

\[ M66 = 17.34 \, \text{t} \]

\[ N67 = 13.26 \, \text{t} \]

One axle = one pair of wheels
Total Resistance -

1. Train Resistance ($R_T$)

   a) Resistance independent of speed ($R_{T1}$)
      (Rolling Resistance)

      It is due to various frictional forces acting on wheels, rails, engine parts etc.

      \[ R_{T1} = 0.0016 \omega \]

      Here, $\omega = \text{weight of train in tonnes}$

      \( \text{weight of locomotive + wagons) in tonnes} \)

   b) Resistance dependent on speed ($R_{T2}$)

      \[ R_{T2} = 0.00008 \omega V \]

      Here, $\omega = \text{weight of train in tonnes}$

      $V = \text{velocity (speed) of train in kmph}$

   c) Atmospheric Resistance: - ($R_{T3}$)

      (even when wind speed = 0)
\[ R_{t_3} = 0.0000006 \omega v^2 \]

Total train Resistance -

\[ R_t = R_{t_1} + R_{t_2} + R_{t_3} \]

\[ R_t = 0.0016\omega v + 0.00008\omega v + 0.0000006\omega v^2 \]

always considered.

2. Resistance due to track profile -
   
   a) Due to Gradient -

   \[ R_g = \omega \tan \theta \]

   for small value of \( \theta \)

   \[ \theta = \sin \theta = \tan \theta \]

   b) Resistance due to curve -

   for \( B_0 \)

   \[ R_c = 0.0004 \omega \theta \]

   for \( M_0 \)

   \[ R_c = 0.0003 \omega \theta \]

   for \( N_0 \)

   \[ R_c = 0.0002 \omega \theta \]
Here \( w = wt \) of train in tonnes.

\( \theta^\circ = \text{degree of curve} \)

Resistance due to starting and acceleration – [102]

a) Due to starting

\[ R_{\text{locomotive}} = 0.15w_1 \]
\[ R_{\text{wagon}} = 0.005w_2 \]

Total resistance

\[ R_{\text{t}} = 0.15w_1 + 0.005w_2 \]

\( w_1 = \text{wt of locomotive in tonnes} \)
\( w_2 = \text{wt of wagons} \)

Note: When train is moving, this resistance is not required to be considered.

b) Due to acceleration:

\[ \square \rightarrow v_1 \quad \square \rightarrow v_2 \]
\[ Ra = 0.028 \omega \left( \frac{V_n - V_1}{t} \right) \]

\[ \omega = \text{rot. of train in tonnes} \]

\[ V_n/V_1 = \text{speed in kmph} \]

\[ t = \text{time in seconds} \]

4. Wind Resistance:

\[ Rw = 0.000017 a V_w^2 \]

\[ a = \text{exposed area in m}^2 \text{ of train} \]

\[ V_w = \text{wind velocity in kmph} \]

\[ Rw = \text{wind resistance in tonnes} \]

For movement of train:

Hauling Capacity = Total Resistance
A locomotive on a 60 ton back with four pairs of driving wheels, carrying an axle load of 20 t each, is required to haul a train at a speed of 80 kmph.

The train is made to run on a level track with curvature of 30°. Calculate max⁰ permissible load that can be pulled by the engine.

\[ m = 1/6 \]

Locomotive

\[ n = 4 \text{ pairs} \]
\[ w_d = 20t \]
\[ m = 1/6 \]

Hauling capacity

\[ H.C. = m \cdot n \cdot w_d \]
\[ = \frac{1}{6} \times 4 \times 20 \]
\[ = 13.33 \text{ t} \]

\[ V = 80 \text{ kmph} \]
\[ D^o = 2^o \]

Hauling capacity = Total Resitance
$$13.33 = R_{T1} + R_{T2} + R_{T3} + 0.0004 \omega D$$

$$= 0.0016 \omega + 0.0008 \omega V +$$
$$0.000006 \omega V^2 +$$
$$0.0004 \omega D$$

$$13.33 = \omega (0.0016 + 0.0008 x 80 +$$
$$0.000006 x 80^2 + 0.0004 x$$

$$m_g = m\omega x$$

$$\omega = 10.546 \text{ tonnes}$$

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9. A locomotive with 8 pairs of driving wheels is to haul a train at a 75 kmph speed. The load on each axle of driving wheels = 100 tons.

1. What is the load the engine can pull on a straight level track?

2. When the same track goes up a slope of 1 in 180, how would the speed be adjusted for same load?

3. If the track, when past the slope goes through a 20° curve on level ground, how would the speed be adjusted?
4. If slope and curve are both, then how much speed will be adjusted.

Hauling capacity = \( u \times n \times w_d \)

\[ 0.20 \times 3 \times 92 = 13.2 \text{ t} \]

1. Speed = 75 kmph

\[ H.C. = R_1 + R_2 + R_3 \]

\[ 13.2 = 0.0016 \omega + 0.00008 \omega V + 0.0000006 \omega V^2 \]

\[ = \omega (0.0016 \pm 0.00008 \times 75 + 0.0000006 \times 75^2) \]

\[ \omega = 120.27 \text{ tonnes} \]

2. Gradient = 1 in 180

\[ R_2 = \omega + \tan \theta = \omega \times \frac{1}{180} \]

\[ H.C. = 0.0016 \omega + 0.00008 \omega V + 0.0000006 \omega V^2 + \omega \tan \theta \]

\[ 18.2 = 0.0016 \times 120.27 \omega + 0.00008 \times 120.27 \omega V + 0.0000006 \times 120.27 \omega V^2 + 120.27 \frac{\omega}{180} \]
13.\( \dot{q} \) = 0.6059 + 0.096V + 0.00072V^2
0.00072V^2 + 0.096V - 4.594 = 0

\[ V = 37.37 \text{ kmph} \]

Reduction of speed = 75 - 87.3 = 37.7 kmph

\[ H.C. = R_1 + R_2 + R_3 + 0.0004 \omega A \]

13.\( \dot{q} \) = 0.0016W + 0.00008WV +
0.00000000WV^2 + 0.0004Wd

= 0.0016x1202.7 + 0.0008x1202.7x1
+ 0.000000x1202.7V^2 +
0.0004x1202.7x2

13.\( \dot{q} \) = 9.886 + 0.096V + 0.00072V^2
0.00072V^2 + 0.096V - 10.314 = 0

\[ V = 70.33 \text{ kmph} \]

\[ H.C. = 0.0016W + 0.00008WV +
0.000000WV^2 + 0.0004Wd +
\omega \tan \theta \]

13.\( \dot{q} \) = 0.0016x1202.7 + 0.00008x1202.7V
+ 0.000000x1202.7V^2 + 0.0004x1202.7
+ 1202.7x1
\[ = 180 \]
A train having 20 wagons weighting 18 tonnes each, is to run at a speed of 50 kmph. The tractive effort of a 2-8-2 locomotive with 22.5 t load on each axle, is 15 tonnes. The weight of locomotive is 120 t. Rolling resistance of wagons and locomotive are 2.5 kg/t and 3.5 kg/t respectively. The resistance which depend upon speed is 2.65 tonnes. Find out the steepest gradient for these conditions.

Weight of locomotive = 120 t
Weight of wagon = 360 t
Total weight of train = 480 t = W
V = speed of train = 50 kmph
Tractive effort = 15 t
Hauling capacity = \( n \cdot m \cdot W_0 \)
\[ = 4 \times 1 \times 22.5 = 15 \text{ t} \]
\[ \text{max} \text{ gradient} = ? = \tan \theta = ? \]
H.C. = RT_1 + RT_2 + RT_3 \pm w + \tan \theta \quad \text{(D)}

Here, Rolling resistance

\[ RT_1 = 2.5 \text{ kg} \times 360 + 3.5 \text{ kg} \times 120 \]
\[ = 1320 \text{ kg} \]
\[ RT_1 = 1.32 \pm \text{(c)} \]

\[ RT_2 = \text{Resistance dependent on speed} \]
\[ RT_2 = 2.65 \text{ tonnes} \]

\[ RT_3 = 0.0000006 \text{ W} \cdot \text{V}^2 \]
\[ = 0.0000006 \times 480 \times 50^2 \]
\[ = 0.72 \text{ tonnes} \]

Putting eqn(D)

\[ 15 = 1.32 + 2.65 + 0.72 + w + \tan \theta \]
\[ \Rightarrow \tan \theta = 15 - 1.32 - 2.65 - 0.72 \]
\[ = 11.5 \]
\[ \Rightarrow \theta = 51^\circ \text{ or } 129^\circ \]

\[ \theta = 51^\circ \]